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Assessing the Design effect of Pressure Vessel Height and Radius on Reactor Stability and Safety

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ABSTRACT

The Design of Reactor Pressure Vessel (RPV) should be that the height of pressure vessel is up to 16.0m and radius is up to 5.6m to ensure safe operation of nuclear reactor. The research conducted safety margin test on the design dimension of RPV in terms of the height and radius, secondly safety margin test was carried out on applied high temperature on the reactor graphite core and thirdly, safety margin test were perform on the cooling problem of the nuclear reactor core in relation to fuel temperature. By applying Linear Regression Analysis Techniques on some typical Reactor Pressure Vessel (RPV) models. The results of the statistical analysis on these types of nuclear reactor models reveals that the RPV models promises stability under application of pressure vessel up to 16.0m height and radius 5.6m. At this point the temperature seems at maximum and the reactor agrees to be more stable as the regression plot was optimized, that is the least squares method finds its optimum when the sum, *S*, of squared residuals is at minimum. The safety margin prediction of 3.1% was validated for a typical RPV model as an advantage over the current 5.1% challenging problem for plant engineers to predict the safety margin limit.
Keywords: Reactor pressure vessel design, height and radius, high temperature effect, fuel element, risk and failure, reactor safety, safety factor, Ỳ, optimization, stability margin, reactor pressure vessel design models, selection of pressure vessel.

INTRODUCTION

The main drivers for reactor development are**:**

- Improved safety (for example by the incorporation of passive safety features)
- Reduced capital cost
- Reduced operating cost
- Improved efficiency and utilization
- Improved design effectiveness
- Reduced build-time
- Minimize the risk of failure

These main drivers shall provide a good, novel approach and method for multi-objective decision-making in the development of the nuclear industry.

The reactor pressure vessels are designed with great care to ensure safe operation when used within their pre-scribed temperature and pressure limits. The selection of pressure vessel must be the one which has the capability, pressure rating, corrosion resistance and design features that are suitable for its intended use. When pressure vessel could not function as to supply water or gas in cooling the reactor the phenomena can cause hydrogen built-up within the reactor and this can eventually melt down the reactor core, as in the case studies of nuclear accident in Japan when the pressure vessel fail to function, heat continue to built-up in the reactor and the reactor meltdown and was damaged [1]. Then Seawater was being pumped into the reactor in an attempt to cool down the radioactive core. Also, there was a recorded explosion which occurred, at the NDK Crystal manufacturing company in Belvidere, Illinois resulted from Stress Corrosion Cracking of High-Pressure Vessel [2], and fatal accident at Goodyear Tire and Rubber Plant in Houston, Texas, following Pressure Vessel Codes, the accident occurred, when an overpressure in a heat exchanger led to a violent rupture of the exchanger [3].
There was a safety concern because counterfeit reactor equipment of the reactor pressure vessel at Koodankulam Nuclear Power Plant in the Tirunelveli district of the southern Indian state of Tamil Nadu [4]. Since these also include safety grade equipment, there is also a potential for accident.

In exercising the responsibility for the selection of pressure equipment, the prospective user is often faced with a choice between over or under-designed equipment. The hazards introduced by under-designed pressure vessels are readily apparent, but the penalties that must be paid for over-designed apparatus are often overlooked[5].
The list of available nominal chemical composition of pressure vessel construction materials are highlighted in Table 1:

Table I: Highlighted the nominal chemical composition of pressure vessel materials

Major Elements (Percent)							
Material	Typical Trade Name	Fe	Ni	Cr	Mo	Mn	Other
T316/316 L Stainless Steel		65	12	17	2.5	2.0	Si 1.0
Alloy 20	Carpenter 20	35	34	20	2.5	2.0	Cu 3.5, Cb 1.0 max
Alloy 400	Monel 400	1.2	66				Cu 31.5
Alloy 600	Inconel 600	8	76	15.5			
Alloy B-2	Hastelloy B-2	2	66	1	28	1	Co 1.0
Alloy C-276	Hastelloy C-276	6.5	53	15.5	16	1	W4.0, Co 2.5
Nickel 200			99				
Titanium Grade 2	Commercially pure titanium					Ti 99 min	
Titanium Grade 2	Commercially pure titanium					Ti 99 min	
Titanium Grade 7		99					0.15 Pd
Zirconium Grade 702		Zr + Hf 99.2 min, Hf 4.5 max					
Zirconium Grade 705		Zr + Hf 99.2 min, Hf 4.5 max					Nb 2.5

A **pressure vessel** is a closed, rigid container designed to hold gases or liquids at a pressure different from the ambient pressure.
In addition to industrial compressed air receivers and domestic hot water storage tanks, other examples of pressure vessels are: diving cylinder, recompression chamber, distillation towers, autoclaves and many other vessels in mining or oil refineries and petrochemical plants, nuclear reactor vessel, habitat of a space ship, habitat of a submarine, pneumatic reservoir, hydraulic reservoir under pressure, rail vehicle airbrake reservoir, road vehicle airbrake reservoir and storage vessels for liquified gases such as ammonia, chlorine, propane, butane and LPG.

