DC Motor with Load Coupled by Gears Speed Control using Modified Ziegler-Nichols Based PID Tunings

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Abstract

The used of DC motor in various applications has been increased due to the ease with which it speed can be controlled to give the desired performance characteristics under various condition. PID controllers are widely used in DC motor speed control due to its simple structure and robustness to the modeling error, however their effectiveness is often limited due to the poor selection (or tuning) of its parameters. To facilitate the determination of the appropriate values of the parameters of the PID controller for the control of DC motor at any set point therefore required using appropriate tuning method. This paper provides a better understanding of how PID controller is tuned using Ziegler-Nichols Step Response, Cohen-Coon Method and Chien–Hrones–Reswick (CHR) method. Experimental results of PID control of DC motor with load coupled by gear shows that CHR-PID tunings gives a much improved performance over Ziegler-Nichols Step Response and Cohen-Coon PID-tuning with settling time of 355sec., 112sec., and 111 sec . respectively at each set point

Keywords: DC motor, Ziegler-Nichols Step Response, Cohen-Coon method and CHR method

1 Introduction

The used of DC motor in various applications such as steel rolling mills, electric trains, electric vehicles, electric cranes and robotic manipulators has been increased in the recent years due to the ease with which it speed can be controlled to give the desired performance characteristics under various condition (Waleed & Khearia 2012). The purpose of DC motor speed controller is to generate a signal representing the demanded speed, and to drive the motor at that speed (Pal &.Mudi 2012). Although, proportional-integral-derivative (PID) controllers are widely used in DC motor speed control due to its simple structure and robustness to the modeling error, their effectiveness is often limited due to the poor selection (or tuning) of its parameters required to satisfy the design requirements as best as possible (Åström & Hägglund 1995). To facilitate the determination of the appropriate values of the parameters of the PID controller for the DC motor under control at any set point therefore required using appropriate tuning method. Controller tuning methods provide the controller parameters in the form of formulae or algorithms. They ensure that the obtained control system would be stable and would meet given objectives.

There are several established techniques for tuning the PID controller parameters (Aborisade & Adewuyi 2014). Ziegler and Nichols proposed the well-known Process Reaction Curve technique and the Closed-Loop Cycling method (Ziegler & Nichols 1942). These tuning methods are very simple, but cannot guarantee to be always effective. Many variants of the traditional Ziegler-Nichols PID tuning methods have been proposed for gaining better and more acceptable control system response based on the desirable control objectives such as percent of overshoot, integral of absolute value of the error (IAE), settling time, manipulated variable behavior and etc. Some of these tuning methods have considered only one of these objectives as a criterion for their tuning algorithm and some of them have developed their algorithm by considering more than one of the mentioned criterion.

Even though PID tuning method using Cohen-Coon (Seborg *et al*. 1989) and Chien–Hrones–Reswick (Neil 2002) techniques have been extensively studied by many researchers, the details on how the techniques are implemented in DC motor speed control are still vague. This paper is intended to provide a better understanding of how PID controller is tuned using Cohen-Coon Method and CHR approach. The paper also compares the transient performance of the DC motor using Cohen-Coon Method and CHR techniques with Ziegler-Nichols method.

This paper is organized as follows. Section 2 gives a brief introduction of the DC motor and its model concepts if external load is into it through a gear train. Section 3 describes the implementation of Cohen-Coon, CHR and Ziegler-Nichols techniques to tune PID controller. The simulation results using MATLAB are presented in detail in Section 4 and Section 5 concludes the paper.

2. Model of a DC Motor with Load Coupled by Gears

DC motor drives are used for many speed and position control systems where their excellent performance, ease of control and high efficiency are desirable characteristics. Essentially, a DC motor consists of a stator, a

rotor and a commutator. The stator is the housing of the motor and contains magnet, bearing and etc. The rotor is the rotating part of the motor, which contains a coil of wire through which current flows. The coil of wire in the rotor connects to the commutator and receives current through brushes. The commutator ensures that the current flows in the proper direction while the rotor turns.

Traditionally, DC motors was considered to be a variable speed motor and are currently used in many industries because of its torque speed characteristics that can be varied over a wide range while retaining high efficiency. One of the major challenges of DC motors is the control of its speed. DC motor control is generally realized by adjusting the terminal voltage applied to the armature but other methods such as adjusting the field resistance, inserting a resistor in series with the armature circuit are also available (Chapman 2005).

