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Towards Determining the Optimum Process Mean using an Exponential Distribution

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Abstract
Manufacturers are often faced with the problem of selecting the optimum process mean. Wen and Mergen (1999) used the unbalanced step loss function for measuring the cost of the non-conforming item and adopted a trade-off model for determining the optimum process mean. They assumed that the quality characteristic is normally distributed, the process variance constant and the process mean is unknown. This paper presents the modified Wen and Mergen model with a step loss function and piecewise function using an exponential distribution. The proposed model is a generalization of Wen and Mergens model.
Keywords: step-loss function, piecewise linear loss function, exponential distribution

1. Introduction
The selection of economic process mean is an important topic for statistical process control. The setting of the optimum process mean will directly affect the total cost to the society including the inspection cost, scrap or rework cost and the loss to the customer. Considerable researches in this area include Li (1997), Wu and Tang (1998), Lin and Chirng (1999), Wen and Mergen (1999), Li and Cherng (2000) Maghsoodloo and Li (2000), Philips and Cho (2000) and Li and Wu (2001).
Wen and Mergen (1999) used the unbalanced step loss function for measuring the cost of the non-conforming item. The normal quality characteristic, the constant process variance, and the unknown process mean are assumed in their model. They selected the optimum process mean based on minimizing the costs of falling below the lower specification limit (TL) and exceeding the upper specification limit (TU).
Cho and Leonard (1997) presented that the piecewise linear quality loss function for product is roughly proportional to the deviation of the quality characteristic from its specification limits. The linear loss function is usually applied in the filling/canning problem for determining the optimum manufacturing target Carlsson (1984), Golhar and Pollock (1998), Misiorek and Barnett (2000) and Lee et. Al (2001).
The lognormal distribution is usually adopted for describing the lifetimes of mechanical and electrical systems and other survival data. It is apparent that the exponential distribution is an important competitor to the lognormal, gamma or weibull distributions as models for non-negative phenomena.
This paper further presents the modified Wen and Mergen's (1999) model with exponential distribution . the step loss and the piecewise linear loss function of product are considered in the modified model.

2. Literature Review
Wen And Mergen's Model With Normal Distribution
By minimizing the unbalanced costs of out-of-specification, Wen and Mergen obtained the optimum process mean. There are three assumptions in their model.
1. The quality characteristic, X, is normally distributed with an unknown mean μ and a known variance σ^2
2. The quality characteristic nominal-is-best.
3. The target value, T, is the middle value of the specification, i.e., $T=(T_l + T_u)/2$

According to Wen and Mergen , the expected total loss per item is

$$C_{To} = D_u \int_{Tu}^{\infty} f(x)dx + D_L \int_{-\infty}^{Tl} f(x)dx = D_u\left[1 - \Phi\left(\frac{Tu-\mu}{\sigma}\right)\right] + D_L \Phi\left(\frac{Tl-\mu}{\sigma}\right) \quad (1)$$

Where
T_u = the upper specification limit
T_l= the lower specification limit
C_T = total loss per item due to exceeding the T_u and T_l
D_u = the monetary loss per item of exceeding T_u
D_l = the monetary loss per item of staying below T_l

$$f(x) = \frac{1}{\sqrt{(2\pi)}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \qquad (-\infty < x < \infty) \quad (2)$$

Φ (z) = the cumulative distribution function for the standard normal random variable with probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z}{2}} \quad , \qquad (-\infty < z < \infty) \quad (3)$$

In order to determine the optimum process mean μ, Wen and Mergen took the derivative of Equation 1. Since the second order derivative of equation 1 is positive, we set the first-order derivative equal to zero. The optimum μ is,

$$\mu^* = T - \frac{\sigma^2}{T_u - T_l} ln\left(\frac{D_u}{D_l}\right) \quad (4)$$

2. Modified Wen And Mergen's Model With Exponential Distribution

Assume that the quality characteristic X follows the exponential distribution. The probability density function of X is as follows

$$f(x) = \lambda e^{-\lambda x} \quad , \quad x \geq 0 , \quad (5)$$

Where λ is the parameter of the exponential distribution.

