# **On The Time Lag of A First-Order Process's Sinusoidal Response**

ABDUL-FATTAH MOHAMED ALI<sup>1</sup>\* GHANIM M. ALWAN<sup>2</sup>\*\*

Environmental Engineering Dept., College of Engineering, Baghdad University
 Chemical Engineering Dept., University of Technology, Baghdad
 \*E-mail: Abdulfattah1948@yahoo.com,\*\*E-mail:Dr\_gm@uotechnology.edu.iq

### Abstract

The time lag (TL) of a linear first-order process's sinusoidal response has been analyzed for a process time constant ( $\tau$ ) range of 5 to 10<sup>3</sup> s and a radian frequency ( $\omega$ ) range of 1x10<sup>-4</sup> to 50 rad/s. The analysis showed that at  $\omega \le 0.1/\tau$ , TL has a maximum value equal to  $\tau$ . As  $\omega$  increases, TL decreases such that over the range  $3 \le \omega \le 50$  rad/s, TL = 1.5456/ $\omega$ ; being independent of  $\tau$ . At  $\omega \ge 50$  rad/sec, TL  $\cong 0$ ; i.e. the response is in phase with the sinusoidal forcing function (SFF) but with a non-existent amplitude. The obtained results are contrary to what had/has been reported in some relevant textbooks in which the response's out-of-phase condition, with respect to the SFF, is shown to be minimum at very low  $\omega$  values, increasing to a maximum as  $\omega \to \infty$ . **Key words:** Process dynamics; Frequency response; Time lag; Time constant.

#### 1. Introduction

Textbooks on process dynamics and control abound with illustrations on the phase shift ( $\varphi$ ) associated with the response of a linear 1<sup>st</sup>-order process to a sinusoidal forcing function (SFF). However, little is mentioned about the response's time lag (TL) and nothing about its significance. Furthermore, in some texts,  $\varphi$  is erroneously illustrated in lieu of TL on the time axis of graphical representation of SFFs and their corresponding responses, giving the wrong impression that as  $\omega$  increases the response's time lag also increases up to a  $\varphi$  value of  $\pi/2$  radians. References [1] to [6] are well-known textbooks in this field, published over a time span of more than four decades (1964-2011); cited here as an example that this is the case.

#### 2. Time Lag

The steady state response of a linear 1<sup>st</sup>-order process to a SFF of the form:  $SFF = A \sin(\omega t)$  (1)

is given by:

$$= \frac{\mathbf{A}\mathbf{K}}{\sqrt{\omega^2 \tau^2 + 1}} \sin\left(\omega t - \varphi\right)$$
(2)

This may be expressed as,

Y(t)

$$Y(t) = \frac{AK}{\sqrt{\omega^2 \tau^2 + 1}} \sin \omega \left( t - \frac{\varphi}{\omega} \right) \qquad (3)$$

where  $(\phi / \omega)$  is the time lag (also called time delay or time shift). As  $\omega \to \infty$ ,  $\phi$  (being  $\tan^{-1}\omega\tau$ ) increases to a limit of  $\pi/2$  radians; however TL decreases to a limit of zero. In other words, increasing  $\omega$  brings the response closer to the forcing function (on a time scale), albeit with a reduced amplitude.

Fig. 1 shows the variation of TL with  $\omega$  over the range  $1 \times 10^{-4} \le \omega \le 50$  rad/s for the practically significant process time constant range of  $5 \le \tau \le 10^3$  s.

Three aspects of Fig. 1 are noteworthy. The first is that for any value of  $\tau$  there is an  $\omega$  value below which TL is simply the process's  $\tau$ . Table 1 gives a sample of such values.



Fig. 1 .Variation of TL with  $\omega$  for  $\tau$  values of 5, 10, 100, and  $10^3~s$ 

1 able 1 while a values at below which $1L - t$ with corresponding t value	Table 1	$\omega$ values at/below which TL	$L = \tau$ with corresponding $\tau$ value
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ω [rad/sec]	$1 \times 10^{-5}$	1 × 10 <sup>-4</sup>	1× 10 <sup>-3</sup>	1× 10 <sup>-2</sup>	0.1
τ [sec]	1 × 10 <sup>4</sup>	1 × 10³	$1 \times 10^2$	10	1

Or , generally for  $\omega \leq 0.1/\tau$  ,  $TL = \tau$  .

This fact can be mathematically proven as follows :

Since  $TL = \phi/\omega = (\tan^{-1}\omega\tau)/\omega$  then for  $\omega = 0.1/\tau$ , TL becomes  $\tau [(\tan^{-1}0.1)/(0.1)] = \tau$ . Values of  $\omega$  smaller than  $0.1/\tau$  (say x/ $\tau$ ) will also lead to this result because the expression for TL will always be  $\tau [(\tan^{-1}x)/x]$  in which the numerator is an angle and the denominator is its tangent which are virtually equal for small angles in the radian measure.

The second aspect of Fig.1 is that over the range  $3 \le \omega \le 50$  rad/s, TL is virtually a function of  $\omega$  only; being independent of  $\tau$ . Fig.2 enlarges the range  $1 \le \omega \le 10$  rad/s of Fig.1 to elucidate this fact.

The third aspect of Fig.1 is that at a  $\omega$  value of 50 rad/s, TL is equal to the negligibly small value of 0.03 s relative to the process  $\tau$  values considered. Over the virtually  $\tau$ -independent  $\omega$  range  $3 \le \omega \le 50$  rad/s, TL is related to  $\omega$  by the following correlation:

$$TL = \frac{1.5456}{\omega} \qquad (R^2 = 0.99998) \qquad (4)$$

With a maximum error of 2.75% over its specified applicability range.





### 3. Verification

The aforementioned treatise was verified by the simulation arrangement using the Simulink of MATLAB (version 2012) as in shown in Fig. 3. Figs. 4 and 5 show samples of the on-line simulated results, which provide and confirm the presented analysis.



# Fig. 3. Simulation arrangement for first-order process's sinusoidal response





Fig. 4. On-line samples of simulated results at  $\tau$ =10sec for different  $\omega$ .













Fig. 5. On-line samples of simulated results at  $\tau$ =1000 sec for different  $\omega$ .

### 4. Conclusions

The time lag of the steady state sinusoidal response of a linear first- order process has a maximum value equal to its time constant at the low  $\omega$  range of  $\omega \leq 0.1/\tau$  rad/s. This value decreases as  $\omega$  increases such that over the range  $3 \leq \omega \leq 50$  rad/s, TL =  $1.5456/\omega$ , being independent of  $\tau$  for all  $\tau \geq 5$  s. The response's TL practically vanishes at  $\omega \geq 50$  rad/s rendering it in phase with the SFF but with a virtually non-existent amplitude.

Analysis of TL in this article indicates that what had/has been reported (explicitly or implied ) in some renowned textbooks on process dynamics and control , over more than four decades (e.g. the ones listed below as references ) that at very small  $\omega$  values the steady state response is virtually in phase with the SFF; becoming increasingly out of phase as  $\omega$  increases is incorrect.

### Nomenclature

- A SFF amplitude
- K Process steady state gain
- $\mathbf{R}^2$  Coefficient of determination

### Greek

- $\tau$  Process time constant (s)
- $\varphi$  Phase shift (rad)
- ω Radian frequency (rad/s)

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