Stabilization of Descriptor System via a Non-Classical Variational Approach

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Abstract
The project of this paper is to present asymptotic stabilization controller feedback for descriptor system with non-classical variational approach, it should be mentioned that the sufficient conditions presented in this paper are dependent of the original system matrices and independent of the partion of the original descriptor system.
Keywords: Consistent Initial Condition, C-Controllable, Descriptor System, Non-classical Variational Approach, Regular System.

1. Introduction

Discussion of descriptor systems originated in 1974 with the fundamental paper of Campbell [4] and later on the anthological paper of Luenberger in 1977 [10].Since that time, considerable progress has been made in investigating such system see surveys,[6],[9]for linear descriptor system and for nonlinear system [1], on the investigation of stability of descriptor system many result have been derived [2],[13].
Control of descriptor systems has been extensively studied in the past year due to the fact that descriptor system better describe physical systems than regular ones [7],[8].

A constructive method to give a variational formulation to every linear equation or a system of linear equations by changing the associated bilinear forms was given in [12], this method has a more freedom of choice a bilinear form that makes a suitable problem has a variational formulation. The solution then may be obtained by using the inverse problem of calculus of variation .To study this problem and its freedom of choosing such a bilinear form and make it easy to be solved numerically or approximately, we have mixed this approach with some kinds of basis, for example Ritz basis of completely continuous functions in a suitable spaces, so that the solution is transform from non direct approach to direct one. The since the linear operator is then not necessary to be symmetric, this approach is named as a non-classical variational approach

2. Description of the Problem

Consider the descriptor system

$$EX'(t) = AX(t) + Bu(t) \qquad \dots (1)$$

Here, $X(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, are constant matrices , with $index\ E = k$, $rank(E^k) = p$.
Singular systems can be named by descriptor variable systems, generalized state space systems, semi state systems, differential-algebraic systems.

3. Some Basic Concept

3.1 Definition [4]
The System (1) is called regular if there exist a constant scalar $\lambda \epsilon \mathbb{C}$ such that $det(\lambda E - A) \neq 0$.

3.2 Remarks

1. The regularity is very important property for descriptor linear system. It's guarantees the existence and uniqueness of solutions to descriptor linear system. see [5].
2. If the descriptor linear system is irregular system , i.e. $det\ (\lambda E - A) = 0$, this leads to no solution or non-unique solution (finite or infinite number of solutions)[3].
3. In [4] if the singular system is irregular. It can transfer our system to regular one as following algorithm :

3.3 Computational Algorithm to Make the System Regular

<u>Step (1):</u> For system (1) find the finite spectrum eigenvalue $\sigma_f\ (E, A)$.
<u>Step (2):</u> Chose $\lambda \notin \sigma_f\ (E, A)$.
<u>Step (3) :</u> Define $\hat{E} = (\lambda E - A)^{-1} E$
$\hat{A} = (\lambda E - A)^{-1} E$
$\hat{B} = (\lambda E - A)^{-1} B$.
<u>Step (4):</u> The new system $\hat{E}\, X'(t) = \hat{A}\, X(t) + \hat{B}\, u(t)$ is regular .

4. Based on previously one have $\hat{E}\hat{A} = \hat{A}\hat{E}$ and $\mathcal{N}(\hat{E}) \cap \mathcal{N}(\hat{A}) = 0$ (where $\mathcal{N}(.)$ is null space of the matrix)

even when the original matrices are not and this condition is necessary and sufficient for existence and uniqueness of solution based on [4].

4. Standard Decomposition

4.1 Theorem [5]

For any singular system $EX'(t) = AX(t) + Bu(t)$, there exist two non-singular matrices Q and P such that the system is restricted system equivalent to :

$$w_1' = A_1 w_1 + B_1 u \quad \ldots(2)$$

$$Nw'_2 = w_2 + B_2 u \quad \ldots(3)$$

With the coordinate transformation $\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = P^{-1} X$, $w_1 \epsilon\ \mathbb{R}^{n1\times n1}$, $w_2 \epsilon\ \mathbb{R}^{n2\times n2}$

And $QEP = diag\ (I_{n1}, N)$, $QAP = diag\ (A_1, I_{n2})$, $QB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$

Where $n_1 + n_2 = n$, $N \epsilon\ \mathbb{R}^{n_2 \times n_2}$ is nilpotent

4.2 Remarks

1. This decomposition is called first equivalent decomposition or standard decomposition [6].
2. The matrices Q and P , which transfer a singular system in to its standard form are not unique [5].
3. For a given regular matrix pair (E, A) , we can find Q, P as follows:

   Step (1): Choose $\boldsymbol{\gamma}$, such that $det\ (\boldsymbol{\gamma} E - A) \neq 0$.