DESIGN AND OPERATION STANDARDS

In the nuclear industrial sector, pressure vessels are designed to operate safely at a specific pressure and temperature, technically referred to as the "Design Pressure" and "Design Temperature". The pressure vessel is designed to a pressure, there is typically a safety valve or relief valve to ensure that this pressure is not exceeded in operation. A vessel that is inadequately designed to handle a high pressure constitutes a very significant safety hazard. Because of that, the design and certification of pressure vessels is governed by design codes such as the ASME Boiler and Pressure Vessel Code in North America[6], the Pressure Equipment Directive of the EU (PED), Japanese Industrial Standard (JIS), CSA B51 in Canada, AS1210 in Australia and other international standards like Lloyd's, Germanischer Lloyd, Det Norske Veritas, Stoomwezen and so on.

SHAPE OF A PRESSURE VESSEL

Theoretically a sphere would be the optimal shape of a pressure vessel. Most pressure vessels are made of steel. To manufacture a spherical pressure vessel, forged parts would have to be welded together. Some mechanical properties of steel are increased by forging, but welding can sometimes reduce these desirable properties. In case of welding, in order to make the pressure vessel meet international safety standards, carefully selected steel with a high impact resistance are be used. Most pressure vessels are arranged from a pipe and two covers. Disadvantage of these vessels is the fact that larger diameters make them relatively more expensive, so that for example the most economic shape of a 1000 litres, 250 bar (25,000 kPa) pressure vessel might be a diameter of 450 mm and a length of 6500 mm.

No matter what shape it takes, the minimum mass of a pressure vessel scales with the pressure and volume it contains. For a *sphere*, the mass of a pressure vessel is

$$M = \frac{3}{2}PV\frac{\rho}{\sigma},$$ ………………………………………Equation (1)

Where**:**

M is mass
p is the pressure difference from ambient- the gauge pressure
V is volume
ρ is the density of the pressure vessel material
σ is the maximum working stress that material can tolerate.

Other shapes besides a sphere have constants larger than 3/2 (infinite cylinders take 2), although some tanks, such as non-spherical wound composite tanks can approach this.

CYLINDRICAL VESSEL WITH HEMISPHERICAL ENDS

This is sometimes called a "bullet" for its shape, although in geometric terms it is a *capsule.*
For a cylinder with hemispherical ends,

$$M - 2\pi R^2(R+W)P\frac{\rho}{\sigma},$$ ……………………………Equation (2)

where

- R is the radius, W is the middle cylinder width only, and the overall width is W + 2R

CYLINDRICAL VESSEL WITH SEMI-ELLIPTICAL ENDS

In a vessel with an aspect ratio of middle cylinder width to radius of 2:1,

$$M = 6\pi R^3 P\frac{\rho}{\sigma}$$ ………………………………………Equation (3)

GAS STORAGE

In looking at the first equation, the factor PV, in SI units, is in units of (pressurization) energy. For a stored gas, PV is proportional to the mass of gas at a given temperature, thus

$$M = \frac{3}{2}nRT\frac{\rho}{\sigma}.$$ ………………………………………Equation (4)

The other factors are constant for a given vessel shape and material. So we can see that there is no theoretical "efficiency of scale", in terms of the ratio of pressure vessel mass to pressurization energy, or of pressure vessel mass to stored gas mass. For storing gases, "tankage efficiency" is independent of pressure, at least for the same temperature.
So, for example, a typical design for a minimum mass tank to hold *helium* (as a pressurant gas) on a rocket would use a spherical chamber for a minimum shape constant, carbon fiber for best possible ρ/σ, and very cold helium for best possible M/pV.

STRESS IN THIN-WALLED PRESSURE VESSELS

Stress in a shallow-walled pressure vessel in the shape of a sphere is

$$\sigma_\theta = \sigma_{\text{long}} = \frac{pr}{2t},$$ …………………………………Equation (5)

where σ_θ is hoop stress, or stress in the circumferential direction, σ_{long} is stress in the longitudinal direction, *p* is internal gauge pressure, *r* is the inner radius of the sphere, and *t* is thickness of the cylinder wall. A vessel can be considered "shallow-walled" if the diameter is at least 10 times (sometimes cited as 20 times) greater than the wall depth.

Stress in a shallow-walled pressure vessel in the shape of a cylinder is

$$\sigma_\theta = \frac{pr}{t}$$,……………………………………………..Equation (6)

$$\sigma_{\text{long}} = \frac{pr}{2t}$$,…………………………………………Equation (7)

where σ_θ is hoop stress, or stress in the circumferential direction, σ_{long} is stress in the longitudinal direction, *p* is internal gauge pressure, *r* is the inner radius of the cylinder, and *t* is thickness of the cylinder wall.
Almost all pressure vessel design standards contain variations of these two formulas with additional empirical terms to account for wall thickness tolerances, quality control of welds and in-service corrosion allowances.
For example, the ASME Boiler and Pressure Vessel Code (BPVC) (UG-27) formulas are:
Spherical shells:

$$\sigma_\theta = \sigma_{\text{long}} = \frac{p(r + 0.2t)}{2tE}$$ ……………………Equation (8)

Cylindrical shells:

$$\sigma_\theta = \frac{p(r + 0.6t)}{tE}$$ ………………………………Equation (9)

$$\sigma_{\text{long}} = \frac{p(r - 0.4t)}{2tE}$$ ………………………….Equation (10)

Where, *E* is the joint efficient, and all others variables as stated above.

