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A Seasonal Arima Model for Nigerian Gross Domestic Product

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Abstract

Time series analysis of Nigerian Gross Domestic Product series is done. A seasonal difference and then a non-seasonal one were obtained. The correlogram of the differenced series revealed seasonality of order 4. It also reveals an autocorrelation structure of a known seasonal model involving a seasonal autoregressive component of order one and a non-seasonal moving average component of order one. The model has been shown to be adequate.

Keywords: Gross Domestic Product, ARIMA modelling, Seasonal models, Nigeria.

1.Introduction

1.1.ARIMA Models.

A time series is defined as a set of data collected sequentially in time. It has the property that neighbouring values are correlated. This tendency is called *autocorrelation.* A time series is said to be stationary if it has a constant mean and variance. Moreover the autocorrelation is a function of the lag separating the correlated values and called *the autocorrelation function* (ACF).

A stationary time series $\{X_t\}$ is said to follow an *autoregressive moving average model of orders p and q* (designated ARMA(p,q)) if it satisfies the following difference equation

$$X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + ... + \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + ... + \beta_q \varepsilon_{t-q} \quad (1)$$

or

$$A(B)X_t = B(B)\varepsilon_t \quad (2)$$

where $\{\varepsilon_t\}$ is a sequence of random variables with zero mean and constant variance, called *a white noise process*, and the α_i's and β_j's constants; $A(B) = 1 + \alpha_1 B + \alpha_2 B^2 + ... + \alpha_p B^p$ and $B(B) = 1 + \beta_1 B + \beta_2 B^2 + ... + \beta_q B^q$ and B is the backward shift operator defined by $B^k X_t = X_{t-k}$.

If p=0, model (1) becomes *a moving average model of order q* (designated MA(q)). If, however, q=0 it becomes *an autoregressive process of order p* (designated AR(p)). An AR(p) model of order p may be defined as a model whereby a current value of the time series X_t depends on the immediate past p values: $X_{t-1}, X_{t-2}, ..., X_{t-p}$. On the other hand an MA(q) model of order q is such that the current value X_t is a linear combination of immediate past values of the white noise process: $\varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_{t-q}$. Apart from stationarity, invertibility is another important requirement for a time series. It refers to the property whereby the covariance structure of the series is unique (Priestley, 1981). Moreover it allows for meaningful association of current events with the past history of the series (Box and Jenkins, 1976).

An AR(p) model may be more specifically written as

$$X_t + \alpha_{p1}X_{t-1} + \alpha_{p2}X_{t-2} + ... + \alpha_{pp}X_{t-p} = \varepsilon_t$$

Then the sequence of the last coefficients $\{\alpha_{ii}\}$ is called *the partial autocorrelation function*(PACF) of $\{X_t\}$. The ACF of an MA(q) model cuts off after lag q whereas that of an AR(p) model is a combination of sinusoidals dying off slowly. On the other hand the PACF of an MA(q) model dies off slowly whereas that of an AR(p) model cuts off after lag p. AR and MA models are known to have some duality properties. These include:

1. A finite order AR model is equivalent to an infinite order MA model.
2. A finite order MA model is equivalent to an infinite order AR model.
3. The ACF of an AR model exhibits the same behaviour as the PACF of an MA model.
4. The PACF of an AR model exhibits the same behaviour as the ACF of an MA model.
5. An AR model is always invertible but is stationary if $\mathrm{A}(B) = 0$ has zeros outside the unit circle.
6. An MA model is always stationary but is invertible if $\mathrm{B}(B) = 0$ has zeros outside the unit circle.

