Design of PID Controller for Higher Order Discrete Systems Based on Order Reduction Employing ABC Algorithm

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ABSTRACT

This paper proposes a new computational simple scheme for Model Order Reduction to design a discrete PID controller for higher order linear time invariant discrete systems. Artificial Bee Colony (ABC) optimization algorithm is employed for both order reduction and controller design. First a successful reduced order model is obtained for original higher order discrete system using ABC optimization algorithm which is based on the minimization of integral square error between the original and reduced order models pertaining to step input. Then a PID controller is designed for reduced order model, based on the minimization of integral square error between the desired response and actual response, pertaining to a unit step input using ABC algorithm. Finally the designed PID controller is connected to the original higher order discrete system to get the desired specifications. The validity of the proposed method is illustrated through a numerical example.

Keywords: Discrete system, Model order reduction, PID controller, Integral square error, Artificial Bee Colony algorithm.

I. INTRODUCTION

Scientists and Engineers are confronted with the analysis, design and synthesis of real life problems. The first step in such studies is the development of a 'Mathematical Model' which can be utilized for the real problem. The mathematical procedure of system modeling often leads to comprehensive description of a process in the form of higher order differential equations. These equations in frequency domain leads to a higher order transfer function which is difficult to use either for Analysis or controller synthesis. Therefore, it is necessary to find lower order transfer function which maintains dominant characteristics of the original higher order. Reduction of higher order systems to low order models has also been an important subject area in control engineering for many years. There are two approaches for the reduction of discrete systems, namely the indirect method and direct method. The indirect method [1-3] uses some transformation and then reduction is carried out in the transformed domain. First the Z-domain transfer function is converted into S-domains by linear or bilinear transformation and then after reducing in S-domain suitably, they are converted back into Z-domain. In the direct method the higher order Z-domain transfer function is reduced to lower order transfer function in same domain without any transformation [4-5].

Design of controller based on reduced order model is called process reduction technique. During the past decades, the process control techniques in the industry have made great advances. The process approach is computationally simpler as it deals with reduced order models. The computational and implementation difficulties involved in design of optimal and adaptive controller for higher order linear time invariant system can also be minimized with the help of reduced order models. Several methods have been developed for designing of PID controller [6-9]. Recently Evolutionary algorithms have been suggested to improve the PID tuning, such as those using Genetic Algorithm (GA) [10], Particle Swarm Optimization algorithm (PSO) [11] and Differential Evolutionary algorithm (DE) [12]. With the advance of computational methods in the recent times, optimization algorithms are proposed to tune the control parameters in order to find the optimal performance.

Recently, the Artificial Bee Colony Algorithm (ABC) appeared as promising evolutionary techniques for handling the optimization problems, which is based on the intelligent foraging behavior of honey bee swarm, proposed by Karaboga in 2005[13-14]. This swarm algorithm is very simple and flexible when compared to the other existing swarm based algorithms. It can be used for solving uni-model and multi-model numerical optimization problems. This algorithm uses only common control parameters such as colony size and maximum cycle number. It is a population based search procedure and can be modified using the artificial bees with time and the aim of the bees is to discover the places of food sources with high nectar amount and finally choose source with the highest nectar amount among the other resources.

In this paper, controller design of a higher order discrete system is presented employing process reduction approach. The original higher order discrete system is reduced to a lower order model employing ABC Algorithm based on the minimization of the integral square error (ISE) between the transient responses of original higher order and the reduced order model pertaining to unit step input. Then a proportional integral derivative (PID) controller is designed for reduced order model. The parameters of the PID controller are tuned by using the same error minimization technique employing ABC. The performance of the designed PID controller is verified by connecting with original higher order discrete system to get the desired specifications.