The factor of safety is often included in these formulas as well, in the case of the ASME BPVC this term is included in the material stress value when solving for Pressure or Thickness.

LINEAR EXPANSION

To a first approximation, the change in length measurements of an object ("linear dimension" as opposed to, e.g., volumetric dimension) due to thermal expansion is related to temperature change by a "linear expansion coefficient". It is the fractional change in length per degree of temperature change. Assuming negligible effect of pressure, we may write**:**

$\alpha L = \frac{1}{L}(\frac{\partial L}{\partial T})$……………………………………………Equation (11)

where L is a particular length measurement and $\frac{\partial L}{\partial T}$ is the rate of change of that linear dimension per unit change in temperature.
The change in the linear dimension can be estimated to be:

$\Delta\frac{L}{L} = \alpha L \Delta T$……………………………………………..Equation.(12)

This equation works well as long as the linear-expansion coefficient does not change much over the change in temperature ΔT. If it does, the equation must be integrated.

EFFECTS ON STRAIN

For solid materials with a significant length, like rods or cables, an estimate of the amount of thermal expansion can be described by the material strain, given by and defined as**:**

εthermal = (Lfinal – Linitial)/Linitial…………………….. Equation.(13)

where Linitial is the length before the change of temperature and Lfinal is the length after the change of temperature.
For most solids, thermal expansion is proportional to the change in temperature:

εthermal $\propto$ ΔT…………………………………………… Equation (14)

Thus, the change in either the strain or temperature can be estimated by:
Ethermal = αL ΔT……………………………………… Equation (15)
Where, ΔT = (Tfinal – Tinitial) …………………………Equation (16)
is the difference of the temperature between the two recorded strains, measured in degrees Celsius or Kelvin, and αL is the linear coefficient of thermal expansion in "per degree Celcius" or "per Kelvin", denoted by $°C^{-1}$ or K^{-1}, respectively.

AREA EXPANSION
The area thermal expansion coefficient relates the change in a material's area dimensions to a change in temperature. It is the fractional change in area per degree of temperature change. Ignoring pressure, we may write:

$\alpha A = \frac{1}{A}\left(\frac{\partial V}{\partial T}\right)$……………………………………………Equation.(17)

where A is some area of interest on the object, and $\frac{\partial A}{\partial T}$ is the rate of change of that area per unit change in temperature.
The change in the linear dimension can be estimated as:

$\Delta\frac{A}{A} = \alpha A \Delta T$……………………………………………Equation (18)

This equation works well as long as the linear expansion coefficient does not change much over the change in temperature δT. If it does, the equation must be integrated.

VOLUMETRIC EXPANSION
For a solid, we can ignore the effects of pressure on the material, and the volumetric thermal expansion coefficient can be written:

$\alpha V = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)$……………………………………………Equation (19)

where V is the volume of the material, and $\frac{\partial V}{\partial T}$ is the rate of change of that volume with temperature.
This means that the volume of a material changes by some fixed fractional amount. For example, a steel block with a volume of 1 cubic meter might expand to 1.002 cubic meters when the temperature is raised by 50 °C. This is an expansion of 0.2 %. If we had a block of steel with a volume of 2 cubic meters, then under the same conditions, it would expand to 2.004 cubic meters, again an expansion of 0.2 %. The volumetric expansion coefficient would be 0.2 % for 50 K, or 0.004 %/K.
If we already know the expansion coefficient, then we can calculate the change in volume

$\Delta\frac{V}{V} = \alpha V \Delta T$…………………………………… Equation.(20)

Where, $\Delta\frac{v}{V}$ is the fractional change in volume (e.g., 0.002) and ΔT is the change in temperature (50° C).

The above example assumes that the expansion coefficient did not change as the temperature changed. This is not always true, but for small changes in temperature, it is a good approximation. If the volumetric expansion coefficient does change appreciably with temperature, then the above equation will have to be integrated:

$\Delta\frac{V}{V} = \int_{TO}^{TO+50} \alpha v(T)\,dT$…………………… Equation (21)

where To is the starting temperature and $\alpha v(T)$ is the volumetric expansion coefficient as a function of temperature *T*.

The temperature of the fuel varies as a function of the distance from the center to the rim. At distance *d* from the center the temperature (T_d) is described by the equation where ρ is the power density (W m^{-3}) and K_f is the thermal conductivity.

$T_d = T_{Rim} + \rho\,(r_{pellet}^2 - d^2)\,(4\,K_f)^{-1}$……………………Equation.(22)

When the nuclear fuel increases in temperature, the rapid motion of the atoms in the fuel causes an effect known as Doppler broadening. When thermal motion causes a particle to move towards the observer, the emitted radiation will be shifted to a higher frequency. Likewise, when the emitter moves away, the frequency will be lowered. For non-relativistic thermal velocities, the Doppler shift in frequency will be**:**

$$f = f_0\left(1 + \frac{v}{c}\right)$$ ………………………………………. Equation (23)

where f is the observed frequency, f_0 is the rest frequency, v is the velocity of the emitter towards the observer, and c is the speed of light.