   Step (2) : let $\hat{E} = (\boldsymbol{\gamma} E - A)^{-1} E$.

   Step (3): Find the non –singular matrix , such that

   $$T \hat{E} T^{-1} = diag\ (\hat{E}_1, \hat{E}_2)$$

   Where, $T \epsilon\ \mathbb{R}^{n\times n}$, and $\hat{E}_1 \epsilon\ \mathbb{R}^{n_1\times n_1}$ non-singular

   $\hat{E}_2 \epsilon\ \mathbb{R}^{n_2\times n_2}$ is nilpotent .

   Step (4) : Set $Q = diag\ (\hat{E}_1^{-1}, (\boldsymbol{\gamma}\hat{E}_2 - I)^{-1}) T\ (\boldsymbol{\gamma} E - A)^{-1}$, $P = T^{-1}$.

   Step (5) : $Q\,E\,P = diag\ (I_{n_1}, N)$

   $Q\,A\,P = diag\ (A_1, I_{n_2})$.

5. Consistent Initial Condition

5.1 Corollary [4]

Suppose that $E\,A = A\,E$ and $\mathcal{N}(E) \cap \mathcal{N}(A) = \{0\}$

Then there exist a solution to $E\,X' + A\,X = Bu(t)$, $X(0) = x_0$, if and only if x_0 is of the form $x_0 = E^D Eq + (I + E\,E^D) \sum_{n=0}^{k-1} (-1)^n (EA^D)^n A^D Bu^{(n)}(0)$. For Some q, Where E^D is the Drazin inverse of E see[4]. Furthermore , the solution is unique.

5.2 Remark

For a given x_0 and class of controls let the set of admissible controls $\Omega(X_0, J)$, $J = [0, \infty)$, be those u such that (1) with $x(0) = x_0$ is consistent.

If the controls are from the set of k-times continuously differentiable function on $[0, \infty)$ i.e. $C^k[0, \infty)$,

$A\Omega(X_0, J) = C^k(J) \cap \{u: (I - EE^D)x_0 = (I - EE^D) \sum_{n=0}^{k-1} (-1)^n (EA^D) A^D B(u)^{(n)}\}$,

And $(I - EE^D)x_0 = (I - EE^D) \sum_{n=0}^{k-1} (-1)^n (EA^D) A^D B(u)^{(n)}$.

5.3 Computational Algorithm to Find Consist initial Space

Step (1): Consider the descriptor system $EX'(t) = AX(t) + Bu(t)$ where E, A are $n \times n$ matrix and f is k-time continuously differentiable in $\mathbb{R}^n$.

Step (2): Find $ind\ (E) = k$.

Step (3): Find the finite spectrum eigenvalue $\sigma_f\ (E, A)$ and choose $\lambda \notin \sigma_f\ (E, A)$.

Step (4): define $\hat{E} = (\lambda E - A)^{-1} E$

$\hat{A} = (\lambda E - A)^{-1} A$

$\hat{B} = (\lambda E - A)^{-1} B$.

Step (5): Our new system $\hat{E}\,X'(t) = \hat{A}\,X(t) + \hat{B}u(t)$ is regular .

Step (6): Find the Drazin inverse of $\hat{E}$ and $\hat{A}$ by using any of the methods described previously .

Step (7): We find the class of consistent initial conditions by solve

$(I - \hat{E}\hat{E}^D)(x(0) - \hat{A}^D Bu) = 0$ or $(I - \hat{E}\hat{E}^D)\,x(0) = 0$.