II. DESCRIPTION OF THE PROBLEM

A. Model order reduction:

Consider an n^{th} order linear time invariant discrete system represented by

$$G_n(Z) = \frac{N(Z)}{D(Z)} = \frac{a_0 + a_1 z + \dots\dots\dots\dots + a_{n-1} z^{n-1}}{b_0 + b_1 z + \dots\dots\dots + b_{n-1} z^{n-1} + b_n z^n} = \frac{\sum_{i=0}^{n-1} a_i z^i}{\sum_{j=0}^{n} b_j z^j} \dots (1)$$

The objective is to find an r^{th} order model that has a transfer function$(r < n)$:

$$R(z) = \frac{N_r(z)}{D_r(z)} = \frac{c_0 + c_1 z + \dots\dots\dots + c_{r-1} z^{r-1}}{d_0 + d_1 z + \dots\dots\dots + d_{r-1} z^{r-1} + d_r z^r} = \frac{\sum_{i=0}^{r-1} c_i z^i}{\sum_{j=0}^{r} d_j z^j} \dots (2)$$

where $a_i(0 \le i \le n-1); b_j(0 \le j \le n); c_i(0 \le i \le r-1)$ *and* $d_j(0 \le j \le r)$ are scalar constants. The derivation of successful reduced order model coefficients for the original higher order model is done by minimizing the error index 'E', known as ISE, employing ABC and is given by [15]:

$$E = \int_0^\infty [y(t) - y_r(t)]^2 dt \qquad \dots (3)$$

Where $y(t)$ and $y_r(t)$ are the unit step response of original and reduced order systems.

B. Controller design:
All the methods for design of a controller by process reduction technique involve the following steps:

Step-1: Determine the lower order model to a given original higher order discrete system by minimizing the integral square error (E).

Step-2: Design a PID controller for the reduced order model. The parameters of the PID controller are optimized using the same error minimization technique between the desired and actual response pertaining to a unit step input, Employing ABC Algorithm.

Step-3: Test the designed PID controller for the reduced order model for which the PID controller has been designed.

Step-4: Test the designed PID controller for the original higher order model.

III. PROPORTIONAL INTEGRAL DERIVATIVE (PID) CONTROLLER

A Proportional-Integral-Derivative controller (PID controller) is a generic control loop feedback mechanism (controller) widely used in industrial control systems. The basic structure of conventional feedback control system is shown in fig (1). The PID controller compares the measured process value $'y'$ with a reference set point value $'y_0'$ the difference (or) error (e) is then processed to calculate a new process input$'u'$. This input will try to adjust the measured process value back to the desired point. A PID controller calculates an 'error' value as the difference between measured process variable (y) and a desired set point(y_0). The error 'e' is defined as $e = y_0 - y$.

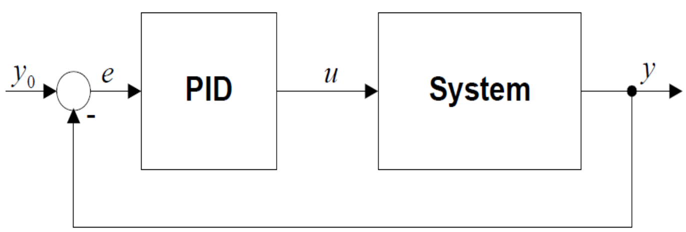


Fig.1.Block diagram of basic feedback controller

The controller attempts to minimize the error by adjusting the process control inputs. In the absence of knowledge of the underlying process, PID controllers are the best controllers. However, for best performance, the PID parameters used in the calculation must be according to the nature of the system while the design is generic. The parameters depend on the specific system. The block diagram of PID controller is shown in fig (2).

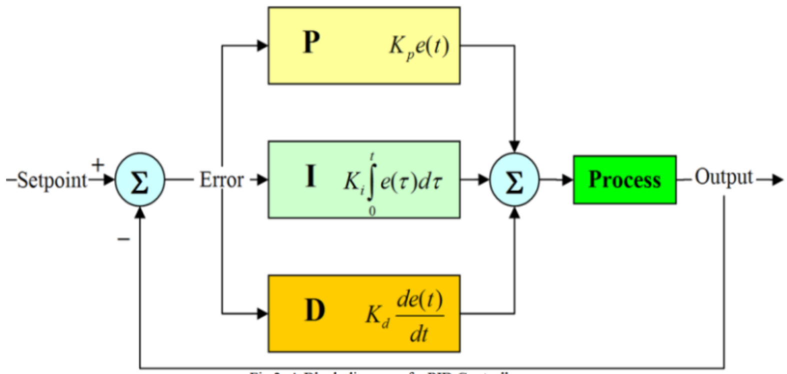


Fig.2. A Block diagram of a PID Controller

The PID controller algorithm involves three constant parameters, and is accordingly sometimes called 'Three-term control': The Proportional, Integral and Derivative values, denoted by P, I, D. Heuristically, these values can be interpreted in terms of time: P depends on present error, I on accumulation of past error, and D is prediction of future errors, based on current rate of change. The weighted sum of these three actions is used to adjust the process via a control element such as the position of a control value. The PID controller output $u(t)$ is defined as

$$u(t) = K_p e(t) + K_i \int_0^t e(t)\, dt + K_d \frac{d}{dt} e(t) \quad \ldots (4)$$

