A String of Disjoint Job Blocks on Two Stage Open Shop Scheduling with Transportation Time

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Abstract
This paper provides a heuristic algorithm for n jobs, 2-machine Open-shop scheduling problem in which Processing times are associated with their respective probabilities. The concepts of disjoint job block in a string and transportation time from one machine to another are also taken into consideration. The specific goal of the study is to obtain an optimal or near optimal String of jobs to minimize the makespan. The heuristic algorithm developed in this paper is very simple and easy to understand. A numerical illustration is provided to demonstrate the computational efficiency of proposed algorithm.
Keywords: Open Shop Scheduling, Equivalent job, Disjoint job block, Transportation Time.

1. Introduction

Scheduling is concerned with allocation of shared resources over time to competing activities. A Good Scheduling help considerably in reducing operational costs, improving customer service and utilizing the resources optimally. In Open Shop Scheduling Problem, the order of jobs processed on a machine and the order of machines to process a job can be selected arbitrarily. In recent years, open shop scheduling problems has attracted many researchers**.** Some attempts in these directions are due to Lal and Maggu [13], Gupta and Singh [7], Gupta and Bala [9]etc. Usually the goal of Scheduling is to find a feasible combination of the order of machines and jobs (i.e., a feasible schedule) in order to minimize the makespan in dynamic scheduling. The earliest results to minimize the makespan in n-jobs, 2-machine Flow Shop Scheduling problem is given by Johnson[11]. His work was extended by Bagga[2], Gonzalez and Sahni [5], Dannenbring [4], Maggu and Das [14], Singh [16], Lal and Maggu [13], Rebaine and Strusevich [15], Anup [1], Gupta and Singh [6,7], Gupta and Sharma [8] by considering various parameters**.** The idea of job block equivalent to a single job was originated by Maggu & Das [14] in order to create a balance between the cost of providing priority in Service to the customer and the cost of giving service if no priority is considered. Anup [1] extended the study by associating probabilities with processing time as the processing time are always not exact. Heydari [11] dealt with a flow shop scheduling problem where the n jobs are processed in two disjoint job blocks in a string consists of one job block in which order of jobs is fixed & other job block in which order of jobs is arbitrary.

The present Work is the extension of the study made by Gupta et.al. [10] by including the concept of transportation time as there are many situations where the transportation times are quite significant and cannot be simply neglected. Thus the problem in the paper has wider and practically more applicable and provides suitable results. An algorithm has been developed to minimize the maximum completion time (makespan). The algorithm is demonstrated through a numerical example.

2. Practical Situation

Many applied and experimental situations occur in day to day working in factories and industrial concern . Open shop scheduling problem has wide use in industries. For example consider repairs of a huge airplane that may be include its engines and electrical system. These operations are both necessary but it is impossible to do them simultaneously. The practical situation of open shop scheduling also may be taken in automobile repair Centers, quality control centers, semiconductor manufacturing, class assignments, examination scheduling, satellite communications etc. In many manufacturing companies different jobs are planted at different places then the transportation time has a significant role in production concern. For example, In computer systems, the output of a job on one processor may require a communication time to become the input of a succeeding job on other processor. Sometimes, the priority of one job over the other is preferred. It may be because of urgency or demand of its relative importance. Hence the job block criteria becomes significant.

3. Notations

- A_i :Processing time of i^{th} job on machine A.
- B_i :Processing time of i^{th} job on machine B.
- A_i' :Expected Processing time of i^{th} job on machine A.
- B_i' :Expected Processing time of i^{th} job on machine B.

- p_i :Probability associated with A_i.
- q_i :Probability associated with B_i.
- G_i :Processing time of i^{th} job on fictitious machine G.
- H_i :Processing time of i^{th} job on fictitious machine H.
- $T_{i,A\to B}$:Transportation time from machine A to B.
- $T_{i,B\to A}$:Transportation time from machine B to A

4. Problem Formulation

Let n jobs 1,2,3,…..n be processed through two machines A and B. Let A_i and B_i be the processing time of i^{th} job (i=1,2,3….n) on machine A and B respectively. Let pi and qi be the probabilities associated with processing time A_i and B_i respectively such that $0 \le p_i \le 1$, $\sum p_i=1$, $0 \le q_i \le 1$, $\sum q_i =1$. Let $T_{i,A\to B}$ be the transportation time of i^{th} job from A machine to the B machine which is same as transportation time from B to A i.e. $T_{i,A\to B}$ is same as $T_{i,B\to A}$. Take two job block α and β such that block α consists of s jobs out of n jobs in which the order of jobs is fixed and β consists of r jobs in which the order of jobs is arbitrary such that r + s = n. Let α ∩ β = ∅ i.e. the two job blocks α & β form a disjoint set. A string ***S*** of job blocks α and β is defined as ***S*** = (α, β).