A common actuator that provides rotary motion in control systems is the DC motor (Ogata 1990, Kuo 1991). The electric circuit of the DC motor with external load coupled to the motor spindle through a gear train is shown in Figure 1. The magnetic flux φ between the stator and the rotor is given by the linear relation

$$\varphi = k_f I_f = \phi \tag{1}$$

where k_f is a constant and I_f is the stator current. The torque T_m developed by the motor is given by the relation:

$$T_m = k_m i_a \phi \tag{2}$$

where k_m is the armature coil constant and i_a is a constant armature current.

Substituting (1) into (2), the torque now has the form

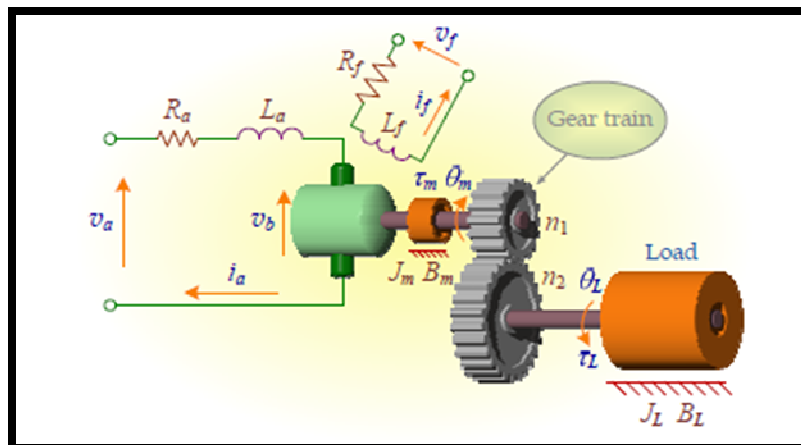$$T_m = k_a i_a \tag{3}$$

where $k_a = k_f k_m I_f$.

Figure 1: Equivalent model of a DC motor with external load coupled by gears (Claudio & John 2011)

The Kirchhoff's law of voltages for the rotor network is

$$v_a = L_a \frac{di_a}{dt} + R_a i_a + v_b \tag{4}$$

The voltage v_b is proportional to the motor speed, i.e.,

$$v_b = k_b \frac{d\theta_m}{dt} = k_b \omega_m \tag{5}$$

where k_b is the back emf constant, θ_m is the angular position or displacement, and ω_m is the angular velocity of the motor. Applying the Laplace transform to (4) with zero initial conditions, the armature current is

$$I_a = \frac{V_a - V_b}{L_a s + R_a} = \frac{V_a - k_b s \Theta_m}{L_a s + R_a} \tag{6}$$

Since, the shaft torque T_m is used for driving load against the inertial and frictional torque, therefore,

$$T_m = J_m \frac{d^2 \theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} = k_a i_a \tag{7}$$

where J_m is the torque inertial and B_m is the coefficient of friction. Apply the Laplace transform to (7) to yield

$$I_a = \frac{J_m s^2 \Theta_m + B_m s \Theta_m}{k_a} \tag{8}$$

where all initial conditions are assumed zero. After some algebraic manipulations the following transfer function is derived

$$\frac{\Theta_m}{V_a} = \frac{k_a}{[(L_a s + R_a)(J_m s + B_m) + k_a k_b]s} \tag{9}$$

Considering the effect of the load coupled by gears to the motor spindle, the block diagram shown in Figure 2 is drawn:
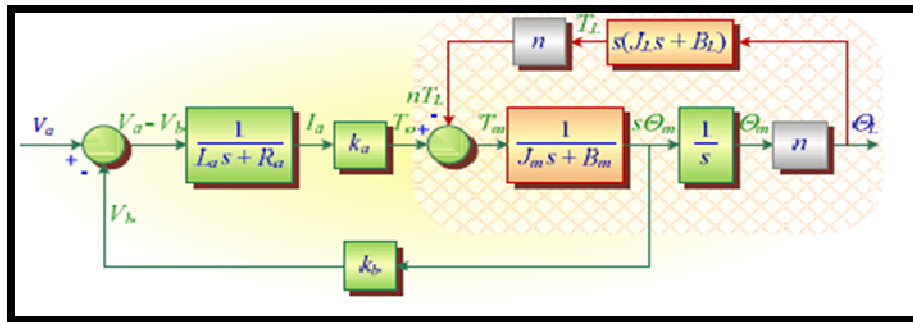