The cumulative distribution function, the expected value, and the variance of the exponential distribution, respectively, are

$$F(x;\lambda) = 1 - e^{-\lambda x} \quad , x \geq 0 \quad (6)$$

$$E(x) = \frac{1}{\lambda} \quad (7)$$

$$Var\ (x) = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right) \quad (8)$$

We now formulate the modified Wen and Mergen model with the step loss function of exponential characteristic for determining the optimum process mean.

Step Loss Function

The expected total loss per item of the modified Wen and Mergen model with the step loss function is

$$CT_1 = D_u \int_{Tu}^{\infty} f(x)dx + D_l \int_0^{Tl} f(x)dx = D_u\, e^{-(\lambda)T_u} + D_l(1 - e^{-(\lambda)T_l}) \quad (9)$$

Where

T_u = the upper specification limit

T_l= the lower specification limit

D_u = the monetary loss per item of exceeding T_u

D_l = the monetary loss per item of staying below T_l

If we assume a Weibull function then

$$CT_1 = D_u \int_{Tu}^{\infty} f(x)dx + D_l \int_0^{Tl} f(x)dx = D_u\left[e^{-[\frac{Tu-\gamma}{\beta}]^{\alpha}}\right] + D_l\left[1 - e^{-[\frac{Tl-\gamma}{\beta}]^{\alpha}}\right] \quad (10)$$

After taking the first order derivative we get the equation as follows

$$f^{'}(x) = \frac{\alpha}{\beta^{\alpha}}\left\{D_u\left[e^{-[\frac{Tu-\gamma}{\beta}]^{\alpha}}\right][(T_u-\gamma)^{\alpha-1}] - D_l\left[e^{-[(Tl-\gamma)/\beta]^{\alpha}}\right][(T_l-\gamma)^{\alpha-1}]\right\} \quad (11)$$

Equation 11 is not a closed function.

In order to determine the optimum μ, using an exponential function we take the derivative of equation 9. Since the second-order derivative of Equation 9 is positive, we set the first-order derivative equal to zero. The optimum parameter lambda is;

$$\lambda = \left(\frac{1}{T_u - T_l}\right) ln\left(\frac{D_u * T_u}{D_l * T_l}\right) \quad (12)$$

Hence, the optimum process mean

$$E(X) = \mu = \frac{1}{\lambda} \quad (13)$$

3. Piecewise Linear Loss Function

Cho and Leonard (1997) presented the piecewise linear quality loss function for the nominal-is best quality characteristic as follows:

$$L(x) = \begin{cases} 0 & if \;\; T_l \leq x \leq T_u \\ D_l(T_l - x), & if \;\; x \leq T_l \\ D_u(x - T_u) & if \;\; x > T_u \end{cases} \quad (14)$$

Where

D_l = the quality loss coefficient when the quality characteristic is less than T_l

D_u= the quality loss coefficient when the quality characteristic exceeds the T_u

The expected total loss per item of the modified Wen and Mergen model with the piecewise linear loss function is.

$$C_{T2} = \int_{Tu}^{\infty} D_u\,(x - T_u)\, f(x)dx + \int_0^{Tl} D_l\,(T_l - x)\, f(x)d\,(x)$$

$$\frac{D_u * e^{-(\lambda)Tu}}{\lambda} + \frac{D_l * e^{-(\lambda)Tl}}{\lambda} + D_l * T_l - \frac{D_l}{\lambda} = 0 \quad (15)$$

Since equation 15 is not closed, one can adopt the simple interval search method for obtaining the optimum parameter λ_* (the optimum process mean) $E(X) = \mu = \frac{1}{\lambda}$

4. Numerical Example

Assume that the quality characteristic follows an exponential distribution. Let the lower specification limit, $T_l = 2$ and the upper specification limit, $T_u = 4$. The monetary loss per item of falling below T_l is $D_l = 1.5$. The monetary loss per item of exceeding T_u is $D_u = 1$. We would like to determine the optimum process mean for minimizing the expected total loss per item.

Step Loss Function For Product

By solving equation (12) we have $\lambda^* = 0.144$. Hence, the optimum process mean $E(X) = 6.952$

Piecewise Linear Quality Loss Function For Product

By solving equation (15) we have $\lambda^* = 0.50209$. Hence the optimum process mean $E(X) = 2$

Step Loss Sensitivity Analysis

Figure 1: Graphical relationship between optimum values of λ^* and the upper specification limit T_u. As T_u increases λ^* increases initially at an increasing rate then gradually decreasing and then constant.