This means

$W_k = \mathcal{N}$ (I - $\hat{E}\hat{E}^D$)
This mean (I - $\hat{E}\hat{E}^D$) x_0 = (I - $\hat{E}\hat{E}^D$)$\sum_{n=0}^{k-1}(-1)^n$ $(\hat{E}\hat{A}^D)^n\hat{A}^D$ $\hat{B}u^{(n)}(0)$

6. Controllability of Descriptor System

6.1 Remark [5]

For every decomposite system (2) (3), we define the controllability matrix pair (A_1 , B_1) as

$$Qc\,[\,A_1\,, B_1] \;=\; [B_1\;\; A_1\,B_1\quad \ldots..\;\; A_1^{n-1}\,B_1\,]$$

And for the matrix pair $(N,\ B_2)$

$$Qc\,[\,N, B_2] \;=\; [B_2\;\; NB_2\;\; \ldots..\,N^{k-1}\,B_2]$$

Since

$$[B_2\;\; NB_2\;\; \ldots\ldots\;\; N^{n_2-1}\;\; B_2\,] = \;[B_2\;\; NB_2\;\; \ldots..N^{k-1}\;\; B_2\;\; 0]$$

6.2 Theorem [5]

Consider the regular descriptor system (1) and it's slow and fast subsystems (2) and (3) respectively.

1- The slow subsystems (2) is C- Controllable if and only if

$$rank\;Qc\;[A_1, B_1] \;=\; n_1$$

i. e. $rank\;[\,\lambda\,I - A_1\quad B_1] \;=\; n_1$, $\forall\lambda \in \mathbb{C}$, λ finite.

2- The fast subsystem (3) is C- Controllable if and only if

$$rank\;\; Qc\;[\,N\,, B_2\,] \;=\; n_2,$$

i. e. $rank\;[\,N\quad B_2] \;=\; n_2,$

3- The system (1) is C- Controllable if and only if it's slow and fast subsystems are C- Controllable.

6.3 Remark [r5]

1- The regular descriptor linear system (1) is R- Controllable if and only if Image $Qc\;[\,A_1, B_1] \;=\; \mathbb{R}^{n1}$ in decomposition system (2)

2- The linear descriptor system with only fast subsystem :

$$N\,X' \;=\; X \;+\; Bu$$

Where N is nilpotent , is always R – Controllable

6.4 Theorem [5]

The regular descriptor linear system (1), with show subsystem (2) and fast subsystem (3) is R- Controllable if and only if the slow subsystem (2) is C- Controllable.

7.Feedback Controller

Consider the regular descriptor system $EX'(t) = AX(t) + Bu(t)$ with state feedback control $u(t) = KX(t)$
Let $\Gamma = \{s_1, s_2, \ldots, s_{n1}\}$ with symmetric about real axis the aim to find $u(t)$ such that Γ is the set of finite pole of closed-loop system $EX'(t) = (A + BK)X(t)$ that is $\sigma(E, A + BK) = \Gamma$
Where σ the set of open-loop finite poles of the system.

7.1 Lemma

The regular descriptor linear system (1) is R- controllable if and only if its standard form decomposition is controllable that mean if $rank[B_1\;\; A_1B_{\,1} \ldots A_1^{n-1}B_1] = n_1$
i.e. for an arbitrary set Γ there exist some K_1 such that $\sigma(A_1 + B_1K_1) = \Gamma$ holds when the descriptor system is R-controllable.

7.2 Theorem

Let (2), (3) be standard decomposition for the system (1) under the transformation $(Q\;, P)$ then when the system (1) is R- controllable $\psi = \{K | K = [K_1\quad 0]P^{-1}, \sigma(A_1 + B_1K_1) = \Gamma\}$ is a state feedback controller in $EX'(t) = (A + BK)X(t)$.

