

Estimation the Reliability of the Productivity Machines and it's Availability for the Exponential Distribution A Case Study of AL-Qastal Factory for Producing Juice and Carbonated Soft Drinks in Jordan

Dr. Hasan Yasien Touama*, Dr. Mazen Hasan Basha
Faculty of Economics and Administrative Sciences, Zarqa University, Jordan
*E-mail: dralfaisel@yahoo.com
E-mail :mazenbasha1959@hotmail.com

Abstract:

This study aimed to apply the parametric methods in order to estimate the reliability of productivity machines and it's availability, also to estimate the failure rate function in AL-Qastal factory for producing juice and carbonated soft drinks in Jordan.

To achieve the study objectives, the study depend on the machines work times and the repair times of the machines, during the period (1/10/2013 – 31/3/2014).

The study findings a number of results, including:

- 1- There exist a statistically significant (positive) relationship at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $\hat{R}(t)$ for the productivity machines and it's availability $A(t)$.
- 2- There exist a statistically significant (negative) relationship at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $\hat{R}(t)$ for the productivity machines and the estimated values of the failure rate function $\hat{h}(t)$.
- 3- There are no statistically significant differences at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $\hat{R}(t)$ for the productivity machines.

Upon the foregoing results, the study reached to a number of conclusions and recommendations.

Keywords: Exponential Distribution, Reliability, Availability, Failure Rate.

1. Introduction:

The productive operation in any organization is based on a certain set of essential inputs (productivity machines, workers and raw materials...). The productivity machines are the most important part thereof, it is doubtless that these machines or any part thereof are vulnerable to breakdown or (technical failure), which leads to materialistic loss and waste of time beside other damages.

Thereupon, reliability evaluation of any machine must be an important base of these machines; for the reliability knowledge of each machine takes us, at the end of that, towards the proper planning of improvement and increase of (quality, productivity and efficiency of maintenance programs and productivity age), so as to produce products and services of high reliability, in harmony with the expectations and needs of the consumer which realizes the competitive advantage of the organization.

In order to show the role of the productive machines reliability and the importance thereon in syrups factory, the study had dealt with the employment of the laboratory ways in productive machines reliability in syrups factory of one of our productive organizations.

The reliability is a probabilistic and statistics term used to analyze the random variables of the positive values and represented with time (T) until (Time to Failure) for any machine (equipment) takes place.

Therefore, the reliability throughout (t), is defined by the potentiality of the machine (equipment) during the period [0, t) without any breakdown (failure).

2. The Methodology:

2.1. The Study Problem:

The entrance of the Hashemite Kingdom of Jordan in the different International treaties, as Free Commercial Treaty with European States and the Arab Protocol, has been reflected on our national institutions performance, which lead to the fact that what had been produced did not find its way to marketing, the thing which caused the decrease sales of the studied institution as an example without being exclusive by 80 % for the year 1998, because of the activation of the Arab protocol and the competition of the Arab product to the Jordanian product from one side, and our productive organization at present faces a case of prescription in the essential industrial and technological modernization operation on the second side, thereupon, the modern scientific bases for the ability of survival. Or ending of certain kinds of machines should be set, i.e. the formation of the industrial and technological base in the way of starting with scientific computation operation of reliability of each kind of machines rate, and each organization of our productive organization, and pursuance of the aforesaid ideas, the

study problem could be briefed as follows:

a. There exist a case of modernization and development of the essential and technological base in AL-Qastal factory for producing juice and carbonated soft drinks in Jordan, for plenty of productivity machines face a condition of economical extinction resulting from the international development of technology.

b. Lack of the operation scientific approach use to estimation the reliability of the productivity machines in them aforesaid syrups factory.

2.2. The Study Objects:

The objects of this study are given as follows:

a- To identify the parametric methods in general, which used to estimate the reliability of the productivity machines and it's availability, in AL-Qastal factory for producing juice and carbonated soft drinks in Jordan.

b- To measure the relationship between the estimated values of the reliability functions for the productivity machines and it's availability and the estimated values of the failure rate function.

c- To measure the differences between the estimated values of the reliability functions for the productivity machines.

d- To offer some importance conclusions and recommendations to the decision making in AL-Qastal factory for producing juice and carbonated soft drinks in Jordan.

2.3. The Study Hypotheses:

The study hypotheses are given in a null form (H_0) as follows:

H₀₁: There is no statistically significant relationship at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $R^{\wedge}(t)$ for the productivity machines and it's availability $A(t)$ and the estimated values of the failure rate function $h^{\wedge}(t)$.

H₀₂: There are no statistically significant differences at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $R^{\wedge}(t)$ for the productivity machines.

3. The Theoretical Part

3.1. The Reliability Function R(t):

The reliability is defined as a probability concept to be used in analyzing the random variable of positive values represented by time ($T \geq 0$) until (time-to-failure) of any equipment, and it has a function of density probability $f(t)$ and the reliability of the equipment through out time (t) is $R(t)$ which takes the following formula (Smith 1976):

$$R(t) = P(T > t) \\ = 1 - \int_0^t f(u) du$$

Generally, the Reliability function $R(t)$ is a decreasing function continuing from left within the period $[0, \infty)$ (Johnson & Christensen, 1988).