Since there is a distribution of speeds both toward and away from the observer in any volume element of the radiating body, the net effect will be to broaden the observed line.

If $P_v(v)dv$ is the fraction of particles with velocity component v to $v + dv$ along a line of sight, then the corresponding distribution of the frequencies is

$$P_f(f)df = P_v(v_f)\frac{dv}{df}df$$,…………………………. ..Equation (24)

where

$$v_f = c\left(\frac{f}{f_0} - 1\right)$$ ……………………………………Equation (25)

is the velocity towards the observer corresponding to the shift of the rest frequency f_0 to

f.

Therefore,

$$P_f(f)df = \frac{c}{f_0}P_v\left(c\left(\frac{f}{f_0} - 1\right)\right)df.$$

…… Equation (26)

We can also express the broadening in terms of the wavelength λ. Recalling that in the non-relativistic limit $\frac{\lambda - \lambda_0}{\lambda_0} \approx -\frac{f - f_0}{f_0}$, we obtain

$$P_\lambda(\lambda)d\lambda = \frac{c}{\lambda_0}P_v\left(c\left(1 - \frac{\lambda}{\lambda_0}\right)\right)d\lambda.$$

…Equation (27)

In the case of the thermal Doppler broadening, the velocity distribution is given by the Maxwell distribution

$$P_v(v)dv = \sqrt{\frac{m}{2\pi kT}}\exp\left(-\frac{mv^2}{2kT}\right)dv$$,……… ..Equation (28)

where,

m is the mass of the emitting particle, T is the temperature and k is the Boltzmann constant.

Then,

$$P_f(f)df - \left(\frac{c}{f_0}\right)\sqrt{\frac{m}{2\pi kT}}\exp\left(-\frac{m\left[c\left(\frac{f}{f_0} - 1\right)\right]^2}{2kT}\right)df$$,… Equation (29)

We can simplify this expression as

$$P_f(f)df = \sqrt{\frac{mc^2}{2\pi kT{f_0}^2}}\exp\left(-\frac{mc^2\,(f-f_0)^2}{2kT{f_0}^2}\right)df$$,……...Equation (30)

which we immediately recognize as a Gaussian profile with the standard deviation

$$\sigma_f - \sqrt{\frac{kT}{mc^2}}f_0$$,…………………………………………… …..Equation.(31)

and full width at half maximum (FWHM)

$$\boxed{\Delta f_{\mathrm{FWHM}} = \sqrt{\frac{8kT\ln 2}{mc^2}}f_0 .}$$

,………………… …..Equation.(32)

The fuel then sees a wider range of relative neutron speeds. Uranium-238, which forms the bulk of the uranium in the reactor, is much more likely to absorb fast or epithermal neutrons at higher temperatures. This reduces the number of neutrons available to cause fission, and reduces the power of the reactor. Doppler broadening therefore creates a negative feedback because as fuel temperature increases, reactor power decreases. All reactors have reactivity feedback mechanisms, except some gas reactor such as pebble-bed reactor which is designed so that this effect is very strong and does not depend on any kind of machinery or moving parts.

FAILURE AND ACCIDENT ANALYSIS

Several reports on the safety of pressure vessel design these include “Reactor Pressure Vessel Task of Light Water Reactor Sustainability Program,”[7] “Prevention of Catastrophic Failure in Pressure Vessels and Pipings,”[8] “Pressurized Thermal Shock Potential at Palisades,”[9], “Stress Analysis & Pressure Vessels,”[10] “AS 1200 Pressure Vessels”[11] and "Stress Analysis & Pressure Vessels”[12].

These accidents may perhaps be as a result of design concept process of some of these reactors (which could involve novel technologies) that have inherent risk of failure in operation and were not well studied/understood. In avoiding such accidents the industry has been very successful. As in over 14,500 cumulative reactor-years of commercial operation in 32 countries, there have been only three major accidents to nuclear power plants – Fukushima, Chernobyl and Three Mile Island. As in other industries, the design and operation of nuclear power plants aims to reduce the likelihood of accidents, and avoid major human consequences when they occur.

However, recent study of the reactor fuel under accident conditions, reveal that after subjecting the fuel to extreme temperatures — far greater temperatures than it would experience during normal operation or postulated accident conditions — TRISO fuel is even more robust than expected. Specifically, the research revealed that **at *1,800 degrees Celsius*** (more than 200 degrees Celsius greater than postulated accident conditions) most fission products remained inside the fuel particles, which each boast their own primary containment system [8].

METHODOLOGY

In this work, Ordinary Least Square (OLS) methodology, which is largely used in nuclear industry for modeling safety, is employed. Some related previous works on the application of regression analysis technique include**:** “Statistical Analysis of Reactor Pressure Vessel Fluence Calculation Benchmark Data Using Multiple Regression Techniques”[13], “Simplified modeling of a PWR reactor pressure vessel lower head failure in the case of a severe accident”[14]. Others are, “Analyses of loads on reactor pressure vessel internals in a pressurized water reactor due to a loss-of-coolant accident considering fluid-structure interaction”[15], “Regression analysis of gross domestic product and its factors in Lithuania,”[16] and “Investigating the Effect of Loss-of-Pressure-Control on the Stability of Water-Cooled Reactor Design Models,”[17].