Parametric parsimony consideration in model building entails preference for the mixed ARMA fit to either the pure AR or the pure MA fit. Stationarity and invertibility conditions for model (1) or (2) are that the equations $\mathrm{A}(B) = 0$ and $\mathrm{B}(B) = 0$ should have roots outside the unit circle respectively. Often, in practice, a time series is non-stationary. Box and Jenkins (1976) proposed that differencing of an appropriate data could render a non-stationary series $\{X_t\}$ stationary. Let degree of differencing necessary for stationarity be d. Such a series $\{X_t\}$ may be modelled as

$$(1 + \sum_{i=1}^{p} \alpha_i B^i)\nabla^d X_t = \mathrm{B}(B)\varepsilon_t \quad (3)$$

where $\nabla = 1 - B$ and in which case $\mathrm{A}(B) = (1 + \sum_{i=1}^{p} \alpha_i B^i)\nabla^d = 0$ shall have unit roots d times. Then differencing to degree d renders the series stationary. The model (3) is said to be an autoregressive integrated moving average model of orders p, d and q and designated ARIMA(p, d, q).

1.2 Seasonal ARIMA models.

A time series is said to be seasonal of order d if there exists a tendency for the series to exhibit periodic behaviour after every time interval d. Traditional time series methods involve the identification , unscrambling and estimation of the traditional components: secular trend, seasonal component, cyclical component and the irregular movement. For forecasting purpose, they are reintegrated. Such techniques could be quite misleading.

The time series $\{X_t\}$ is said to follow a multiplicative $(p, d, q)x(P, D, Q)_s$ *seasonal ARIMA model* if

$$\mathrm{A}(B)\Phi(B^s)\nabla^d \nabla_s^D X_t = \mathrm{B}(B)\Theta(B^s)\varepsilon_t \quad (4)$$

where Φ and Θ are polynomials of order P and Q respectively and s is the seasonal period. That is,

$$\Phi(B^s) = 1 + \phi_1 B^s + ... + \phi_P B^{sP}, \quad (5)$$

$$\Theta(B^s) = 1 + \theta_1 B^s + ... + \theta_Q B^{sQ}, \quad (6)$$

where the ϕ_i and θ_j are constants such that the zeros of the equations (5) and (6) are all outside the unit circle for stationarity and invertibility respectively. Equation (5) represents the autoregressive operator whereas (6) represents the moving average operator.

Existence of a seasonal nature is often evident from the time plot. Moreover for a seasonal series the ACF or correlogram exhibits a spike at the seasonal lag. Box and Jenkins (1976) and Madsen (2008) are a few authors that have written extensively on such models. A knowledge of the theoretical properties of the models provides basis for their identification and estimation. The purpose of this paper is to fit a seasonal ARIMA model to Nigerian Gross Domestic Product Series (NGDP).

2.Materials and Methods:

The data for this work are quarterly Gross Domestic Products(NGDP) from 1980 to 2007 obtainable from Abstracts of the National Bureau of Statistics of Nigeria.

2.1. *Determination of the orders d, D, p, P, q and Q:*

Seasonal differencing is necessary to remove the seasonal trend. If there is secular trend non-seasonal differencing will be necessary. To avoid undue model complexity it has been advised that orders of differencing d and D should add up to at most 2 (i.e. d + D < 3). If the ACF of the differenced series has a positive spike at the seasonal lag then a seasonal AR component is suggestive; if it has a negative spike then a seasonal MA component is suggestive. Box and Jenkins(1976) and Madsen(2008) have given a catalogue of seasonal models and their covariance structures for possible use for modelling.
As already mentioned above, an AR(p) model has a PACF that truncates at lag p and an MA(q)) has an ACF that truncates at leg q. In practice ±2/√n where n is the sample size are the non-significance limits for both functions.

2.2. *Model Estimation***:**

The involvement of the white noise terms in an ARIMA model entails a nonlinear iterative process in the estimation of the parameters. An optimization criterion like least error of sum of squares, maximum likelihood or maximum entropy is used. An initial estimate is usually used. Each iteration is expected to be an improvement of the previous one until the estimate converges to an optimal one. However,for pure AR and pure MA models linear optimization techniques exist (See for example Box and Jenkins (1976), Oyetunji (1985)). There are attempts to adopt linear methods to estimate ARMA models (See for example, Etuk(1996). We shall use Eviews software which employs the least squares approach involving nonlinear iterative techniques.