3.2. The Failure Rate Function h(t):

The failure rate is defined as a ratio of the failed units through a certain temporal period to that period which remained till time (t), the failure rate function $h(t)$ takes the following formula (Kapur & Laberson, 1977).

$$h(t) = f(t) / R(t)$$

3.3. The Probability Density Function of the Machine Work Times:

The probability density function $f(t)$ of the machines work times till that occurrence of failure is distributed as exponential distribution and takes the following formula (Michael 1991).

$$f(t) = (1/\lambda) e^{-t/\lambda}, t > 0 \quad \dots(1)$$

Whereas:

λ : Scale parameter, and defined as a (Mean Time Between Failures -MTBF).

3.4. The Maximum Likelihood Estimation Method (MLE):

This part used the Maximum Likelihood Estimation Method (MLE) to estimate the parameter (λ), the reliability function $R(t)$ and the failure rate function $h(t)$ of the exponential distribution, as follows:

3.4.1. Estimation the Parameter of the Exponential Distribution:

Throughout the Maximum Likelihood Estimation Method (MLE), we can get a formula for the estimator of the exponential distribution parameter (λ), depend on the probability density function $f(t)$ as in the relation (1). Thereupon the estimator of maximum likelihood of the parameter (λ) is given as follows (Pollock & Conroy 1989):

$$\hat{\lambda} = \sum t_i / n \\ = t^- \\ = \text{MTBF} \quad \dots(2)$$

3.4.2. Estimation the Reliability Function R(t):

The reliability function $R(t)$ of the machine work times which is distributed as exponential distribution, takes the following formula:

$$R(t) = e^{-t/\lambda}, t > 0$$

Thereupon, the maximum likelihood estimation of the reliability function $R(t)$, is given as follows (Sinha & Kale 1980):

$$R^{\wedge}(t) = e^{-t/\lambda^{\wedge}}, t > 0 \quad \dots(3)$$

The above estimator $R^{\wedge}(t)$ of the reliability function is considered to be a biased estimator, and by the theory (Lehmann & Sheffe), we can get the unbiased estimator of the reliability function $R^{\wedge}(t)$ as follows (Mood, Graybill & Boes, 1974):

$$R^{\wedge}(t) = [1 - (1/\sum t_i)]^{\wedge(n-1)}, t > 0 \quad \dots(4)$$

The above estimator $R^{\wedge}(t)$ has a minimum variance unbiased estimator (MVUE) of the reliability function $R(t)$.

3.4.3. Estimation the Failure Rate Function $h(t)$:

The failure rate function $h(t)$, which considered a constant value of the exponential distribution takes the following formula:

$$h(t) = 1 / \lambda$$

Thereupon, the maximum likelihood estimator of the function $h(t)$ is written as follows:

$$h^{\wedge}(t) = 1 / \lambda^{\wedge} = n / \sum t_i \quad \dots(5)$$

The estimator $h^{\wedge}(t)$ in relation (5) is considered a biased estimator, whereas the unbiased estimator of the failure rate function is written as follows:

$$h^{\wedge}(t) = (n - 1) / \sum t_i \quad \dots(6)$$

The above estimator $h^{\wedge}(t)$ has a minimum variance unbiased estimator (MVUE) for the failure rate function $h(t)$.

3.4.4. Estimation the Availability $A(t)$:

We can calculate the availability $A(t)$ for each machine depending on the mean time between failure (MTBF) and the Mean Time To Repair (MTTR), and by using the following formula:

$$A(t) = MTBF / (MTBF + MTTR) \quad \dots(7)$$

Whereas:

MTTR: Mean Time To Repair.

4. The Applied Part:

4.1. Collection Data Method:

The study data depends upon the (work and repair) times of the machines in (Al-Qastal) factory for Producing Juice and Carbonated Soft Drinks during the period (1/10/2013 – 31/3/2014)(Appendix (1)). The factory offers packaging solutions, filling machines and processing solutions for dairy, beverages, cheese, ice-cream and prepared food, including distribution tools like accumulators, cap applicators, conveyors, crate packers, film wrappers, line controllers and straw. With head offices in Lund, Sweden and Lausanne, Switzerland .

And in our Factory (Al-Qastal) for Producing Juice and Carbonated Soft Drinks, we have five production lines: three of them (TBA/19-200 ML) which have capacity (7500) packages/hour for each line, (TBA/19-250ML) which have capacity (7500) packages/hour and (Tetra Pak A1 -150ML) which have capacity (10500) packages/hour, and contains three kinds of machines for any production line which are (Tetra Cardboard Packer 70, Tetra Straw Applicator 21 and Tray Shrink 51).

Thereafter, the actual times for the machines work to be studied was registered, till the damage breakdown occurrence, where the machine damage should be repaired and return for work.