8. Solvability of Singular System Using non-Classical Variational Method

8.1 Theorem [12]

Consider the descriptor system $EX'(t) = AX(t) + Bu(t)$ with $x(0) = x_0$, $u(t) \in A\Omega(X_0, J)$ and $ind(E) = k$. Define a linear operator L with domain $D(L)$ and range $\mathcal{R}(L)$ such that

$$Lx = Bu(t) \qquad \ldots.(4)$$

If the conditions with $EA = AE$ and $\mathcal{N}(E) \cap \mathcal{N}(A) = \{0\}$ satisfied and L is symmetric with respect to a certain bilinear then the solution of equation (4) are critical points of functional

$$J[x] = 0.5 < Lx, x > - < Bu\,, x >$$

Moreover, if the chosen bilinear form $< x, y >$ is non-degenerate on $D(L)$ and $\mathcal{R}(L)$ it is also true that the crtical points of the functional $J[x]$ are solution to the given equation .

8.2 Theorem [12]
For the descriptor system $EX'(t) = AX(t) + Bu(t)$ with $x(0) = x_0$, $x_0 \in W_k$ (the class of consistent initial condition) , $u(t) \in A\Omega(X_0, J)$ and $ind(E) = k$.

If the solution $x(t)$ has been approximated by a linear combination of a suitable basis i.e. $x(t) = x(0) + \sum_{i=1}^{n} a_i G_i$ satisfies

1- $x_0 \in W_k$.
2- $G_i(x_0) = 0$.
3- G_i are continuous as required by the variational statement being.
4- $\{G_i\}_i$ Must be linearly independent .
5- Satisfies the homogeneous from of the specified condition.

Then the solution for the system $\frac{dJ}{da_j} = 0$, $\forall j = 1,2, \dots, n$ is the approximate solution for the descriptor system.

9. Stability of control singular system

9.1 Theorem
For the control descriptor system

$$Ex'(t) = Ax\,(t) + Bu\,(t) \qquad \dots(5)$$
$$where\ x(0) = x_0, \quad ind(E) = k\ with$$
$$u(t) \in A\Omega\,(x_0\,, J) = c^k(J) \cap \{u: (I - EE^D)x_0\}$$

If the system (5) is R-controllable and the control u defined by
$u = kx$, where $k = [k_1 \ \ 0]p^{-1}$, and p is non- singular matrix then the solution $x(t)$ is stable.

Proof

Consider the descriptor system

$$Ex'(t) = Ax\,(t) + Bu\,(t)$$
$$where\ x(0) = x_0, \quad ind(E) = k\ with$$
$$u(t) \in A\Omega\,(x_0\,, J) = c^k(J) \cap \{u: (I - EE^D)x_0\}$$

1-The system (5) is regular, according to theorem (4.1) there exists two non-singular Matrices P, Q.

2-Decompose the system (5) into two subsystems by setting

$$w = p^{-1}x = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \quad \begin{matrix} w_1 \in R^{n_1} \\ w_2 \in R^{n_2} \end{matrix}$$

Where $n_1 = deg(det(sE - A))$

$$n_2 = n - n_1$$

The new decomposite system is

$$w_1'(t) = A_1 w_1 + B_1 u \qquad \dots(6)$$
$$Nw_2'(t) = w_2 + B_2 u \qquad \dots(7)$$

Where $QEP = diag\,(I_{n_1}, N)$, $QAP = diag(A_1, I_{n_2})$, $QB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$

3- According to theorem (7.2)

$$set\ \ u = kx \quad , \quad where\ k = [k_1 \ \ 0]p^{-1}$$
$$k_1 \in \mathbb{R}^{r \times n_1}, 0 \in \mathbb{R}^{n_2 \times n}$$

4- By theorem (6.4) check the R-controllability of the global system by checking the controllability of the slow subsystem (6).

5- One can find $k_1 \in \mathbb{R}^{r \times n_1}$ using lemma (7.1).

6- The new homogenous linear system became as

$$Ex'(t) = \tilde{A}x \quad , \quad where\ \tilde{A} = (A + Bk)$$
$$k = [k_1 \ \ 0]p^{-1} \text{ and } x = pw$$

7- Define the bilinear $< x\,, y >$ where the symmetric bilinear given by

$$< x\,, y > = (x\,, Ly)$$

and determined the functional as

$$J[x] = 0.5 < Lx\,, Lx > - < Bu, Lx >$$

from theorem (8.1) one can find the critical point for the functional $J[x]$ is the solution for the linear equation $Lx = Bu(t)$ where $L = \left(E\frac{d.}{dt} - \tilde{A}.\right)$

8- From theorem (8.2) If the solution $x(t)$ has been approximated by a linear combination of a suitable basis satisfies the conditions of theorem (8.2) then the solution for the system $\frac{dJ}{da_j} = 0, \forall j = 1,2,\dots,n$ gave us the parameters a_j imples the solution of descriptor system $x(t) = x(0) + \sum_{j=1}^{n} a_j G_j$, $x(0) \in W_k$ is the stable solution.