Figure 2: Equivalent block diagram of a DC motor with external load coupled by gear (Claudio & John 2011)

The transfer function that relates the unloaded motor torque T_O with the load position Θ_L is:

$$\frac{\Theta_L}{T_O} = \frac{\dfrac{n}{(J_m s + B_m)s}}{1 + \dfrac{n}{(J_m s + B_m)s} \cdot ns(J_L s + B_L)} \tag{10}$$

$$= \frac{n}{[(J_m + n^2 J_L)s + (B_m + n^2 B_L)]s}$$

where

$$n = \frac{n_1}{n_2} \tag{11}$$

n_1 : Number of teeth of the input (motor)

n_2 : Number of teeth of the output gear (load)

J_L : Inertial momentum of the load

B_L : Viscous friction of the load

$n^2 J_L$: Inertial momentum of the load referred to the motor spindle

$n^2 B_L$: Viscous friction of the load referred to the motor spindle

$$J_O = J_m + n^2 J_L$$
$$B_O = B_m + n^2 B_L \tag{12}$$

where J_O is the total inertial momentum of the motor spindle while B_O is the total viscous friction of the motor spindle. By putting (12) in (10), the new transfer function relating the unloaded motor torque T_O with the load position Θ_L is computed as

$$\frac{\Theta_L}{T_O} = \frac{n}{(J_O s + B_O)s} \tag{13}$$

The complete equivalent block diagram of the system is shown in Figure 3
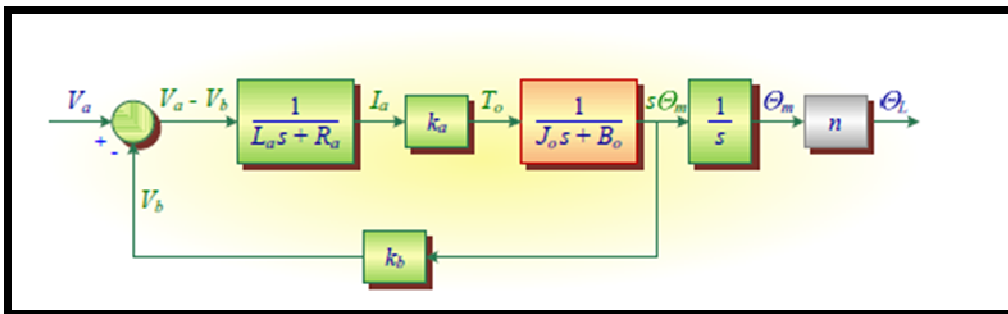
Figure 3: Equivalent block diagram of a DC motor with external load (Claudio & John 2011)

The associated transfer function of the equivalent block diagram of the system shown in Figure 3 is computed as

$$G(s) = \frac{\Theta_L}{V_a} = \frac{k_a n}{[(L_a s + R_a)(J_O s + B_O) + k_a k_b]s} \tag{14}$$

With the parameters of the DC motor shown in Table 1, the transfer function describing the dynamic behavior of the DC motor become

$$G(s) = \frac{\Theta_L}{V_a} = \frac{0.256}{0.02697 s^3 + 2.49662 s^2 + 1.63382 s} \tag{15}$$

Table 1: Parameters of DC Motor with external load coupled by gears

Parameter	Value
Rated motor speed, N	1750 rpm
Rated terminal voltage	240 V
R_a	11.2 Ω
L_a	0.121 H
J_m	0.222 kg - m^2
B_m	0.0030 N - m - sec/rad
J_L	0.022 kg - m^2
B_L	0.0005 N - m - sec/rad
K_a	1.28 N - m/amp
K_b	1.25 V - sec/rad
n_1/n_2	1/5