The PID controller transfer function $G_c(s)$ is given as:

$$G_c(s) = \frac{u(s)}{e(s)} = \frac{K_d S^2 + K_p S + K_i}{S}$$

By using linear transformation $S = Z - 1$ in the above equation we get $G_c(z)$ which is given as:

$$G_c(Z) = \frac{K_d Z^2 + \left(K_p - 2K_d\right)Z + (K_d + K_i - K_p)}{Z - 1}$$

Where K_p: proportional gain, a tuning parameter; K_i: integral gain, a tuning parameter; K_d: derivative gain, a tuning parameter; e =error between actual output and reference input. By tuning these constants in the PID controller algorithm, the controller can provide control action designed for specific process requirements. The response of the controller can be described in terms of the responsiveness of the controller to an error, the degree to which the controller overshoot signal over shoots, the set point and the degree of system oscillations.

IV. ARTIFICIAL BEE COLONY (ABC) ALGORITHM:

ABC is a population based optimization algorithm based on intelligent behavior of honey bee swarm [13]. In the ABC algorithm, the foraging bees are classified into three categories; Employed bees, Onlookers and Scout bees. A bee waiting on the hive for making decision to choose a food source is called an Onlooker and a bee going to the food source visited by it previously is named an Employed bee. A bee carrying out random search is called a Scout. The employed bees exploit the food source and they carry the information about the food source back to the hive and share information with onlookers. Onlooker bees are waiting in the hive at dance floor for the information to be shared by the employed bees about their discovered food sources and scouts bees will always be searching for new food sources near the hive. Employed bees share information about food sources by dancing in the designated dance area inside the hive. The nature of dance is proportional to the nectar content of food source just exploited by the dancing bee. Onlooker bees watch the dance and choose a food source according to the probability proportional to the quality of that food source. Therefore, good food sources attract more onlooker bees compared to bad ones. Whenever a food source is exploited fully, all the employed bees associated with it abandon the food source and become scout. Scout bees can be visualized as performing the job of exploration, where as employed and onlooker bees can be visualized as performing the job of exploitation.

In the ABC algorithm, each food source is a possible solution for the problem under consideration and the nectar amount of a food source represents the quality of the solution which further represents the fitness value. The number of food sources is same as the number of employed bees and there is exactly one employed bee for every food source. At the first step, the ABC generates a randomly distributed initial population *P* (*C=0*) of SN solutions (food sources position), where SN denotes the size of population. Each solution (food sources) $X_i\,(i = 1, 2\ \dots SN)$ is a *D*-dimension vector. Here *D* is number of optimization parameters. After initialization, the population of the position (solution) is subjected to repeated cycles, $C = 1, 2\ \dots C_{max}$ of the search process of the employed bees, onlookers and scouts. The production of new food source position is also based on comparison process of food source's position. However, in the model, the artificial bees do not use any information in comparison. They randomly select a food source position and produce a modification on the existing, in their memory as described in Eq.(6) provided that the nectar amount of the new source is higher than that of the previous one of the bee memorizes the new position and forgets the old position. Otherwise she keeps the position of the previous one. An onlooker's bees evaluate the nectar information taken from all employed bees and choose a food source depending on the probability value associated with that food source p_i, calculated by the following equation(5):