THE RESEARCH OBJECTIVES

To apply the linear regression technique on *reactor pressure vessel design models* for the determination of their safety margin that is to examine the effect of pressure vessel height and radius on the stability and safety of the nuclear reactor during operation.

THE RESEARCH MOTIVATION

The purpose of this work is to assist countries wishing to include nuclear energy for the generation of electricity, like Nigeria, to secure a reactor that is better and safe. Also, the studies intended to provide guidance in developing practical catalytic materials for power generation reactor and to help researchers make appropriate recommendation for Nigeria nuclear energy proposition as one of the solutions to Nigeria energy crisis. Moreover, the study is to provide a good, novel approach and method for multi-objective decision-making based on six dissimilar objectives attributes**:** evolving technology, effectiveness, efficiency, cost, safety and failure. Furthermore, this is to help Nigeria meet its international obligations to use nuclear technology for peaceful means. Finally, the achievement is to make worldwide contribution to knowledge.

RESEARCH DESIGN/APPROACH

Theory and experiment have shown that for some reactor, the design of pressure vessel plays significant role in the safety of the reactor during operation in preventing overheating of the reactor and reactor meltdown during accident. Hence, in this work, a statistical analysis of a design input parameter of a typical reactor pressure vessel was investigated for safety. More specifically, the studies concentrated on technical factors that limit the achievement of higher burn-up of fuel in various design of reactor pressure vessels, such as the pressure height and radius mechanical interaction. Furthermore, the study examined the temperature of the fuel behaviour under reactor accident conditions are also included.

The research approach involves adjusting the parameters of a model function to best fit a data set. A simple data set consists of *n* points (data pairs) (x_i, y_i), *i* = 1, ..., *n*, where x_i is an independent variable and y_i is a dependent variable whose value is found by observation. The model function has the form *f* (*x*,β), where the *m* adjustable parameters are held in the vector β. The goal is to find the parameter values for the model which "best" fits the data. The least squares method finds its optimum when the sum, *S*, of squared residuals

$$S = \sum_{i=1}^{n} r_i^2$$

is a minimum……………………………………………………………(25)

Table 2: Input data for safety margin against pressure vessel height and pressure vessel diameter in a typical graphite moderated reactor design model. **Source :** [18]

Nos. of trial (j)	Pressure vessel height (m)	Pressure vessel radius (m)
1	4	5
2	8	5.1
3	12	5.2
4	16	5.6
5	20	5.9
6	24	6.3
7	28	6.6
8	32	6.8
9	34	7.5
10	36	7.9

RESULTS AND ANALYSES

1. Pressure Vessel Design (PVD)

The results of the application of the linear regression analysis of the data in Table 6 of a typical Pressure Vessel Design (PVD) models are presented as follows**:**

(i) Empirical Expression for Safety Factor, Ỳ

In assessing the effect of pressure vessel height and radius on the Stability and Safety of the nuclear reactor during operation, the data obtained in Table 6 which represents a typical parameter for Pressure Vessel Design (PVD) models was used in order to obtain the best fit for the model. The new conceptual fuel design for reactor operation could optimize the performance of the water-cooled reactor.
The linear regression model equation to be solved is given by**:**

$$\grave{Y} = B_0 + B_1X_j + e_j \quad \text{... Equation (33)}$$

where,
B_0 is an intercept, B_1 is the slope, X_j is the rate of increase in fuel volume
e_j = error or residual, j = 1,2,3,…,k and k is the last term.

Empirical Expression for Safety Factor, Ỳ for Normal Pressure Reading
The model empirical expression is the equation of the straight line relating heat in the reactor and the volume of fuel in the reactor as a measure of safety factor estimated as**:**

$$\boxed{\grave{Y} = (4.3231) + (0.0872)*(X_j) + e_j} \quad \text{...... Equation (34)}$$

- the equation (34) is the estimated model or predicted
where,
Ỳ = Dependent Variable, Intercept = 4.3231,
Slope = 0.0872, X = Independent Variable,
e = error or residual, j = 1,2,3,…,10 and 10 is the last term of trial.