9.2 Example

Consider the singular system

$$Ex' = A\,x + Bu$$

Where

$$E = \begin{bmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0\\ 1&0&0&0&0&0 \end{bmatrix},\quad A = \begin{bmatrix} 0&0&1&0&0&0\\ 1&0&0&0&0&0\\ 0&1&0&1&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 1&0&0&0&0&1 \end{bmatrix}, B = \begin{bmatrix} 1&0\\ 0&0\\ 0&0\\ 0&0\\ 1&1\\ 0&0 \end{bmatrix}$$

Solution:

There exists a non- singular matrices Q , P s. t.

$$Q = \begin{bmatrix} 1&0&0&-1&-1&0\\ 2&-1&-1&1&-2&0\\ 0&\sqrt{3}&-\sqrt{3}&\sqrt{3}&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ -1&0&0&0&0&1 \end{bmatrix},\quad P = \begin{bmatrix} \frac{1}{3}&\frac{1}{3}&1&0&0&0\\ \frac{1}{3}&\frac{-1}{6}&\frac{\sqrt{3}}{6}&0&0&0\\ \frac{1}{3}&\frac{-1}{6}&\frac{-\sqrt{3}}{6}&0&1&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&\frac{-1}{2}&\frac{-\sqrt{3}}{6}&0&1&1 \end{bmatrix}$$

And

$$QEP = diag\left(I_{n_1}, \quad N\right) \quad , N = \begin{bmatrix} 0&1&0\\ 0&0&0\\ 0&0&0 \end{bmatrix}, n_1 = 3$$

$$QAP = \left(A_1, I_{n_2}\right),\quad A_1 = \begin{bmatrix} 1&0&0\\ 0&\frac{-1}{2}&\frac{\sqrt{3}}{2}\\ 0&\frac{\sqrt{3}}{2}&\frac{-1}{2} \end{bmatrix}, n_2 = 3$$

$$QB = \begin{bmatrix} B_1\\ B_2 \end{bmatrix} \quad , B_1 = \begin{bmatrix} 0&-1\\ 0&-2\\ 0&0 \end{bmatrix} \quad , B_1 = \begin{bmatrix} 0&0\\ 1&1\\ -1&0 \end{bmatrix}$$

$w_1' = A_1 w_1 + B_1 u$ take $u = k_1 w_1$

And since $k \in \mathbb{R}^{2\times 6}$ then $k_1 \in \mathbb{R}^{2\times 3}$

First the global system is R-controllable since the slow subsystem is controllable .

$$Rank\,[B_1 \;\; A_1B_1 \;\; A_1^2B_1] = 3$$

To find k_1

Using pole placement method we find

$$k_1 = [\propto_3 - a_3 : \propto_2 - a_2 : \propto_1 - a_1]T^1$$
$$= [-50 - 0 : 15 - 0 : -8 + 1]T^1$$
$$= [-50 \;\; -15 \;\; -7]T^1$$

Hence $k_1 = \begin{bmatrix} 0&0&0\\ 8&-1&\frac{2\sqrt{3}}{3} \end{bmatrix}$ and Since $k = [k_1 \quad 0]p^{-1}$

Then $k = \begin{bmatrix} 0&0&0&0&0&0\\ 6&11&7&0&-7&0 \end{bmatrix}$.