$$P_i = \frac{fit_i}{\sum_{n=1}^{SN} fit_n} \quad \dots (5)$$

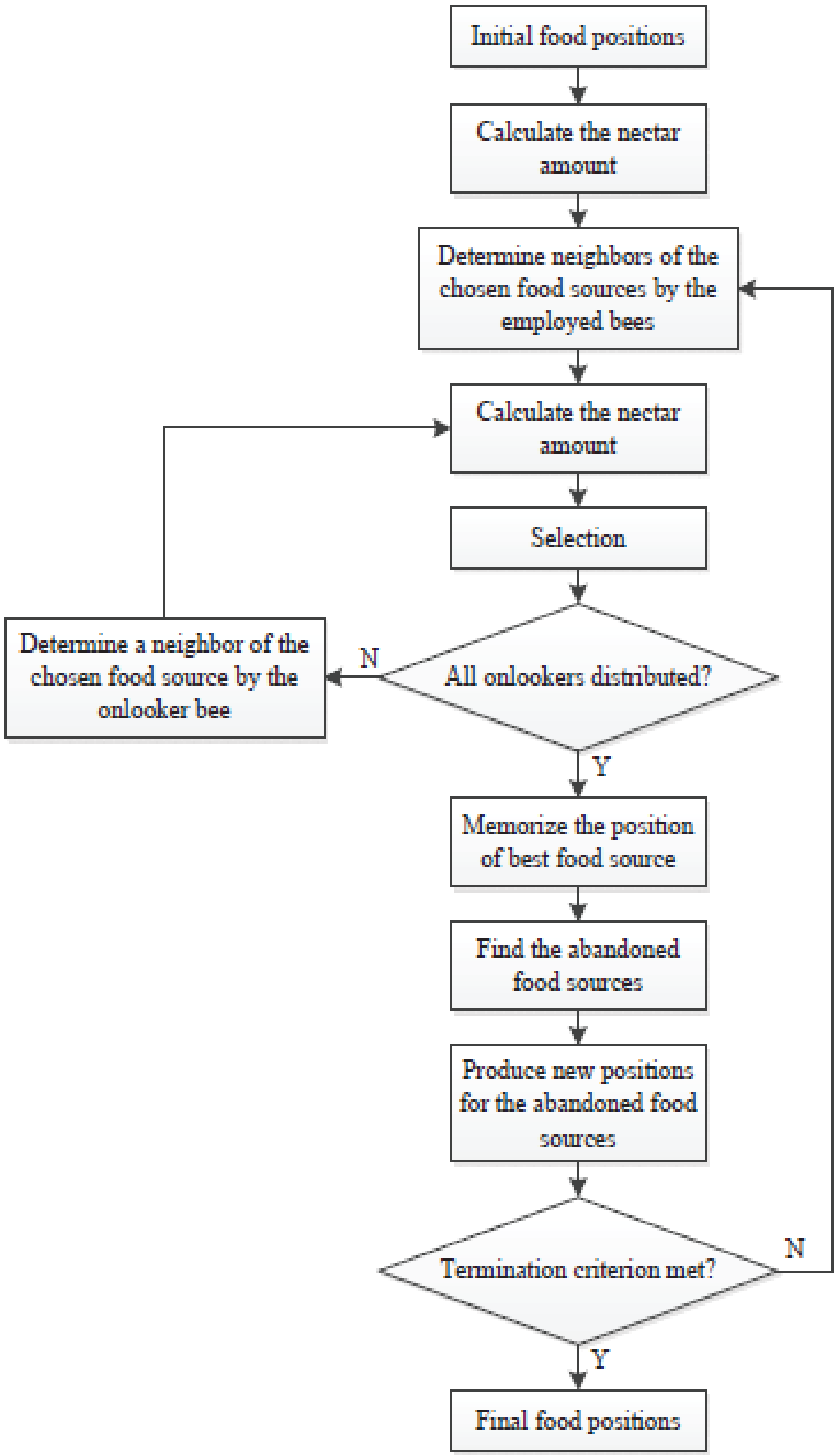
Where fit_i is the fitness value of the solution 'i' evaluated by its employed bee, which is proportional to the nectar amount of food source in the position $'i'$ and SN. In this way, the employed bees exchange their information with the onlookers. In order to produce a new food position from the old one, the ABC uses following expression (6):