The Figure 1 shows the linear regression plot section

(ii) Linear regression plot section

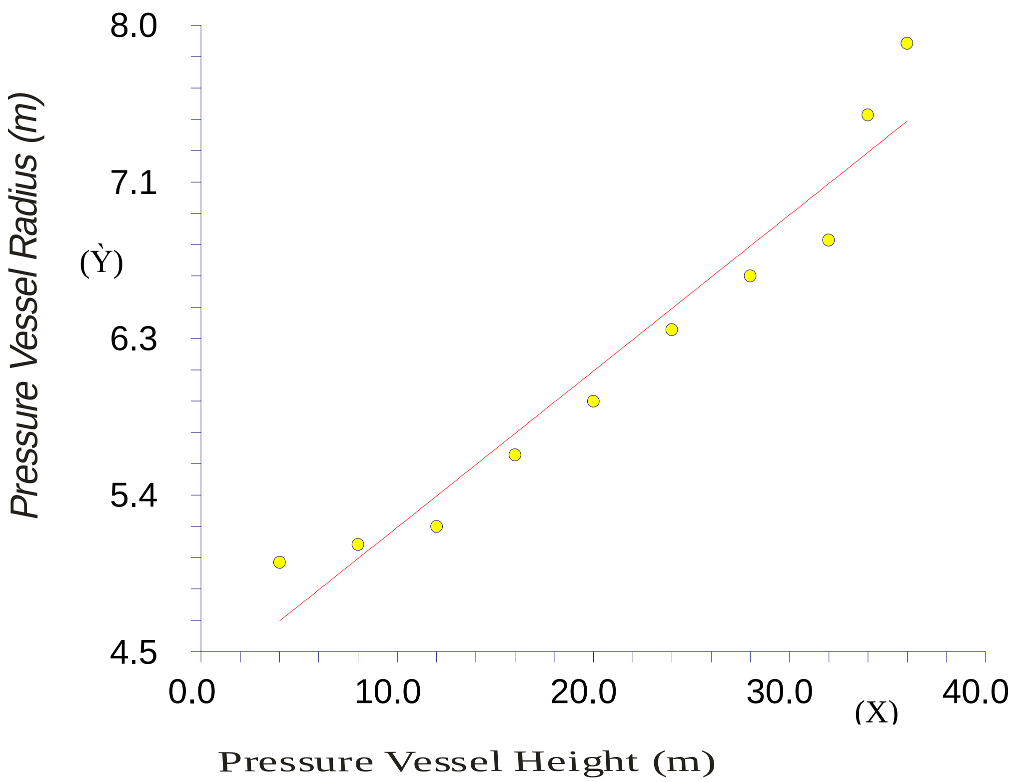

Figure 1: pressure vessel height and radius design

(iii) F-test Result

Table 3: Summary of F-test Statistical Data

Parameter	Value
Dependent Variable	Ỳ (Decay heat or temperature)
Independent Variable	X (fuel volume)
Intercept(B_0)	4.3231
Slope(B_1)	0.0872
R-Squared	0.9392
Correlation	0.9691
Mean Square Error (MSE)	6.999824×10^{-2}
Coefficient of Variation	0.0427
Square Root of MSE	0.2645718

Table 4: Descriptive Statistics Section

Parameter	Dependent	Independent
Variable	Heat (^{0}c)	Fuel (g)
Count	10	10
Mean	6.1900	21.4000
Standard Deviation	1.0115	11.2368
Minimum	5.0000	4.0000
Maximum	7.9000	36.0000

The Table 5 is the regression estimation section results that show the least-squares estimates of the intercept and slope followed by the corresponding standard errors, confidence intervals, and hypothesis tests. These results are based on several assumptions that are validated before they are used.

Table 5: Regression Estimation Section

Parameter	**Intercept B(0)**	**Slope B(1)**
Regression Coefficients	4.3231	0.0872
Lower 95% Confidence Limit	3.8904	0.0691
Upper 95% Confidence Limit	4.7558	0.1053
Standard Error	0.1876	0.0078
Standardized Coefficient	0.0000	0.9691
T-Value	23.0392	11.1158
Prob Level (T-Test)	0.0000	0.0000
Reject H0 (Alpha = 0.0500)	Yes	Yes
Power (Alpha = 0.0500)	1.0000	1.0000
Regression of Y on X	4.3231	0.0872
Inverse Regression from X on Y	4.2022	0.0929
Orthogonal Regression of Y and X	4.3221	0.0873

In Table 6 the analysis of variance shows that the F-Ratio testing whether the slope is zero, the degrees of freedom, and the mean square error. The mean square error, which estimates the variance of the residuals, was used extensively in the calculation of hypothesis tests and confidence intervals.

Table 6: Analysis of Variance Section

Source	**DF**	**Sum of Squares**	**Mean Squares**	**F-Ratio**	**Prob Level**	**Power(5%)**
Intercept	1	383.161	383.161			
Slope	1	8.649014	8.649014	8.649014	123.5605	1.0000
Error	8	0.5599859	$6.999824X10^{-2}$			
Adj. Total	9	9.209	1.023222			
Total	10	392.37				
S = Square Root($6.999824X10^{-2}$) = 0.2645718						

In Table 7 Anderson Darling method confirms the rejection of H_0 at 20% level of significance but all of the above methods agreed that H_0 Should not be rejected at 5% level of significance. Hence the normality assumption is satisfied as one of the assumptions of the Linear Regression Analysis is that the variance of the error variable δ^2 has to be constant.