2.3. *Diagnostic Checking*:

The model that is fitted to the data should be tested for goodness-of-fit. We shall do some analysis of the residuals of the model. If the model is adequate, the autocorrelations of the residuals should not be significantly different from zero.

3.Results and Discussions:

The time plot of the original series NGDP in Figure 1 shows secular trend and marked seasonality that grows with time, which is an indication of a multiplicative seasonal model. Seasonal (i.e. 4-month) differencing of the series produces a series SDNGDP with much lesser trend and seasonality. See Figure 2 for the time plot. Non-seasonal differencing yields a series DSDNGDP with some seasonality but no trend (see Figure 3). Its ACF as shown in Figure 4 has the following features:

1. There are significant (positive) spikes at lags 4, 8 and 12 indicating seasonality of order 4 as well as a seasonal AR component.
2. The spikes immediately before and after each seasonal lag are of the same sign and equal.
3. As the lag increases autocorrelation generally decreases.

These features fit the description of the $(0, 0, 1)x(1, 0, 0)_s$ model (5.131) of page 132 of Madsen(2008) which is given by

$$(1 - \Phi B^s)\, X_t = (1 - \theta B)\varepsilon_t$$

With s=4, this translates into

$$X_t = \alpha_4 X_{t-4} + \beta_1\varepsilon_{t-1} + \varepsilon_t \qquad (7)$$

where X represents DSDNGDP. The estimation of the model summarized in Table 1 yields the model

$$X_t = 0.2356X_{t-4} - 0.9043\varepsilon_{t-1} + \varepsilon_t \qquad (8)$$

The estimation involved 7 iterations. We note that both coefficients are significantly different from zero, each being larger than twice its standard error. Moreover, there is considerable agreement between the actual and the fitted models as shown in Figure 5. The correlogram of the residuals in Figure 6 also depicts the adequacy of the model since all the autocorrelations are not significantly different from zero.

4.Conclusion:

The DSDNGDP series has been shown to follow the seasonal model (8). This model has been shown to be adequate.

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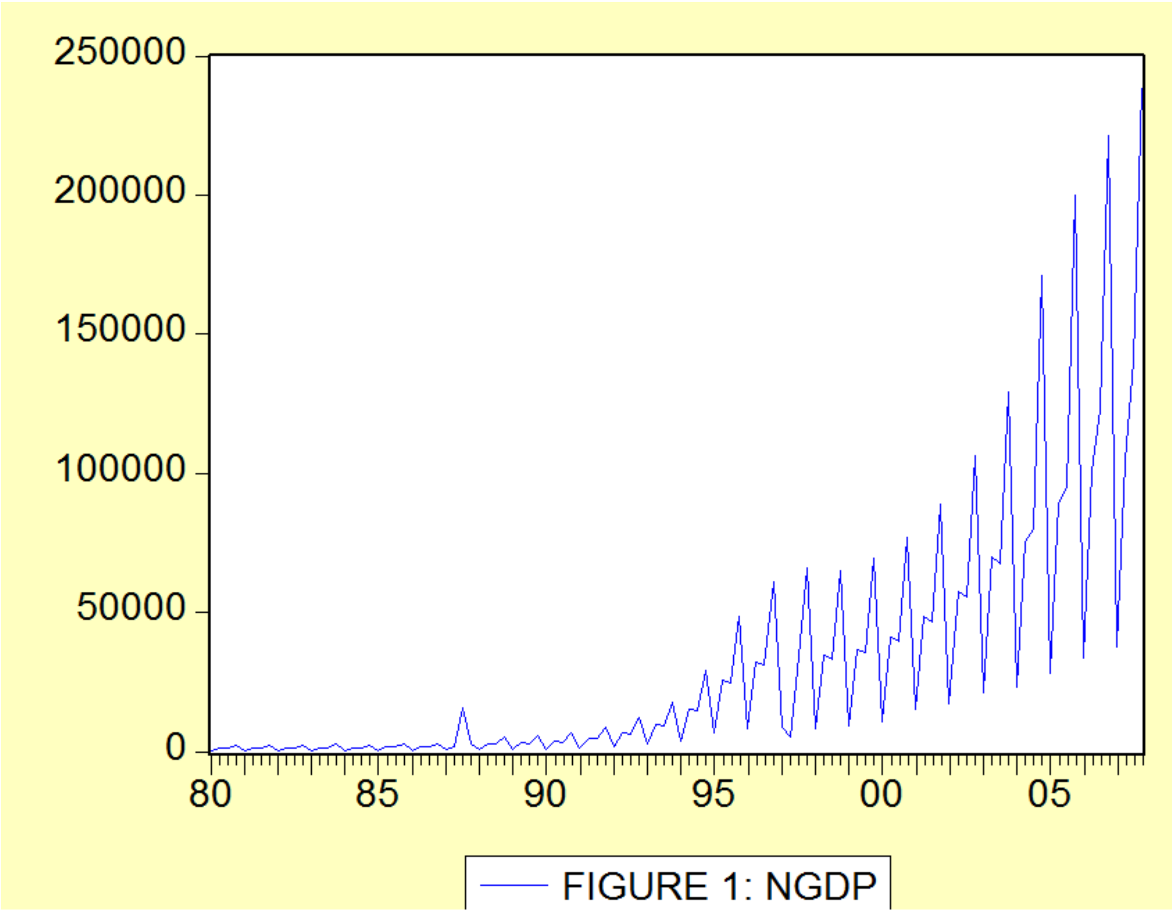
Table 1: Model Estimation