$$v_{ij} = x_{ij} + \emptyset_{ij}\left(x_{ij} - x_{kl}\right) \quad \dots (6)$$

Where $k \in (1, 2, \ldots BN)$ and $l \in (1, 2 \ldots D)$ are randomly chosen indexes. Although 'k' and 'l' are determined randomly, it has to be different from $'i' and\ 'j'$. $\emptyset_{ij}$ is a random number between[-1, 1]. It controls the production of neighbor food source position around $x_{i,j}$ and the modification represents the comparison of the neighbor food positions visualized by the bee. Equation (6) shows that as the difference between the parameters of the $x_{i,j} and\ x_{k,l}$ decreases, the perturbation on the position $x_{i,j}$ decreases too. Thus, as the search approaches to the optimum solution in the search space, the step length is adaptively reduced. If its new fitness value is better than the best fitness value achieved so far then the bee moves to this new food source abandoning the old one, otherwise it remains in its old food source. When all employed bees have finished this process, they share the fitness information with the onlookers, each of which selects a food source according to probability given in Eq. (5). With this scheme, good food sources will get more onlookers than the bad ones. Each bee will search for better food source around neighborhood path for a certain number of cycles (limit), and if the fitness value will not improve then that bee becomes scout and discover a new food source to be replaced with $x_{i,j}$. This operation can be defined as

$$x_{i,j} = x_j^{min} + rand(0,1) * \left(x_j^{max} - x_j^{min}\right) \qquad \cdots (7).$$

V. FLOW CHART FOR ABC ALGORITHM


Initial food positions
Calculate the nectar amount
Determine neighbors of the chosen food sources by the employed bees
Calculate the nectar amount
Selection
All onlookers distributed?
N
Determine a neighbor of the chosen food source by the onlooker bee
Y
Memorize the position of best food source
Find the abandoned food sources
Produce new positions for the abandoned food sources
Termination criterion met?
N
Y
Final food positions

VI. NUMERICAL EXAMPLES

Consider the transfer function of plant from references [3] as:

$$G(z) = \frac{N(z)}{D(z)} = \frac{0.1625z^7 + 0.125z^6 - 0.0025z^5 + 0.00525z^4 - 0.02263z^3 - 0.00088z^2 + 0.003z - 0.000413}{z^8 - 0.6307z^7 - 0.4185z^6 + 0.078z^5 - 0.057z^4 + 0.1935z^3 + 0.09825z^2 - 0.0165z + 0.00225} \quad \ldots (8)$$

For which a controller is to design to get desired output.

Application of ABC for model order reduction:

To obtain lower order model for higher order model ABC is employed. The objective function 'E' defined as an integral squared error of difference between the responses given by equation (3) is minimized by ABC. In the present study, a population size of *SN=50*, and maximum number of cycles $(C_{max}) = 200$ have been used.

Finally the successful reduced 2^{nd} order model employing ABC technique is obtained as given in equation (9):