Therefor $Ex' = (A + Bk)x = \tilde{A}\,x$

$$\begin{bmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0\\ 1&0&0&0&0&0 \end{bmatrix} x' = \begin{bmatrix} 0&0&1&0&0&0\\ 1&0&0&0&0&0\\ 0&1&0&1&0&0\\ 0&0&0&1&0&0\\ 6&11&7&0&-6&0\\ 1&0&0&0&0&1 \end{bmatrix} x$$

Find the class of consistent initial condition Since $|sE - A| = -6 - 11\,s - 6s^2 - s^3$

$$\left.\begin{array}{r}\Rightarrow s=-1\\ s=-3\\ s=-2\end{array}\right\}\text{ the finit eigenvalue , set }\lambda=1$$

$$\hat{E}=(\lambda E-A)^{-1}E=(E-A)^{-1}E=\left(\begin{bmatrix}1&0&-1&0&0&0\\-1&1&0&0&0&0\\0&-1&1&-1&0&0\\0&0&0&-1&1&0\\-6&-11&-7&0&6&0\\0&0&0&0&0&-1\end{bmatrix}^{-1}\right)E$$

$$\hat{E}=\begin{bmatrix}0.0417&-0.7083&-0.25&0&0.25&0\\0.0417&-0.2917&-0.25&0&0.25&0\\-0.9583&-0.7083&-0.25&0&0.25&0\\-1&-1&-9&0&0&0\\-1&-1&-9&0&1&0\\-1&0&0&0&0&0\end{bmatrix},\quad rank\,(\hat{E})=2\quad \hat{E}^D=$$

$$\begin{bmatrix}6.9952&10.9904&5.9952&0&-5.9952&0\\-1&1&0&0&0&0\\-30.94733&-70.8849&-41.9377&0&41.9377&0\\115.6502&297.1854&185.5352&0&-185.5352&0\\-30.9473&-69.8849&-42.9377&0&42.9377&0\\-37.9425&-81.8753&-47.9329&0&47.9329&0\end{bmatrix}$$

And $$\hat{E}^D\hat{E}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\-5.9952&-10.9904&-5.9952&0&5.9952&0\\24.9521&58.8945&35.9425&0&-33.9425&0\\-5.9952&-10.9904&-6.9952&0&6.9952&0\\-6.9952&-10.9904&-5.9952&0&5.9952&0\end{bmatrix}$$

$$\gamma=I-\hat{E}^D\hat{E}=\begin{bmatrix}-0&0&0&0&0&0\\-0&0&0&0&0&0\\5.9952&10.9904&6.9952&0&-5.9952&0\\-24.9521&-58.8945&-35.94&1&35.9425&0\\5.9952&10.9904&6.9952&0&-5.9952&0\\5.9952&10.9904&5.9952&0&-5.9952&1\end{bmatrix}$$

Since the elementary row operations does not change the null space, convert γ to reduce row echelon form, one gets:

$$\begin{bmatrix}1&0&0&0.1147&0.4377&0.1770\\0&1&0&-0.1356&-1.0629&0.4273\\0&0&1&0.1147&0.4377&-0.8230\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}x_1(0)\\x_2(0)\\x_3(0)\\x_4(0)\\x_5(0)\\x_6(0)\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}$$

Therefor $x_1(0)+0.1147x_4(0)+0.4377\,x_5(0)+0.1770\,x_6(0)=0$

$x_2(0)-0.1356\,x_4(0)-1.0629\,x_5(0)+0.4273\,x_6(0)=0$

$x_3(0)+0.1147x_4(0)+0.477\,x_5(0)-0.8230\,x_6(0)=0$

Set $x_4(0)=1$, $x_5(0)=-2$, $x_6(0)=-1$, then

$$x_1(0)=-0.1147+0.8754+0.1770=0.9377$$
$$x_2(0)=0.1356-2.1258+0.4263=-1.5629$$
$$x_3(0)=-0.147+0.8754-0.8230=-0.0623$$

Now let $L=\left(E\frac{d.}{dt}-\tilde{A}.\right)x$, where

$$J[x]=0.5\int_0^1\begin{bmatrix}x_1'-x_3\\x_2'-x_1\\x_3'-x_2-x_4\\x_5'-x_4\\-6x_1-11x_2-7x_3+6x_5\\x_1'-x_1-x_6\end{bmatrix}\cdot\begin{bmatrix}x_1'-x_3\\x_2'-x_1\\x_3'-x_2-x_4\\x_5'-x_4\\-6x_1-11x_2-7x_3+6x_5\\x_1'-x_1-x_6\end{bmatrix}^T dt$$