The mathematical model of the problem can be stated in the matrix form as:

Jobs	**Machine A**		**$T_{i,A\to B}$ or $T_{i,B\to A}$**	**Machine B**	
i	A_i	p_i	$T_{i,A\to B}$ or $T_{i,B\to A}$	B_i	q_i
1	A_1	p_1	$T_{1,A\to B}$ or $T_{1,B\to A}$	B_i	q_1
2	A_2	p_2	$T_{2,A\to B}$ or $T_{2,B\to A}$	B_i	q_2
3	A_3	p_3	$T_{3,A\to B}$ or $T_{3,B\to A}$	B_i	q_3
.	.	.	.	.	.
.	.	.	.	.	.
n	A_n	p_n	$T_{n,A\to B}$ or $T_{n,B\to A}$	B_i	q_n

Tableau-1

Our objective is to find an optimal or near optimal String of all the jobs which minimize the makespan.

5. Assumptions

1. Priority is given to the i_1 job over $i_2….i_k$ in job block $(i_1,i_2………i_k)$.
2. Pre-emption is not allowed i.e. once a job started on a machine, the process on that machine can't be stopped unless the job is completed.
3. Transporting device is always available.
4. Transportation time from machine first to second and second to first is same.

6. Algorithm

The heuristic algorithm for the problem discussed here, is as follow as:

Step1: Calculate the expected processing times A'_i and B'_i on machines A & B respectively as follows:

(a) $A'_i = A_i \times p_i$

(b) $B'_i = B_i \times q_i$

Step2: Define two fictitious machines G and H with processing times G_i and H_i as follows:

(a) $G_i = A'_i + T_{i,A\to B}$

(b) $H_i = B'_i + T_{i,B\to A}$

Step3: Calculate the expected processing time for the equivalent job α for the job block (k, m) on fictitious machines G and H using [14]

For the machine order G→H

(a) $G_\alpha = G_k + G_m - \min(G_m, G_k)$

(b) $H_\alpha = H_k + H_m - \min(G_m, H_k)$

For the machine order H→G

(a) $H_\alpha = H_k + H_m - \min(G_k, H_m)$

(b) $G_\alpha = G_k + G_m - \min(G_k, H_m)$

Step4: Obtain the order of jobs in the job block β in an optimal manner using Johnson's [12] technique by treating job block β as sub flow shops Scheduling problem of the main problem. Let γ be the new job block. Define its processing time G'_γ and H'_γ on the lines of Maggu & Das[14] as defined in step2.

Step5: Reduce the given problem into new problem replacing

(i) 's' jobs by job block α with processing times G'_α & H'_α on machine G & H respectively as defined in step 2.

(ii) 'r' jobs of job block β by γ with processing times G'_γ & H'_γ on machine G & H respectively as defined in step above. The new problem can be represented as:-

Jobs	Machine G	Machine H
(i)	G'_i	H'_i
α	G'_α	H'_α
γ	G'_γ	H'_γ

Tableau-2

Step6: For machine order H→G, Construct a set S_G of all the processing time G_i where $G_i \leq H_i$ and S'_G of all the processing time G_i where $G_i \geq H_i$.

Step7: Let S_1 denote a sub optimal sequence of jobs corresponding to non decreasing times S_G and similarly a sequence S_2 corresponding to set S'_G.

Step8: The augmented ordered sequence ***S*** = (S_1, S_2) gives optimal or near optimal sequence for processing the jobs on machine A for the given problem.

Step9: For the order H→G, Construct the set S_H and S'_H of processing times H_i Where $H_i \leq G_i$ and of processing times H_i where $H_i \geq G_i$ respectively .

Step10: Let S_2 denote a sub optimal sequence of jobs corresponding to the non decreasing processing times in the set S_H. Similarly S'_2 corresponding to S'_H.

Step11: The augmented ordered sequence ***S'*** = (S'_1, S'_2) gives the optimal or near optimal sequence for processing the jobs on the machine B for the given problem.

Step12: Prepare in-out tables for sequences ***S*** & ***S'*** and compute total elapsed time.

7. Numerical Illustration: Let 5 jobs are processed in a String S on two machines A & B where ordering of machines for a job operation is immaterial. Let the string S consist of two job blocks α = (2,4) & β = (1,3,5) where α is a fixed order job block and β is arbitrary order job block. The processing time with their respective probabilities and transportation time are given in the following tableau-3

Jobs	A_i	p_i	$T_{i,A\rightarrow B}$ or $T_{i,B\rightarrow A}$	B_i	q_i
1	15	0.3	4	10	0.2
2	14	0.2	3	13	0.1
3	17	0.2	5	14	0.1
4	16	0.2	2	9	0.3
5	13	0.1	6	5	0.3

Tableau-3

Our Objective is to obtain an Optimal String for above said problem in which jobs 2, 4 are to be processed as a group job (2, 4).

Solution:

As Per Step 1: Expected processing time A'_i and B'_i on machine A & B are shown in tableau- 4.

