

# H2 Optimal and $\mu$ -synthesis Design of Quarter Car Active Suspension System

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## Abstract

Better journey comfort and controllability of automobile are pursued via car industries with the aid of considering using suspension system which plays a very crucial function in handling and ride comfort characteristics. This paper presents the design of an active suspension of quarter automobile system using robust H2 optimal controller and robust  $\mu$  - synthesis controller with a second order hydraulic actuator. Parametric uncertainties have been additionally considered to model within the system. Numerical simulation become completed to the designed controllers. Results display that during spite of introducing uncertainties, the designed  $\mu$  - synthesis controller improves ride consolation and road protecting of the automobile while as compared to the H2 optimal controller.

**Index Terms---** Quarter car active suspension system, H2 optimal controller,  $\mu$  - synthesis controller

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## 1 Introduction

An automobile suspension system is the mechanism that bodily separates the automobile body from the wheels of the car. The cause of suspension system is to enhance the experience comfort, road handling and stability of automobiles. Apple backyard and Well stead have proposed several performance traits to be taken into consideration so one can obtain an excellent suspension system. Suspension includes the system of springs, shock absorbers and linkages that connects a vehicle to its wheels. In other that means, suspension system is a mechanism that bodily separates the car body from the automobile wheel. The principal feature of automobile suspension system is to limit the vertical acceleration transmitted to the passenger which directly affords road consolation.

The fundamental idea of land vehicle transportation has not modified tons inside the previous few many years, although a great deal progress become made in enhancing and optimizing automobile design and era. The quest to continually go quicker, similarly and extra effortlessly, has lead in latest years to the improvement of advanced suspension structures. A progressed suspension system allows a vehicle to attain better speeds over rougher terrain, and results in better handling, in addition to improved trip consolation.

A first step in vehicle suspension design is estimating a preferred spring and damper function, and verifying that characteristic the use of software program simulation. The models used at some point of this step are generally low-order, simple fashions, which hampers short development progress. To predict car response before automobile prototype finishing touch, many researchers have tried to use complex and superior damper fashions to simulate the car's dynamics, however these fashions all be afflicted by a few disadvantage – it's far both based on empirical statistics, giving no indication of the bodily parameters of the design sought; it could be overly complex, having many parameters and accordingly rendering software impractical; or it can be quick however based totally on the idea that there's no hysteresis in the damping character.

## 2 Mathematical Models

### 2.1 Active Suspension System Mathematical Model

The quarter car active suspension system model is shown in Figure 1.

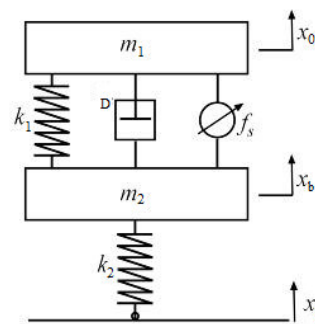


Figure 1: Quarter model of active suspension system.

The quarter car active suspension system mathematical model for a two degrees of freedom motion differential equations will be as follow:

$$m_1 \ddot{x}_0(t) + D[\dot{x}_0(t) - \dot{x}_2(t)] + k_1[x_0(t) - x_2(t)] = u \quad (1) \quad m_2 \ddot{x}_2(t) - D[\dot{x}_0(t) - \dot{x}_2(t)] + k_1[x_2(t) - x_0(t)] + k_2[x_2(t) - x_1(t)] = -u \quad (2)$$

### 3 Road Profiles

#### 3.1 Random Road Profile

The random road disturbance input has a maximum peak of 15 cm and minimum peak of -15 cm as shown in Figure 2.

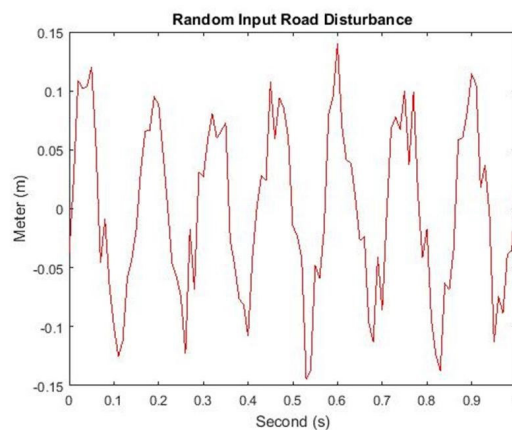


Figure 2: Random road profile

### 4 The Proposed Controller Design

The active suspension system with H2 optimal and  $\mu$ -synthesis controllers with a second order hydraulic actuator system block diagram is shown in Figure 3.

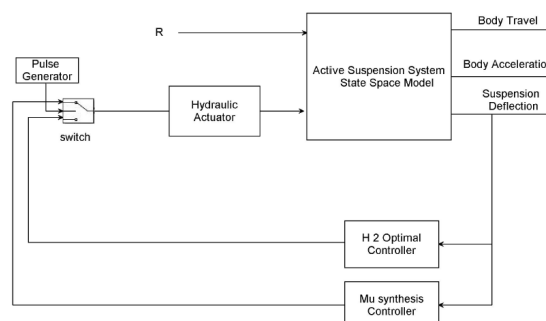


Figure 3: Active suspension system with H2 optimal and  $\mu$  - synthesis controller's system block diagram  
 The switching system switch in micro seconds. The hydraulic actuator model is a second order transfer function defined as:

$$actuator = \frac{4s + 23}{s^2 + 26s + 14} \quad (3)$$

## 5 Result and Discussion

The quarter car active suspension system parameter values are shown in Table 1.