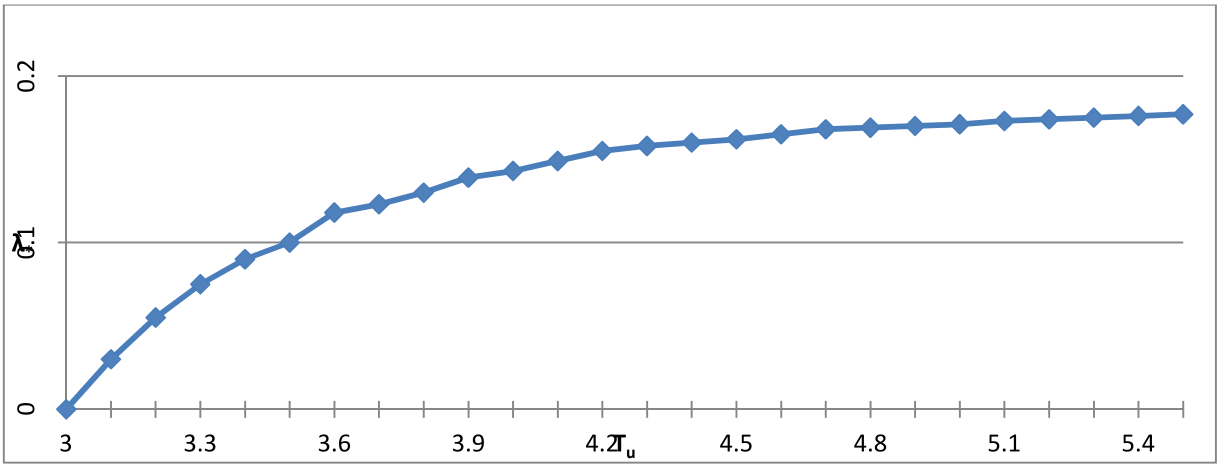


Figure 2: Graphical relationship between optimum values of λ^* and the upper specification limit T_l. λ^* decreases at a decreasing rate leveling off at higher values of T_l.

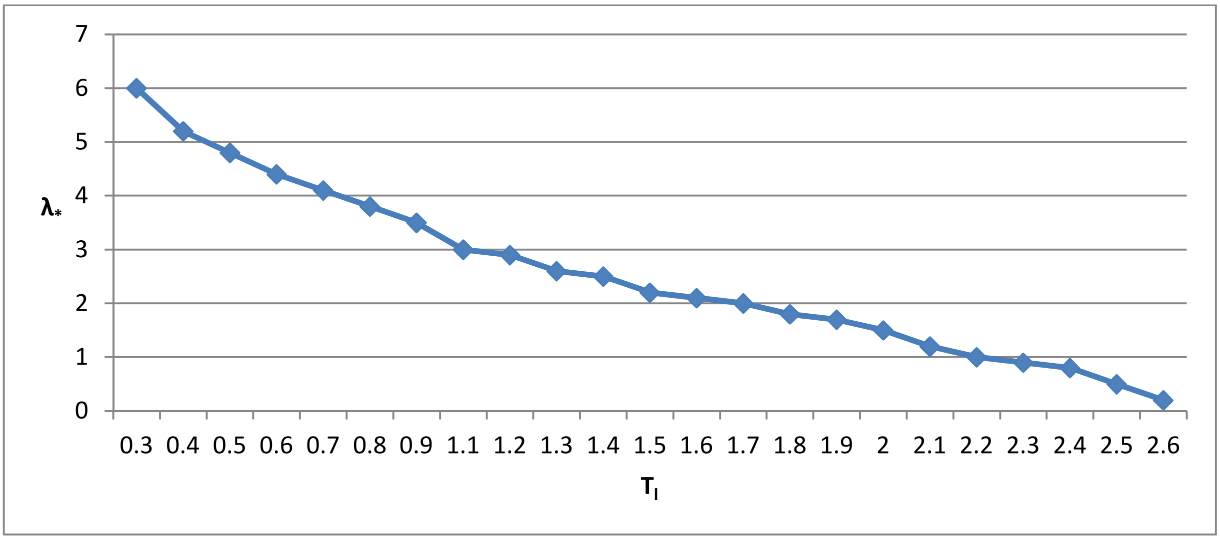


Figure 3: Graphical relationship between optimum values of λ^* and the monetary loss per item D_u. λ^* increases at a decreasing rate with increases in D_u.