Table 7: Tests of Assumptions Section

Assumption/Test Residuals follow Normal Distribution?	**Test Value**	**Prob Level**	**Is the Assumption Reasonable at the 20% or 0.2000 Level of Significance?**
Shapiro Wilk	0.8901	0.169812	No
Anderson Darling	0.5842	0.128324	No
D'Agostino Skewness	1.0600	0.289166	Yes
D'Agostino Kurtosis	-0.5545	0.579233	Yes
D'Agostino Omnibus	1.4310	0.488954	Yes
Constant Residual Variance?			
Modified Levene Test	0.3515	0.569628	Yes
Relationship is a Straight Line?			
Lack of Linear Fit F(0, 0) Test	0.0000	0.000000	No

Notes:

A 'Yes' means there is not enough evidence to make this assumption seem unreasonable.

A 'No' means that the assumption is not reasonable

(iv) Residual Plots Section

The plot section is used as further check on the validity of the model to satisfy all the assumptions of the linear regression analysis.
Amir D. Aczel (2002, P528) have stated that the normality assumption can be checked by the use of plot of errors against the predicted values of the dependent variable against each of the independent variable and against time (the order of selection of the data points) and on a probability scale.
The diagnostic plot for linear regression analysis is a scatter plot of the prediction errors or residuals against predicted values and is used to decide whether there is any problem in the data at hand Siegel F (2002, p.578).

The Figure 2 is for the plot of errors against the order to selection of the data points (e = 1,2,…,12). Although the order of selection was not used as a variable in the mode, the plot reveal whether order of selection of the data points should have been included as one of the variables in our regression model. This plot shows no particular pattern in the error as the period increases or decreases and the residuals appear to be randomly distributed about their mean zero, indicating independence. The residuals are randomly distributed with no pattern and with equal variance as volume of fuel increases.

Note:
1. Residual = original value for heat (Y) minors predicted value for heat, Ỳ
2. Count = the design number (design 1, 2, 3, …, 12)

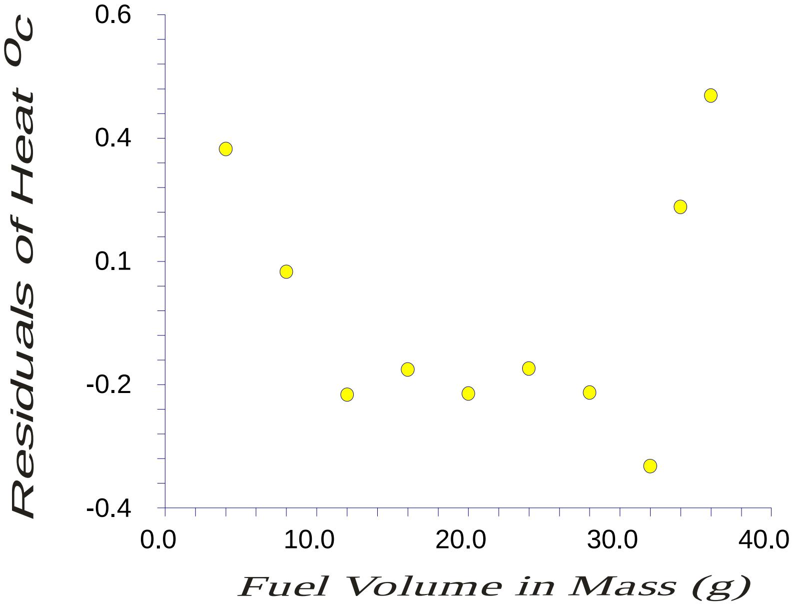

Figure 2**:** Residuals of Heat (^{0}C) versus Fuel (g)

Figure 3 shows the histogram of residuals of error (e_t) and this is nearly skewed to the right but the software used indicated that the plot is normal.

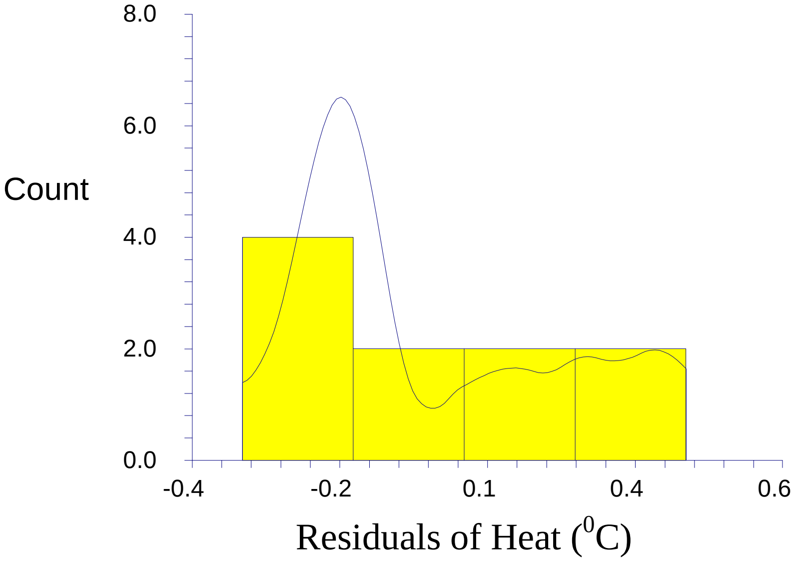

Figure 3**:** Histogram of Residuals of Heat (^{0}C)

While Figure 4 is the result on plot graph of experimental errors. The residuals are perfectly normally distributed as most of the error terms align themselves along the diagonal straight line with some error terms outside the arc above and below the diagonal line. This further indicates that the estimated model is valid.