As per Step2: The Expected processing time G_i & H_i on the machines G & H are given in the following tableau-5

As Per Step 3: For Machine Order G → H

The Expected Processing time for equivalent job α = (2,4) on the lines of Maggu and Das [14] are calculated as follows:

$G_\alpha = G_2 + G_4 - \min(G_4, H_2) = 6.7$

$H_\alpha = H_2 + H_4 - \min(G_4, H_2) = 4.7$

As Per Step 4: Here β = (1,3,5)

Now using Johnson technique [12] by treating job block β as sub flow shop scheduling problem of the main problem. Let γ be new job block. Here we get γ = (5,3,1)

Also γ = (5,3,1) = ((5,3),1) = (μ,1) where μ = (5,3)

$G_\mu = G_5 + G_3 - \min(H_5, G_3) = 8.2$

$H_\mu = H_5 + H_3 - \min(H_5, G_3) = 6.4$

$G_\gamma = G_\mu + G_1 - \min(H_\mu, G_1) = 10.3$

$H_\gamma = H_\mu + H_1 - \min(H_\mu, G_1) = 6.0$

As Per Step 5: The new reduced problem for machine order A→B is shown in tableau-6

As Per Step 5, 6 &7:

Optimal String S can be obtained by using Johnson's method then the new reduced problem give us S = {γ, α} = **{5, 3, 1, 2, 4}.**

As Per Step11: In-out table for String ***S*** and total completion time is shown in tableau - 7

Hence the makespan for String ***S*** is 19.9 units of time.

As Per Step 2: For Machine Order H→ G

The Expected Processing time for equivalent job $\alpha = (2, 4)$ are calculated as follows:
$H_\alpha = H_2 + H_4 - \min(G_5, H_2) = 4.3$
$G_\alpha = G_2 + G_4 - \min(G_5, H_2) = 6.3$
As Per Step 3: Here $\beta = (1,3,5)$
Now using Johnson technique [12] by treating job block β as sub flow shop scheduling problem of the main problem. Let γ be new job block. Here we get $\gamma = (1,3,5)$
Also $\gamma = (1,3,5) = ((1,3),5) = (\mu,5)$ where $\mu = (1,3)$
$H_\mu = H_1 + H_3 - \min(G_1, H_3) = 6.0 + 6.4 - \min(8.5, 6.4) = 6.0$
$G_\mu = G_1 + G_3 - \min(G_1, H_3) = 8.5 + 8.4 - \min(8.5, 6.4) = 10.5$
$H_\gamma = H_\mu + H_5 - \min(G_\mu, H_5) = 6.0 + 7.5 - \min(10.5, 7.5) = 6.0$
$G_\gamma = G_\mu + G_5 - \min(G_\mu, H_5) = 10.5 + 7.3 - \min(10.5, 7.5) = 10.3$
As Per Step4:
The new reduced problem for machine order B→A is shown in Tableau-8
As Per Step 8, 9 & 10:
Optimal String S' can be obtained by using Johnson's method then the new reduced problem give us $S = \{\alpha, \gamma\}$ = **{2, 4, 1, 3, 5}.**
As Per Step11: In-out table for string ***S'*** and total completion time is shown in tableau- 9
Hence the makespan for String ***S'*** is 19.5 units of time**.**
Hence the Optimal String which minimize the makespan is ***S'* =** {2, 4, 1, 3, 5} and minimum makespan is 19.5 units of time.

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Tables

Tableau-4: Expected processing time on machines A & B

Jobs	A'_i	B'_i
1	4.5	2.0
2	2.8	1.3
3	3.4	1.4
4	3.2	2.7
5	1.3	1.5

Tableau-5: The Expected processing time on machines G & H

Jobs	G_i	H_i
1	8.5	6.0
2	5.8	4.3
3	8.4	6.4
4	5.2	4.7
5	7.3	7.5

Tableau-6: The new reduced problem for machine order A→B

Jobs	Machine G	Machine H
(i)	G_i	H_i
α	6.7	4.7
γ	10.3	6.0

Tableau-7: In-out table for machine order A →B

For the machine order A →B		
Jobs	Machine A	Machine B
	In - out	In - Out
5	0.0 - 1.3	7.3 - 8.8
3	1.3 - 4.7	9.7 - 11.1
1	4.7 - 9.2	13.2 - 15.2
2	9.2 - 12.0	15.2 - 16.5
4	12.0 - 15.2	17.2 - 19.9

Tableau-8: The new reduced problem for machine order B →A

Jobs	Machine A	Machine B
(i)	A'_i	B'_i
α	4.3	6.3
γ	6.0	10.3

Tableau-9: In-out table for machine order B →A

For the machine order B →A						
Jobs	**Machine B**			**Machine A**		
	In	**-**	**out**	**In**	**-**	**Out**
2	0.0	-	1.3	4.3	-	7.1
4	1.3	-	4.0	7.1	-	10.3
1	4.0	-	6.0	10.3	-	14.8
3	6.0	-	7.4	14.8	-	18.2
5	7.4	-	8.9	18.2	-	19.5