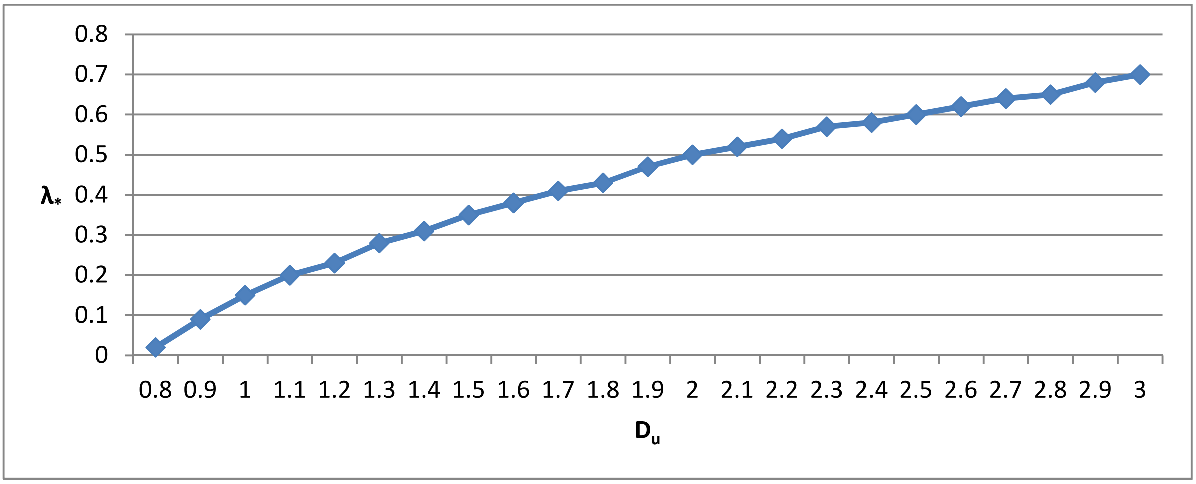
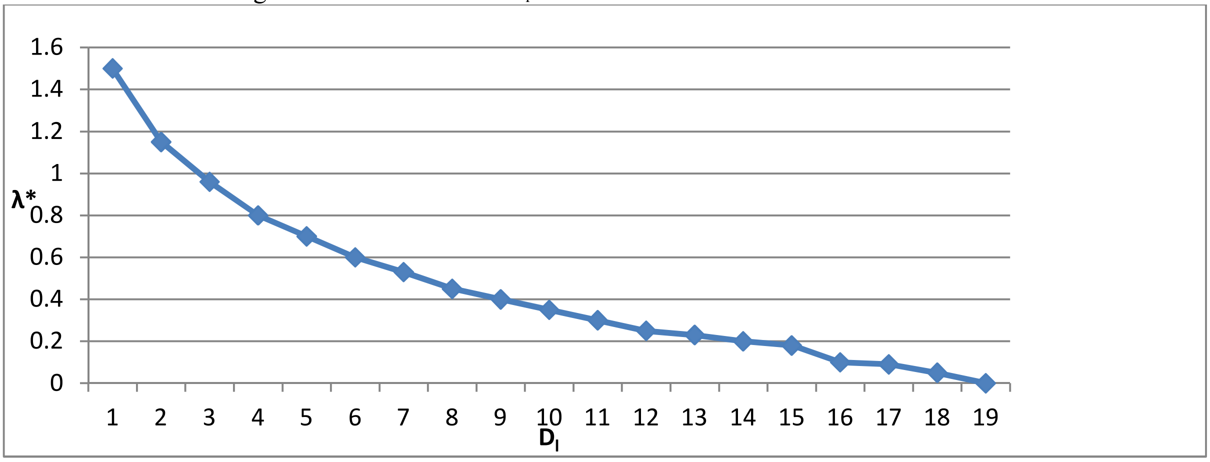


Figure 4: Graphical relationship between optimum values of λ* and the monetary loss per item D_l . λ* decreases at a decreasing rate with increases in D_l.