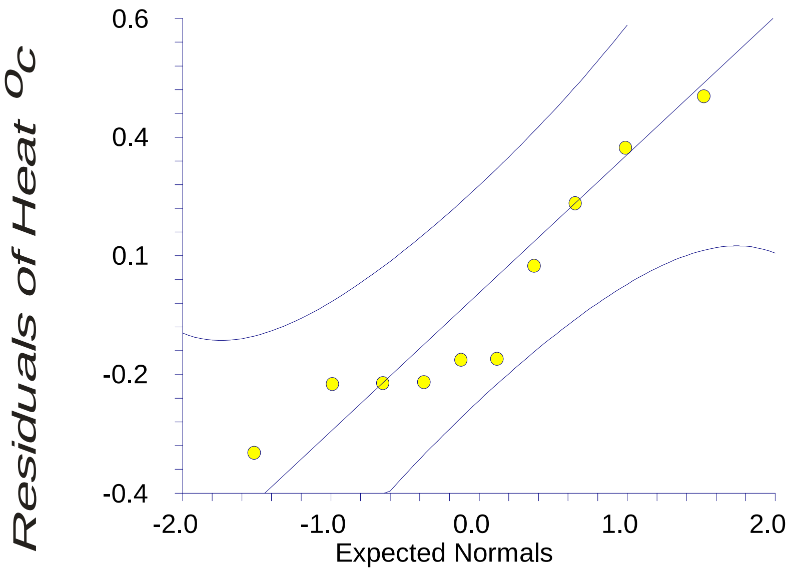

Figure 4**:** Normal Probability Plot of Residuals of Heat (^{0}C)

2. SUMMARY/CONCLUSION

In conclusion the research conducting safety margin test on the design dimension of graphite core of graphite moderated reactor in terms of the height and diameter, secondly safety margin test was carried out on applied high temperature on the reactor graphite core and thirdly, safety margin test were perform on the cooling problem of the nuclear reactor core in relation to fuel temperature. By applying Linear Regression Analysis Techniques on some typical Pressure Vessel Design (PVD) models. The results of the statistical analysis on these types of nuclear reactor models reveals that the PVD models promises stability under application of pressure vessel up to 16m height and radius 5.6m. At this point the temperature seems at maximum and the reactor agrees to be more stable as the regression plot was optimized, that is the least squares method finds its optimum when the sum, *S*, of squared residuals

$S = \sum_{i=1}^{n} r_i^2$ is a minimum at the given dimension of PV height 16.0metre and radius 5.6metre. Meanwhile, at anything below height of 16metre and radius of 5.6metre the fuel element seems to be unstable in the reactor as the regression plot could not find it optimal.

The research implication is that the design dimension of graphite moderated reactor core could be significant to the nuclear fuel temperature and safety of the reactor during operation or accident. Secondly, the safety margin prediction of up to 3.1% has been validated for reactor design models on water-cooled reactor regarding the design dimension of graphite moderated reactor core parameter, core temperature and fuel temperature. The research effort served as an advantage over the current 5.1% challenging problem for plant engineers to predict the safety margin limit. According to Xianxun Yuan (2007, P49) in “Stochastic Modeling of Deterioration in Nuclear Power Plants Components” a challenging problem of plant engineers is to predict the end of life of a system safety margin up to 5.1% validation.

The current design limits for various reactors safety in a nuclear power plant, defined by the relative increase and decrease in the parametric range at a chosen operating point from its original value, varies from station to station. However, the finding in the work would suggest that the design of the plant should ensure that operating reactor core are made up of large graphite core in order to minimize core melting in an extreme high temperature condition which can damaged the reactor.
The Design of Reactor Pressure Vessel (RPV) should be that the height of pressure vessel is up to 16.0m and radius is up to 5.6m are maintained in their construction and possibly provision for extra pressure vessel in the design to ensure safe operation of nuclear reactor. But the final responsibility for selecting a reactor or pressure vessel that will perform to the user’s satisfaction in any particular reaction or test must rest with the user.

If pressure vessels technology solution must be addressed properly then the following areas of applicable pressure technology needs to be well study these include dynamic and seismic analysis; equipment qualification; fabrication; fatigue and fracture prediction; finite and boundary element methods; fluid-structure interaction; high pressure engineering; elevated temperature analysis and design; inelastic analysis; life extension; lifeline earthquake engineering; PVP materials and their property databases; NDE; safety and reliability; and verification and qualification of analysis with relevant software.

Finally, the discoveries on high temperature effect on graphite moderated reactor safety either in terms of design graphite dimension parameter or fuel temperature on the power reactor stability and safety factor should provide a new method for reactor design conceptualization. This shall also provide a good, novel approach and method for multi-objective decision-making based on six dissimilar objectives attributes**:** evolving technology, effectiveness, efficiency, cost, safety and failure. The implication of this research effort to Nigeria’s nuclear power project drive.
It is therefore recommended that for countries wishing to include nuclear energy for the generation of electricity, like Nigeria, the design input parameters of the selected nuclear reactor should undergo test and analysis using this method for optimization and choice.

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