3. PID Tunings Technique

The Proportional-plus-Integral-plus-Derivative (PID) controllers have found wide acceptance and applications in the industries for the past few decades. It has a simple control structure which was understood by plant operators and which they found relatively easy to tune. In spite of the simple structures, PID controllers are proven to be sufficient for many practical control problems and hence are particularly appealing to practicing engineers. A typical structure of a PID control system is shown in Figure 4, where it can be seen that in a PID controller, the error signal *e(t)* is used to generate the proportional, integral, and derivative actions, with the resulting signals weighted and summed to form the control signal *u(t)* applied to the plant model.
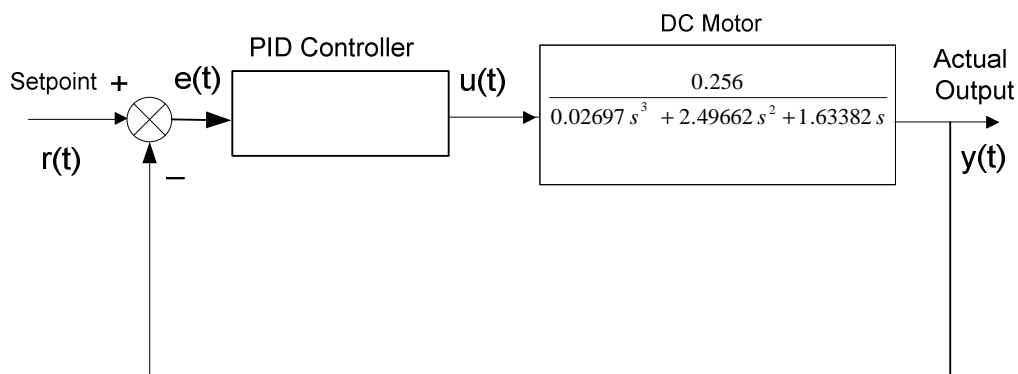
Figure 4: PID control structure

A mathematical description of the PID controller is

$$u(t) = k_p \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \frac{de(t)}{dt} \right] \qquad (16)$$

where $u(t)$ is the input signal applied to the motor model to obtain the output signal $y(t)$, the error signal $e(t)$ is defined as $e(t) = r(t) - y(t)$, and $r(t)$ is the reference input signal.

An abundant amount of research work has been reported in the past on the tuning of PID controllers. In this study the following methods for tuning are used.

3.1 The Ziegler Nichol's Step Response
A simple method of computing the parameters of a PID controller developed by Ziegler and Nichols and published in 1942 is known as Ziegler-Nichols step response method. This method is applied to the open loop step response of the load coupled DC motor displayed in Figure 4. This response is approximated by the transfer function given by a first-order plus dead time (FOPDT) given by (Nagrath & Gopal 2002):

$$G(s) = \frac{b}{1+sT} e^{-sL} \qquad (17)$$

where L is the apparent time delay and T is the apparent time constant, which are determined by drawing a tangent line at the inflection point of the curve and finding the intersections of the tangent line with the time axis and the steady-state level line.

The PID parameters of the Ziegler-Nichols step response method is determine as given in Table 2 where a is computed as (Åström & Wittenmark 1995).

$$a = bL/T \qquad (18)$$

Table 2: Ziegler-Nichols step response method

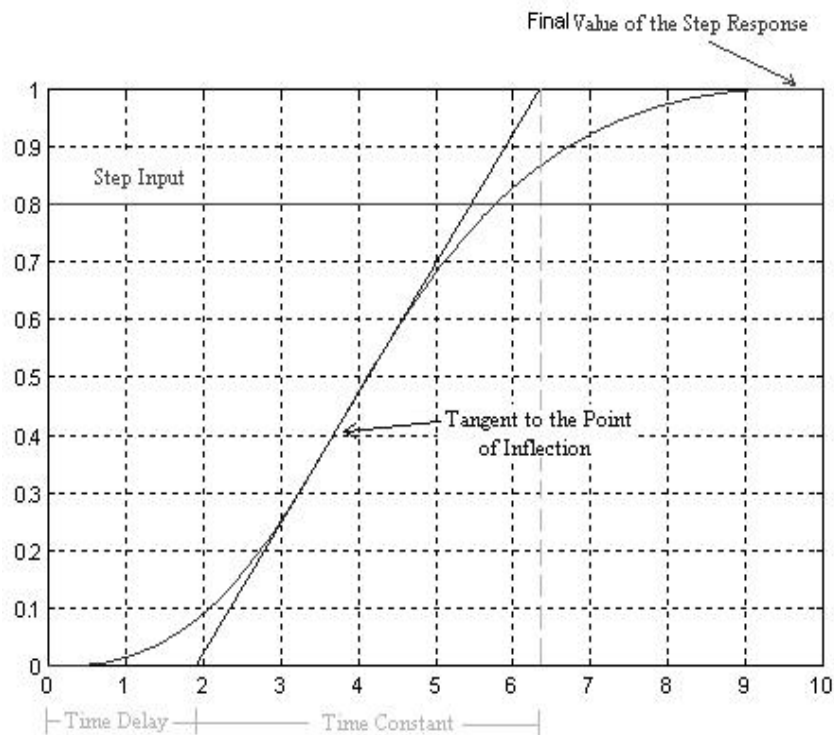| Controller Parameter	k_p	T_i	T_d
PID Controller	$1.2/a$	$2L$	$L/2$