Table 1: Parameters of quarter vehicle model

Model parameters	symbol	symbol Values
Vehicle body mass	m 1	4550 Kg
Wheel assembly mass	m 2	63 Kg
Suspension stiffness	k 1	18,500 N/m
Tire stiffness	k 2	165,000 N/m
Suspension damping	D	800 N-s/m

### 5.1 Control Targets Output Specifications

The control targets output specifications of the quarter car active suspension system is shown in Table 2 below.

Table 2. Control targets output specifications

No	Control Targets	Output
1	Body travel	Minimum
2	Body acceleration	Minimum
3	Suspension deflection	Same as Road Profile

### 5.2 Simulation of a Random Road Disturbance

The body travel, body acceleration and suspension deflection simulation output is shown in Figure 4, Figure 5 and Figure 6 respectively.

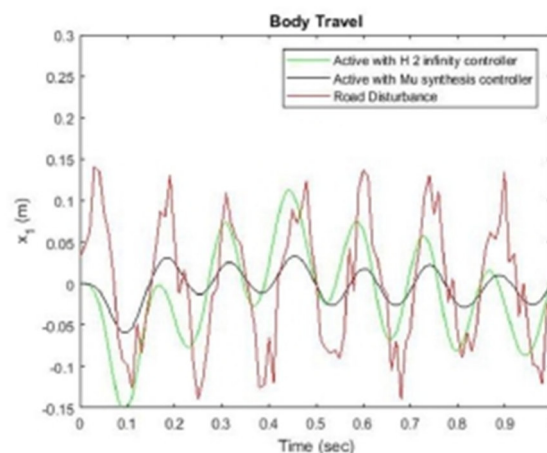


Figure 4: Body travel for random road disturbance

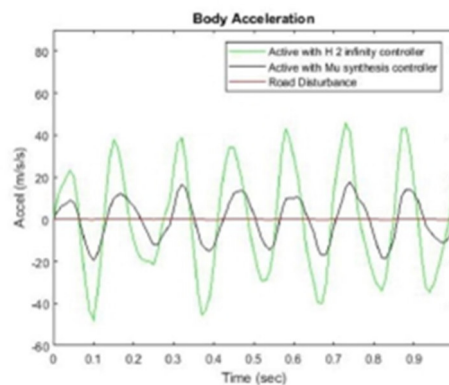


Figure 5: Body acceleration for random road disturbance

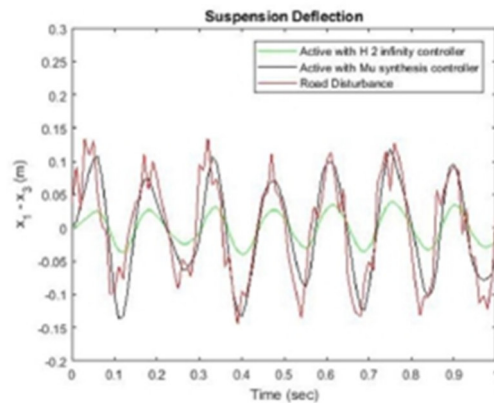


Figure 6: Suspension deflection for random road disturbance

### 5.3 Numerical Values of the Simulation Outputs

The numerical values of the simulation output for the control targets body travel, body acceleration and suspension deflection is shown in Table 3, Table 4 and Table 5 bellow.

Table 3: Numerical values of the body travel simulation output

No	Systems	Random
1	Road Profile	0.15m
2	H2 Optimal	0.04m
3	$\mu$ - synthesis	0.01m

Table 2 shows us the active suspension system with  $\mu$  - synthesis controller have the minimum body travel amplitude in the random road profile.

Table 3: Numerical values of the body acceleration simulation output

No	Systems	Random
1	Road Profile	$5 \frac{m}{s^2}$
2	H2 Optimal	$40 \frac{m}{s^2}$
3	$\mu$ - synthesis	$20 \frac{m}{s^2}$

Table 3 shows us the active suspension system with  $\mu$  - synthesis controller have the minimum body acceleration amplitude in the random road profile.

Table 4: Numerical values of the suspension deflection simulation output

No	Systems	Random
1	Road Profile	0.14m
2	H2 Optimal	0.04m
3	$\mu$ - synthesis	0.14m

Table 4 shows us the active suspension system with  $\mu$  - synthesis controller have the suspension deflection amplitude the same as the road profile input in the random road profiles.

### 6 Conclusion

The technique turned into advanced to design an active suspension for a passenger vehicle with the aid of designing a controller, which improves overall performance of the system with respect to design desires compared to active suspension system. Mathematical modeling has been accomplished using a two degree-of-freedom model of the quarter vehicle model for active suspension system considering best vertical movement to evaluate the overall performance of suspension with recognize to various contradicting design goals. H2 optimal and mu-synthesis

controllers design method has been tested for the active suspension system using random road profile. The capability for advanced experience comfort and better road managing using H2 optimal and mu-synthesis controllers design is examined. The objectives of this undertaking were achieved. Dynamic model for linear quarter car active suspension systems has been formulated and derived for the 2 types of the proposed controllers. The two proposed controllers are compared and the simulation consequences prove the effectiveness of the provided active suspension system with mu synthesis controller.

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