Approximate

$$x_1 = 0.9377 + \sum_{1}^{5} a_i t^i$$

$$x_2 = -1.5629 + \sum_{1}^{5} b_i t^i$$

$$x_3 = -0.0623 + \sum_{1}^{5} c_i t^i$$

$$x_4 = 1 + \sum_{1}^{5} d_i t^i$$

$$x_5 = -2 + \sum_{1}^{5} e_i t^i$$

$$x_6 = -1 + \sum_{1}^{5} f_i t^i$$

Step(16) : equalize the result to zero to find these a_i , b_i, c_i, d_i, e_i, f_i.

We get algebraic system $A.a = b, n \times n$ system, where A is invertible matrix then $a = A^{-1}b$.

Step (17) : find an approximate solution x_i.

Step (18) : stop.

Then one can compute the approximation solution which is stable as in the following figure

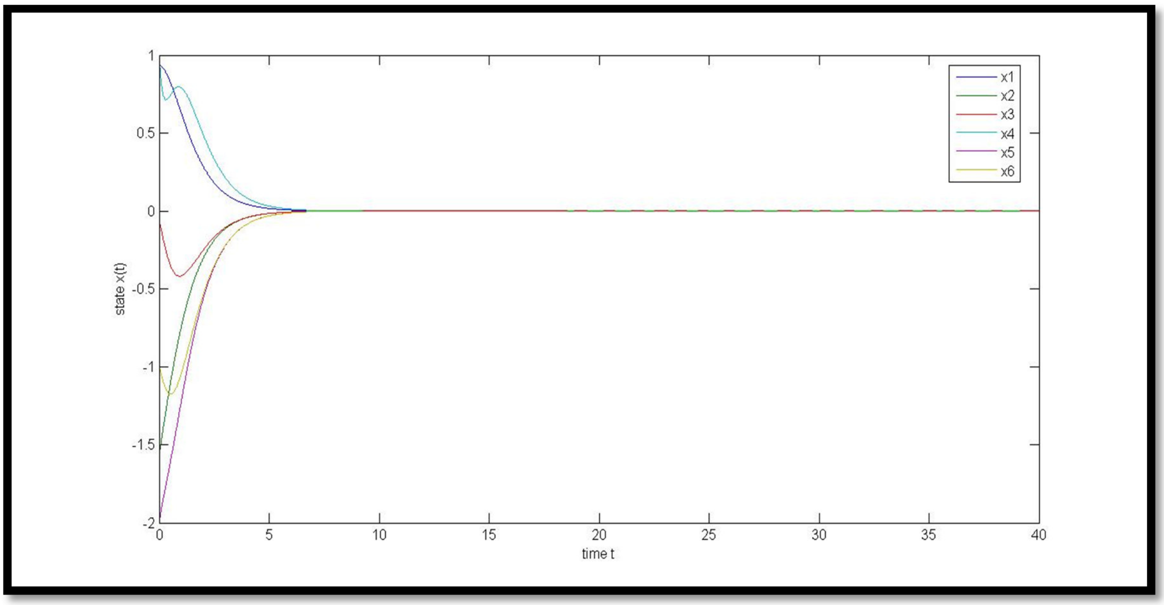

Figure 1. The uncertain closed loop system represent the states $[X_1(t) \;\; X_2(t) \;\; X_3(t) \;\; X_4(t) \;\; X_5(t) \;\; X_6(t) \;]$ with consistent initial condition $[X_{10} \;\; X_{20} \;\; X_{30} \;\; X_{40} \;\; X_{50} \;\; X_{60}]= [0.9377 \;\; -1.5629 \;\; -0.0623 \;\; 1 \;\; -2 \;\; -1] \in W_k$.

Conclusion

When a descriptor system is decomposed into slow and fast subsystem , C-controllability means that both subsystems are controllable .The controllability of the slow subsystem is equivalent to the descriptor system is R-controllable. In this paper survey was presented of stabilizing of descriptor systems using the non-classical variational approach.

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