Figure 4: Open loop step response of the load coupled DC motor

3.2 Cohen-Coon Method

Cohen-Coon method is another version of the Ziegler-Nichols PID tuning method. This method is also based on a delayed first order rise, and the method of tuning the PID gains to achieve good response more sensitive than the Ziegler-Nichols method. Referring to the FOPDT model (17), denote $a = kL/T$ and $\tau = L/(L+T)$, the controller parameters with this method is designed by the direct use of Table 3.

Table 3: Cohen-Coon method

Controller Parameter	k_p	T_i	T_d
PID Controller	$\dfrac{1.35}{a}\left(1+\dfrac{0.18\tau}{1-\tau}\right)$	$\dfrac{2.5-2\tau}{1-0.39\tau}L$	$\dfrac{0.37-0.37\tau}{1-0.81\tau}L$

3.3   Chien–Hrones–Reswick (CHR) Method

Chien-Hrones-Reswick (CHR) method is also another modified version of the Ziegler-Nichols method (Neil 2002). This method, developed in 1952 by Chien-Hrones-Reswick provides a better way to select a compensator for process control applications. Compared with the traditional Ziegler-Nichols tuning formula, the CHR method uses the time constant T of the plant explicitly. The CHR-PID controller tuning formulas are summarized in Table 4 for set-point regulation.

Table 4: CHR tuning formulae for set-point regulation

PID Controller	with 0% overshoot			with 20% overshoot		
	k_p	T_i	T_d	k_p	T_i	T_d
	$0.6/a$	T	$0.5L$	$0.95/a$	$1.4T$	$0.47L$

4.   Experimental Results

The simulink model of the PID control of the load coupled DC motor is shown in Figure 5. In the experiment to evaluate the performance of PID control tuning technique, the parameters of Ziegler-Nichols Step Response, Cohen-Coon and CHR methods are calculated. To be able to define PID parameters of the three analyzed tuning methods, open loop step response of the load coupled DC motor was examined. Figure 4 shows the simulation result of the open loop step response of the load coupled DC motor through which L and T

values used in calculating the PID parameters for the three tuning methods are generated.
The three tuning methods are implemented using Matlab-SIMULINK. Experimental results of PID control of DC motor with load coupled by gear using Ziegler-Nichols Step Response, Cohen-Coon and CHR tuning methods at three different set points are given in Figure 6, 7, 8 and 9 respectively. First set point is 1600 rpm, second set point is 1750 rpm and the third set point is 1900 rpm. The second set point change occurs at 2650th second and the third set point change occurs at 5000th second.

Comparisons of the percentage overshoot and settling time for each tuning method at various set points are presented in Table 5. As seen in the figure and table it is clear that CHR-PID-tunings gives a much improved performance with smallest settling time of 355sec., 112sec., and 111 sec respectively at each set point than Ziegler-Nichols Step Response and Cohen-Coon PID-tuning.