Piecewise Sensitivity Analysis

Figure 5: Graphical relationship between optimum values of λ* and the upper specification limit T_u. λ* decreases in varying rates with increases in T_u.

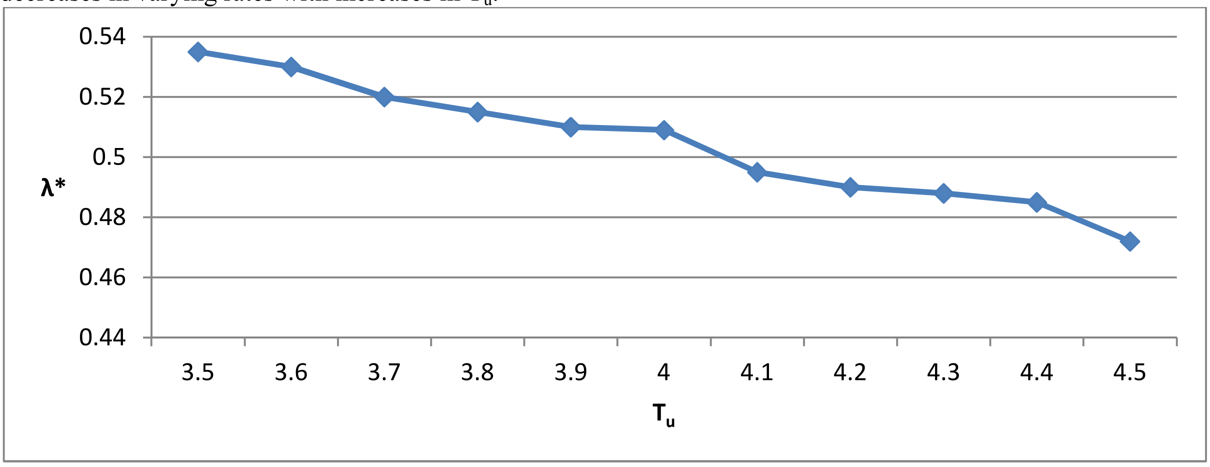


Figure 6: Graphical relationship between optimum values of λ* and the monetary loss per item D_u. λ* increases with increases with increases in D_u and decreasing gradually.

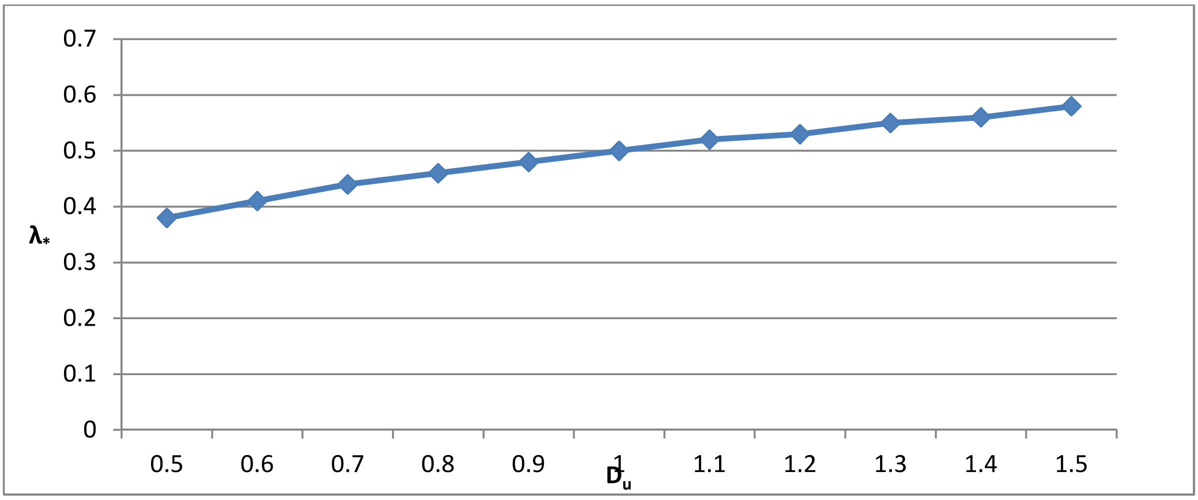


Figure 7: Graphical relationship between optimum values of λ* and the upper specification limit T_u. λ* decreases at a decreasing rate with increases in T_u.