Dependent Variable: DSDNGDP
Method: Least Squares
Date: 01/15/12 Time: 17:53
Sample(adjusted): 1982:2 2007:4
Included observations: 103 after adjusting endpoints
Convergence achieved after 7 iterations
Backcast: 1982:1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(4)	0.235595	0.106221	2.217975	0.0288
MA(1)	-0.904255	0.046402	-19.48739	0.0000

R-squared	0.481697	Mean dependent var	167.0825
Adjusted R-squared	0.476565	S.D. dependent var	10022.60
S.E. of regression	7251.232	Akaike info criterion	20.63496
Sum squared resid	5.31E+09	Schwarz criterion	20.68612
Log likelihood	-1060.700	F-statistic	93.86654
Durbin-Watson stat	2.018328	Prob(F-statistic)	0.000000

Inverted AR Roots	.70	.00 -.70i
Inverted MA Roots	.90	


250000
200000
150000
100000
50000
0
80
85
90
95
00
05
FIGURE 1: NGDP

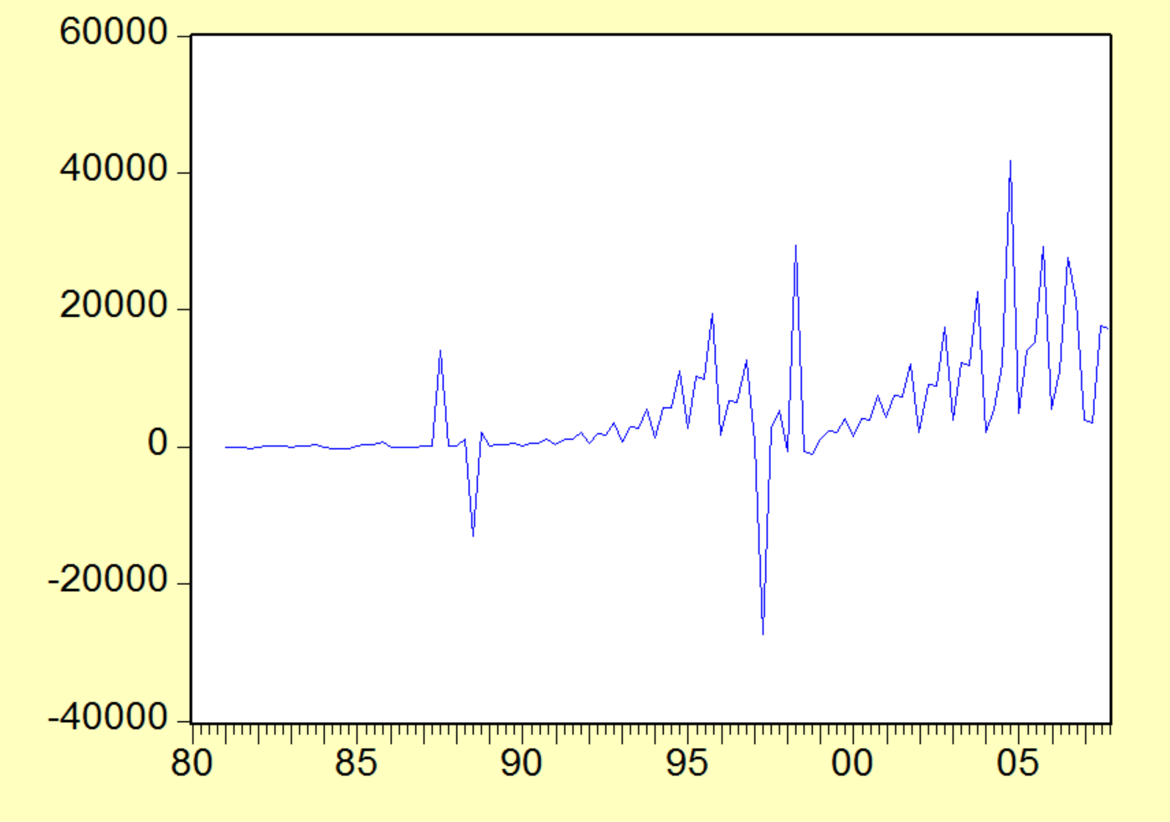


FIGURE 2: SDNGDP

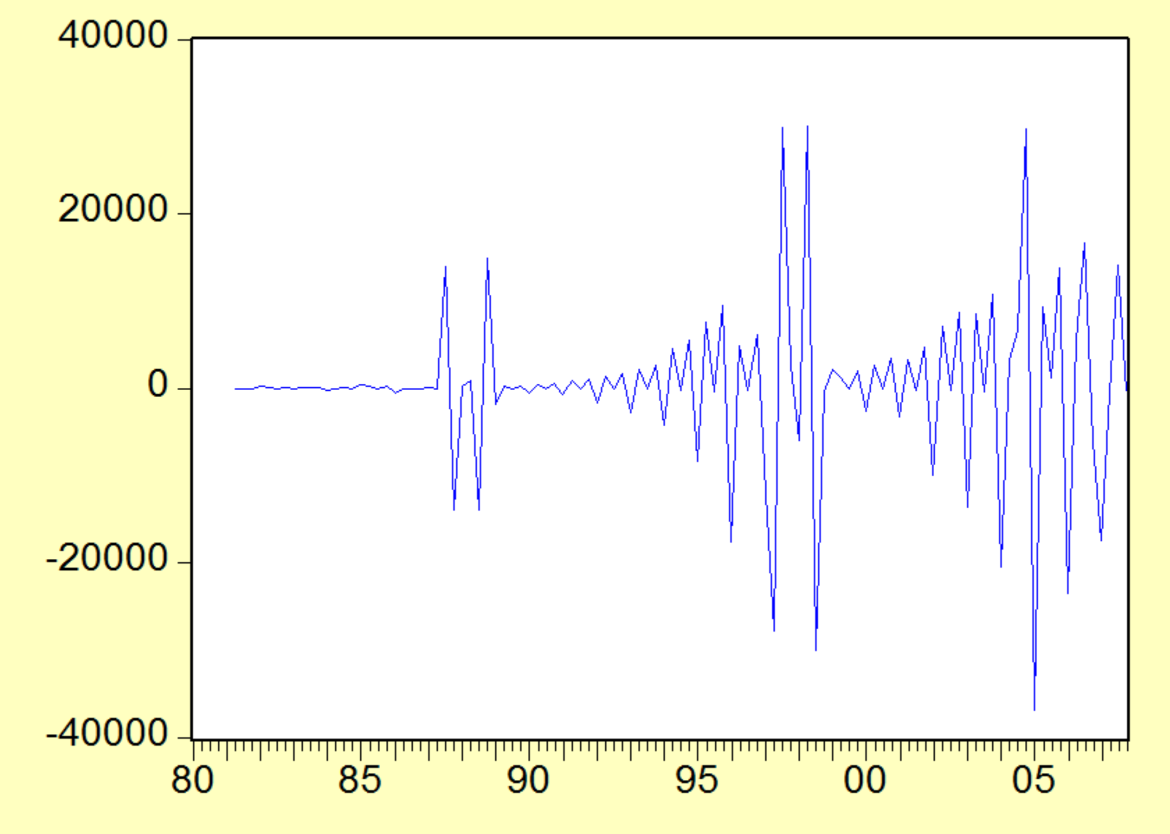


FIGURE 3: DSDNGDP

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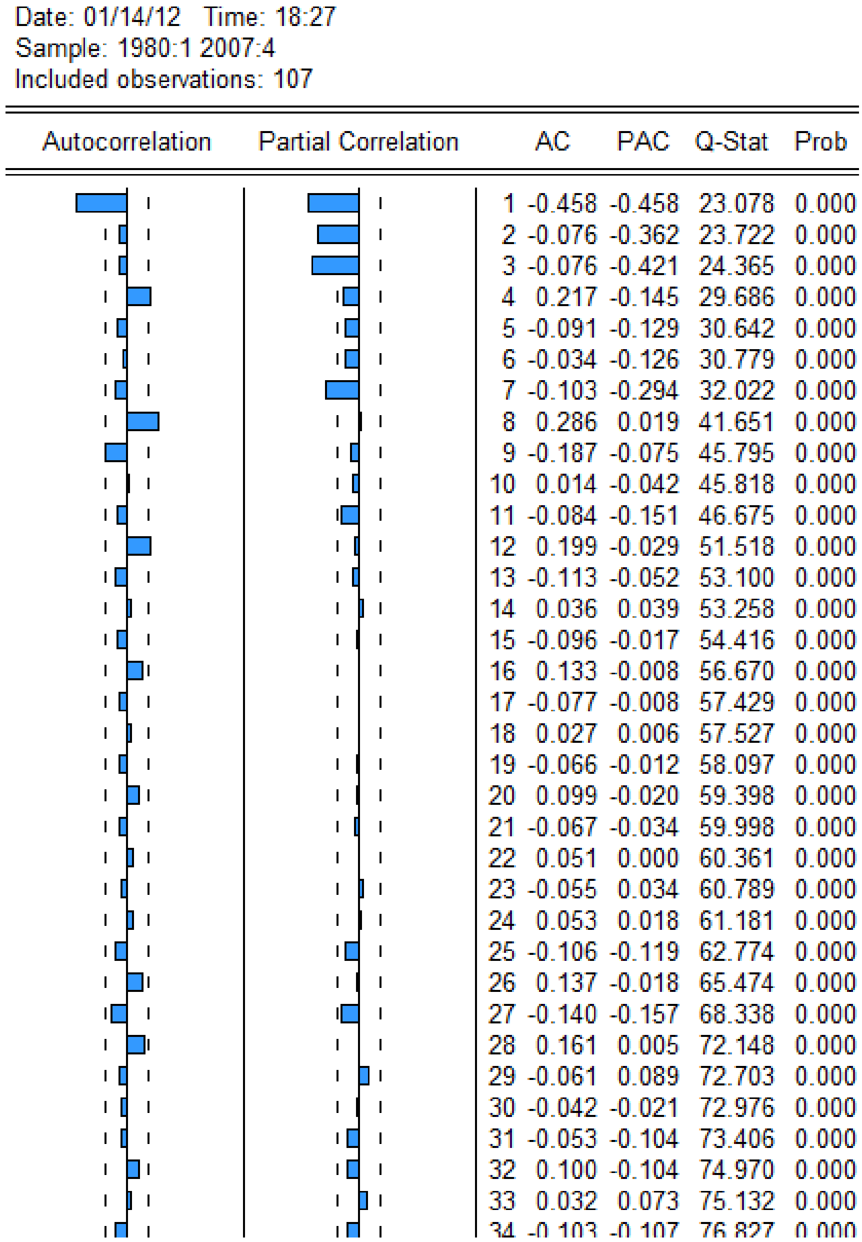