5.   Conclusion

In this paper the speed of a DC motor with load coupled by gears is controlled using PID controller. Regarding to the tuning of PID controller modified Ziegler-Nichols methods has been presented. These methods are implemented using MATLAB/SIMULINK simulation. From the obtained results shown in Figure 9 and Table 5 it is concluded that CHR based tuning of PID controller outperformed the Ziegler-Nichols and Cohen-Coon tuning methods at each set-point. The performance of CHR-PID based tuning is at the cost of its less settling-time to reach desired value.
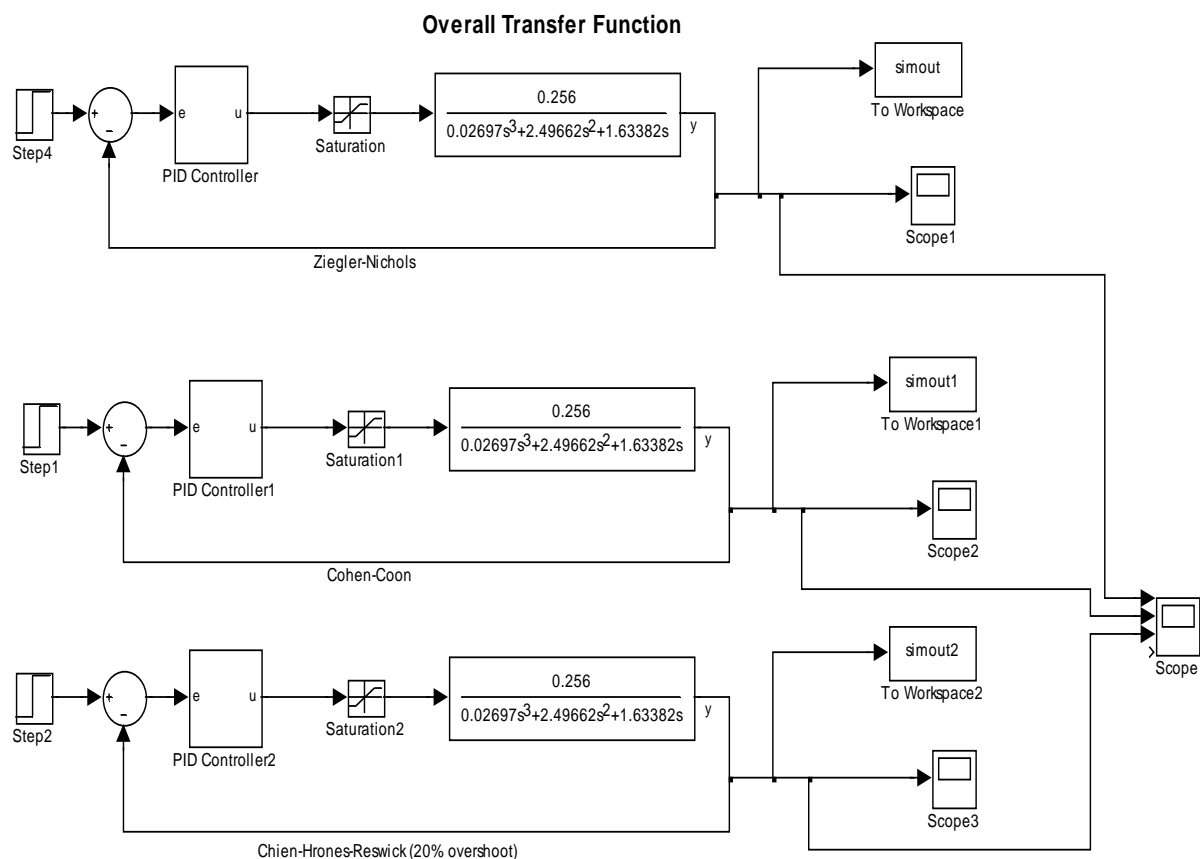