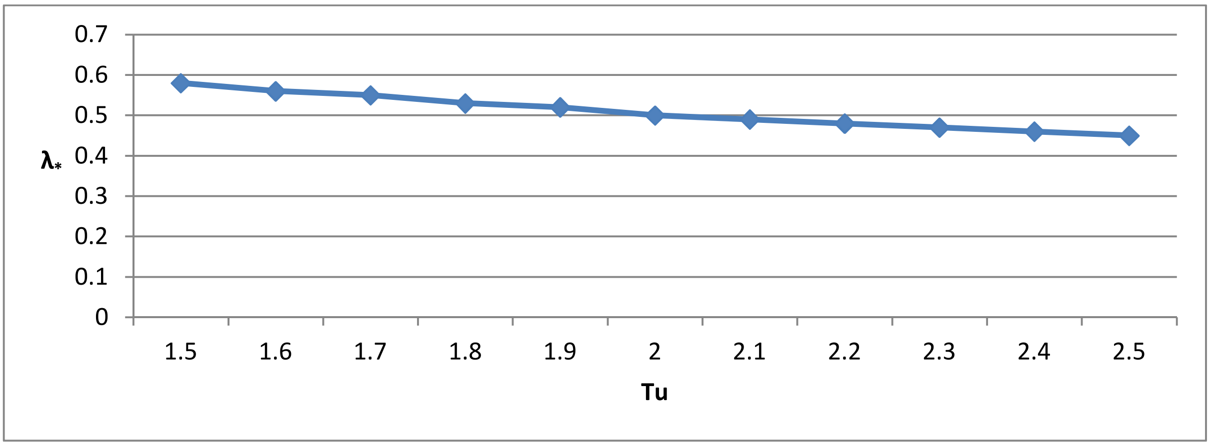
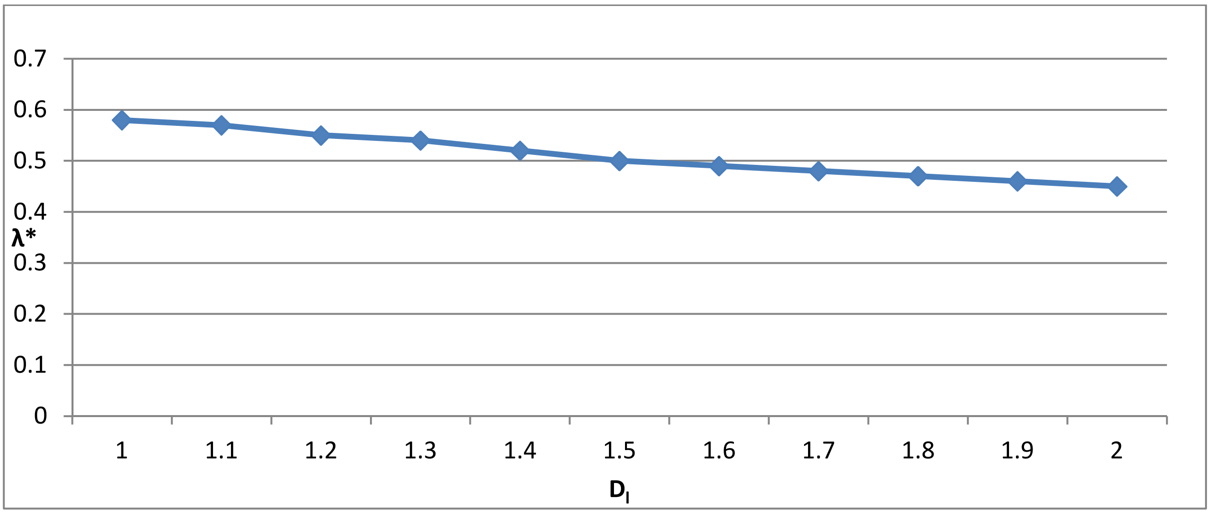


Figure 6: Graphical relationship between optimum values of λ* and monetary loss per item D_l. λ* decreases steadily with increases in D_l.



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