Date: 01/14/12 Time: 18:27
Sample: 1980:1 2007:4
Included observations: 107

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.458	-0.458	23.078	0.000
		2	-0.076	-0.362	23.722	0.000
		3	-0.076	-0.421	24.365	0.000
		4	0.217	-0.145	29.686	0.000
		5	-0.091	-0.129	30.642	0.000
		6	-0.034	-0.126	30.779	0.000
		7	-0.103	-0.294	32.022	0.000
		8	0.286	0.019	41.651	0.000
		9	-0.187	-0.075	45.795	0.000
		10	0.014	-0.042	45.818	0.000
		11	-0.084	-0.151	46.675	0.000
		12	0.199	-0.029	51.518	0.000
		13	-0.113	-0.052	53.100	0.000
		14	0.036	0.039	53.258	0.000
		15	-0.096	-0.017	54.416	0.000
		16	0.133	-0.008	56.670	0.000
		17	-0.077	-0.008	57.429	0.000
		18	0.027	0.006	57.527	0.000
		19	-0.066	-0.012	58.097	0.000
		20	0.099	-0.020	59.398	0.000
		21	-0.067	-0.034	59.998	0.000
		22	0.051	0.000	60.361	0.000
		23	-0.055	0.034	60.789	0.000
		24	0.053	0.018	61.181	0.000
		25	-0.106	-0.119	62.774	0.000
		26	0.137	-0.018	65.474	0.000
		27	-0.140	-0.157	68.338	0.000
		28	0.161	0.005	72.148	0.000
		29	-0.061	0.089	72.703	0.000
		30	-0.042	-0.021	72.976	0.000
		31	-0.053	-0.104	73.406	0.000
		32	0.100	-0.104	74.970	0.000
		33	0.032	0.073	75.132	0.000
		34	-0.103	-0.107	76.827	0.000

FIGURE 4: CORRELOGRAM OF DSDNGDP

FIG 5: Residual

Autocorrelation	Partial Correlation		AC	PAC
		1	-0.029	-0.029
		2	-0.092	-0.093
		3	-0.029	-0.035
		4	-0.082	-0.093
		5	0.005	-0.008
		6	-0.039	-0.058
		7	-0.009	-0.019
		8	0.186	0.171
		9	-0.089	-0.086
		10	-0.036	-0.015
		11	-0.004	-0.012
		12	0.123	0.148
		13	-0.026	-0.041
		14	-0.028	0.006
		15	-0.058	-0.065
		16	0.035	0.019
		17	-0.048	-0.039
		18	-0.038	-0.033
		19	-0.035	-0.060
		20	0.041	-0.017
		21	-0.025	-0.015
		22	-0.025	-0.032
		23	-0.055	-0.055
		24	-0.063	-0.121
		25	-0.069	-0.075
		26	0.097	0.082
		27	-0.035	-0.041
		28	0.100	0.074
		29	-0.058	-0.067
		30	-0.111	-0.087
		31	-0.051	-0.061
		32	0.062	0.080
		33	[illegible]	[illegible]

FIGURE 6: CORRELOGRAM OF THE RESIDUALS