Figure 5: Open loop step response of the load coupled DC motor

Table 5: Comparison of results at various set points

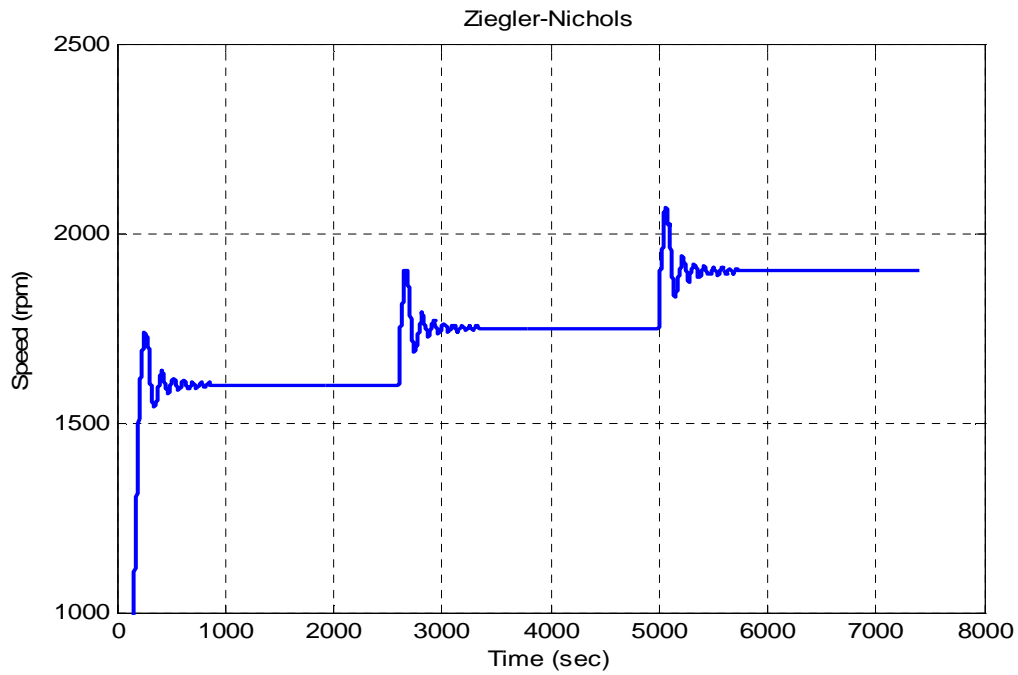| Set Point	Maximum Overshoot (%)			Rise Time (sec.)			Settling Time (sec.)		
	1600 1750 1900 (rpm)			1600 1750 1900 (rpm)			1600 1750 1900 (rpm)		
Ziegler-Nichols	8.80	8.80	8.80	142	173	58	428	229	209
Cohen-Coon	13.81	13.81	13.81	89	224	44	355	195	164
CHR Method	7.47	7.47	7.47	181	11	61	329	112	111

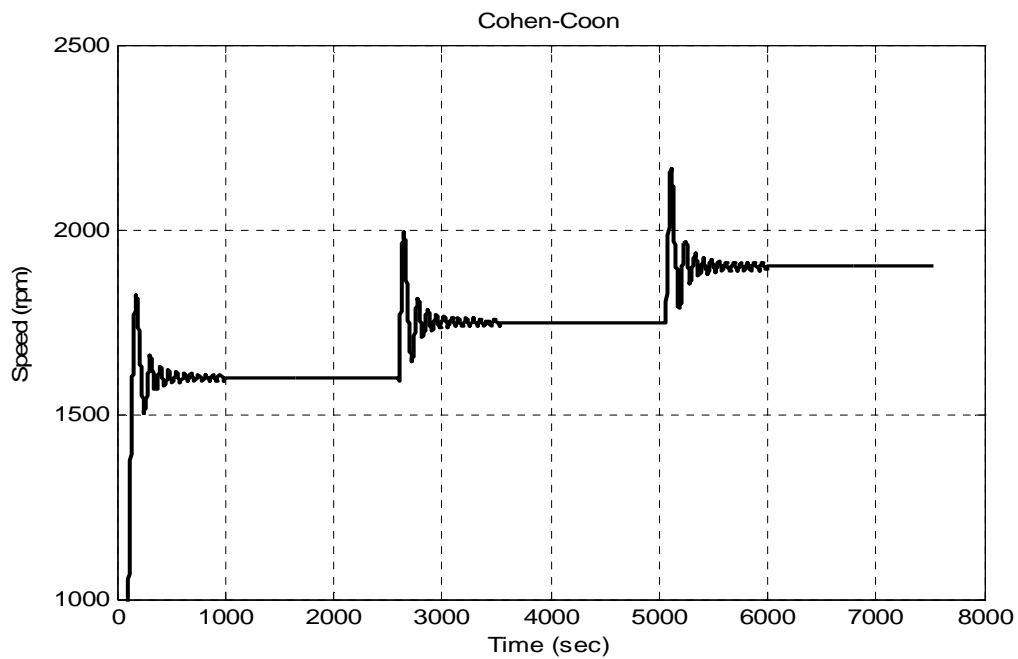**Figure 6: Experimental result of PID control for Ziegler-Nichols Step Response**