$$R(z) = \frac{0.167816z - 0.087777}{z^2 - 1.755589z + 0.829677} \cdots (9)$$

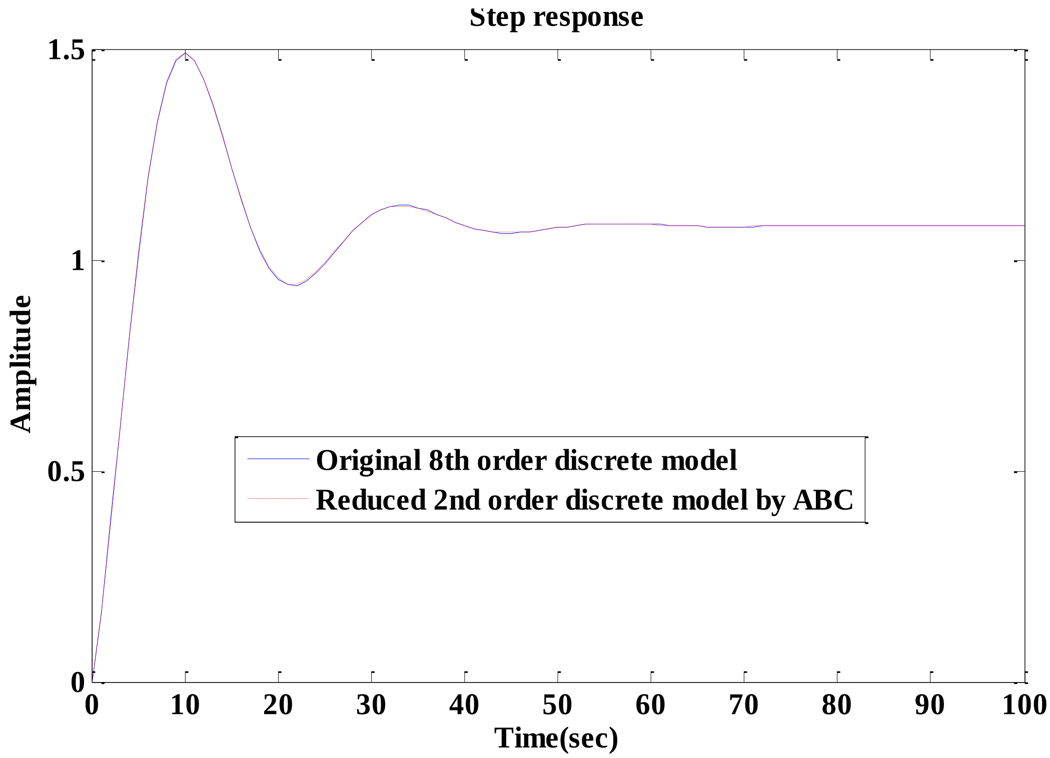
The unit step response of original and reduced systems is shown in fig (3). It can be seen that the steady state responses of proposed reduced order model is exactly matching with that of original model. Also the transient response of proposed reduced model by ABC is very close to that of original model.

Applications of ABC for PID controller design:

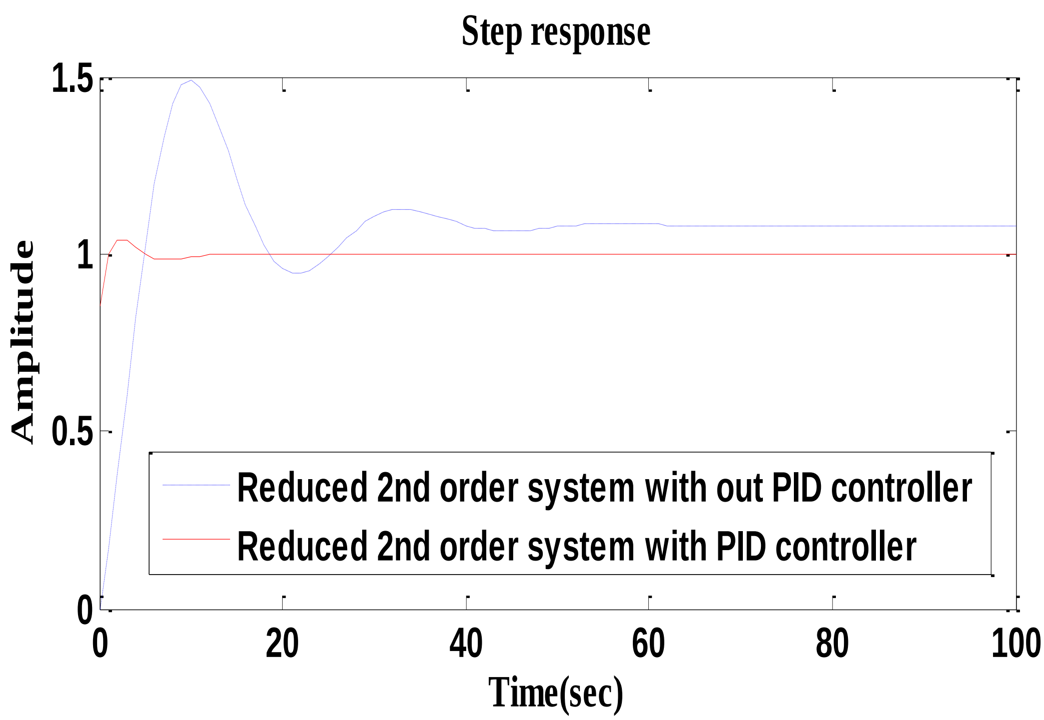
In this study, the PID controller has been designed employing process reduction approach. The original higher order discrete system given by equation (8) is reduced to lower order model employing ABC technique given by equation (9). Then the PID controller is designed for lower order model. The parameters of PID controller are tuned by using same error minimization technique employing ABC as explained in section IV the optimized PID controller parameters are:$K_p = 31.859,\ K_d = 34.651,\ K_i = 8.825;$ The transfer function of the designed PID controller is as follows:

$$G_c(z) = \frac{34.651z^2 - 37.443z + 11.617}{z - 1}$$

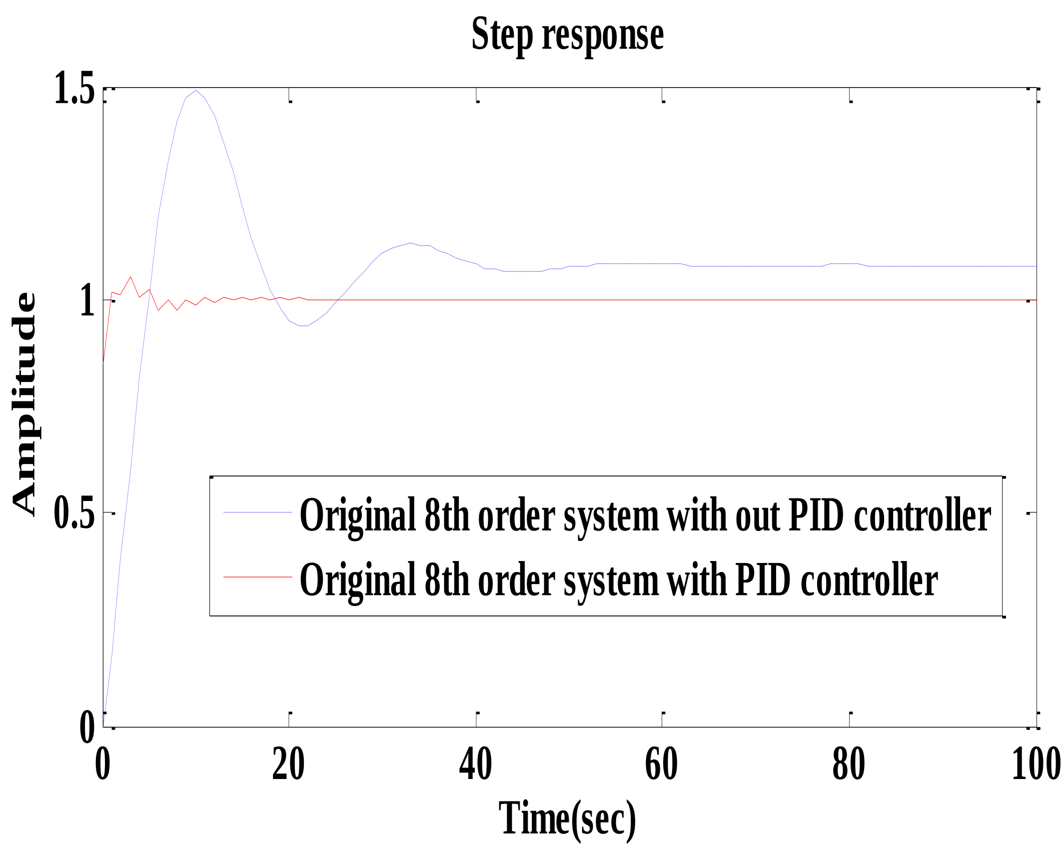
The unit step responses of the original, reduced system with and without PID controller are shown in fig (4) and fig (5). It is obvious from the fig (5). that the design of PID controller using the proposed ABC optimization technique helps to obtain the designer's specification in transient as well as steady state responses for the original system.



Fig(3) Step Response of Original system and Reduced model



Fig(4) step response of reduced with PID controller



Fig(5) step response of original system with PID controller

VII. CONCLUSION

The proposed model reduction method uses the swarm intelligence and population based Artificial Bee Colony algorithm in its procedure to formulate the stable and approximate reduced order model for the original higher order discrete-time systems. The quality of a formulated reduced order model is judged by designing the discrete PID controller. PID controller of the formulated reduced order system efficiently controls the original higher order system. This approach minimizes the complexity involved in direct design of PID controller. The algorithm is simple to implement and computer oriented.

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