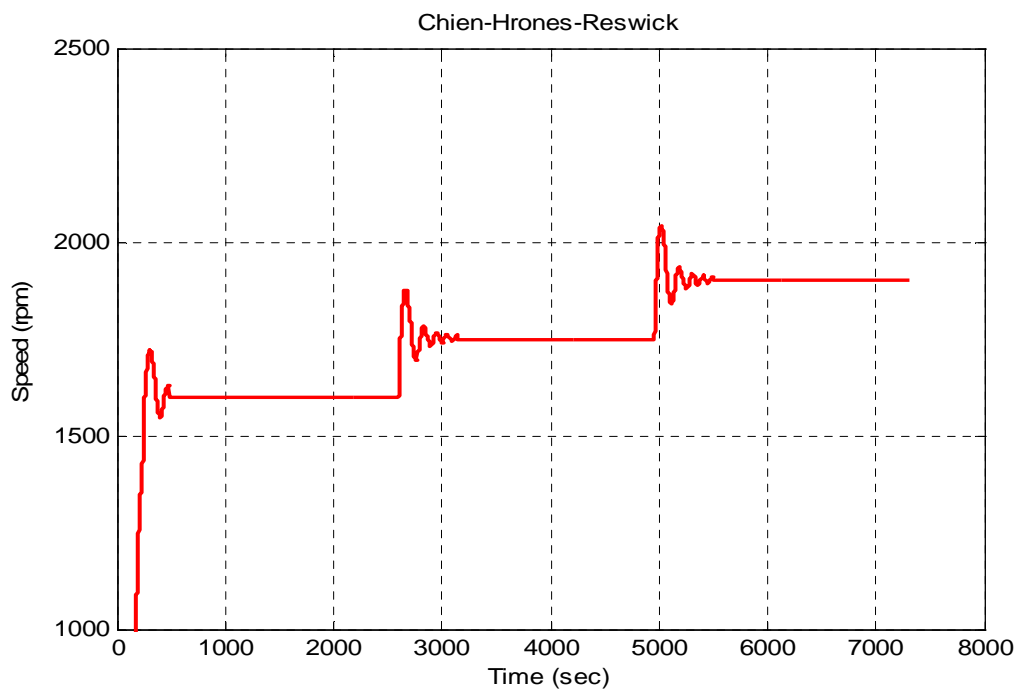**Figure 7: Experimental result of PID control for Cohen-Coon tuning method**

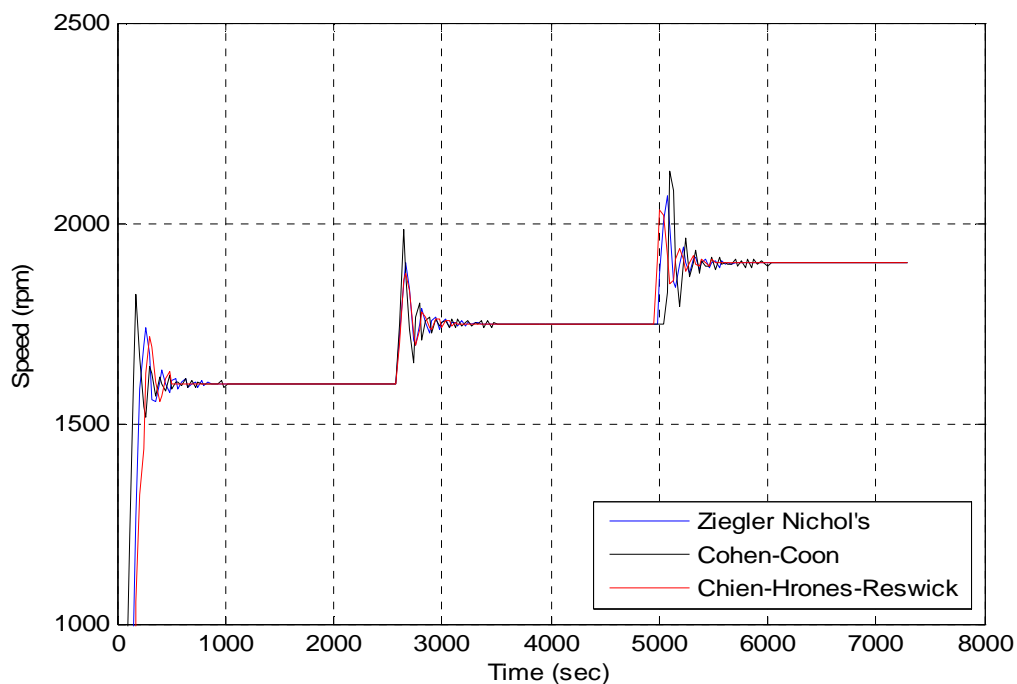**Figure 8: Experimental result of PID control for CHR tuning method**

Figure 9: Comparison of PID control of all Tuning Methods

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