4.2. Statistical Analysis:

4.2.1. The Mean Time between Failures (MTBF):

We can find the mean time between failures (MTBF) for each machine by using the relation (2), as shown in table (1) as follows:

Table 1. The Mean Time Between Failures (MTBF) for the Machines

Production Lines	Machine	MTBF = λ^{\wedge} / hrs
Line (1) TBA/19-200 ML	Tetra Cardboard Packer 70: M ₁	616.7
	Tetra Straw Applicator 21: M ₂	360.4
	Tray Shrink 51: M ₃	952.5
Line (2) TBA/19-250ML	Tetra Cardboard Packer 70: M ₄	452.5
	Tetra Straw Applicator 21: M ₅	633.7
	Tray Shrink 51: M ₆	633.0
Line (3) Tetra Pak A1 -150ML	Tetra Cardboard Packer 70: M ₇	955.5
	Tetra Straw Applicator 21: M ₈	463.3
	Tray Shrink 51: M ₉	625.3

4.2.2. Estimation the Reliability Function $R^{\wedge}(t)$:

Through out the relation (4) we got the minimum variance unbiased estimator (MVUE) of the reliability function $R^{\wedge}(t)$ of the machines and the production lines R^{\wedge}_L , shown in table (2) as follows:

Table 2. The Estimated Reliability Functions $\hat{R}(t)$ for the Machines

Production Lines	Machine	$\hat{R}(t)$	\hat{R}_L
Line (1) TBA/19-200 ML	Tetra Cardboard Packer 70: M_1	0.999	0.996
	Tetra Straw Applicator 21: M_2	0.998	
	Tray Shrink 51: M_3	0.999	
Line (2) TBA/19-250ML	Tetra Cardboard Packer 70: M_4	0.998	0.996
	Tetra Straw Applicator 21: M_5	0.999	
	Tray Shrink 51: M_6	0.999	
Line (3) Tetra Pak A1 -150ML	Tetra Cardboard Packer 70: M_7	0.999	0.996
	Tetra Straw Applicator 21: M_8	0.998	
	Tray Shrink 51: M_9	0.999	

It is clear throughout the results of table (2), that the machines reliability $\hat{R}(t)$ are equals approximately, also the production lines reliability \hat{R}_L are equals.

4.2.3. Estimation the Failure Rate Function $h(t)$:

We can estimate the unbiased estimator of the failure rate function $h(t)$ for all machines using the relation (6), shown in the table (3) as follows:

Table 3. The Estimated Failure Rate Functions $h(t)$ for the machines

Production Lines	Machine	$h(t)$
Line (1) TBA/19-200 ML	Tetra Cardboard Packer 70: M_1	0.0016
	Tetra Straw Applicator 21: M_2	0.0022
	Tray Shrink 51: M_3	0.0010
Line (2) TBA/19-250ML	Tetra Cardboard Packer 70: M_4	0.0017
	Tetra Straw Applicator 21: M_5	0.0011
	Tray Shrink 51: M_6	0.0011
Line (3) Tetra Pak A1 -150ML	Tetra Cardboard Packer 70: M_7	0.0005
	Tetra Straw Applicator 21: M_8	0.0016
	Tray Shrink 51: M_9	0.0011

The results of the table (3), refers to the machine (Tetra Cardboard Packer 70: M_7) of the production line (Tetra Pak A1 -150ML) has lowest failure rate (0.0005) comparing thereof with the failure rate of the another machines in the production lines.

4.2.4. Estimation the Availability $A(t)$:

We can estimate the Availability function $A(t)$ for all machines using the relation (7), shown in the table (4) as follows:

Table 4. The Estimated Functions of Availability $A(t)$ for the machines

Production Lines	Machine	$A(t)$
Line (1) TBA/19-200 ML	Tetra Cardboard Packer 70: M_1	0.946
	Tetra Straw Applicator 21: M_2	0.924
	Tray Shrink 51: M_3	0.985
Line (2) TBA/19-250ML	Tetra Cardboard Packer 70: M_4	0.925
	Tetra Straw Applicator 21: M_5	0.985
	Tray Shrink 51: M_6	0.984
Line (3) Tetra Pak A1 -150ML	Tetra Cardboard Packer 70: M_7	0.991
	Tetra Straw Applicator 21: M_8	0.954
	Tray Shrink 51: M_9	0.966

It is clear throughout the results of table (4), that the availability of the machine (Tetra Cardboard Packer 70: M_7) of the production line (Tetra Pak A1 -150ML) is considered to be the highest and equals to (0.991), comparing thereof with the availability of the another machines in the production lines.

5. Test the Hypothesis:

Before starting to test the study hypothesis, some tests to be made on the study data must be verified, these tests are given as follows:

a- The Variance Inflation Factors (VIF) Test:

This test was used to verify whether Multicollinearity existed between the independent variables or not. Table (5) shows the final results of (VIF) test:

Table 5. The results of Variance Inflation Factors (VIF) Test

No.	Variables	Tolerance	VIF
1	Reliability $R^{\wedge}(t)$	0.911	1.098
2	Failure Rate $h^{\wedge}(t)$	0.411	2.433

The results in table (5), show that there is no Multicollinearity between the variables. This is asserted by the values of (VIF) for the variables (reliability and failure rate) which are (1.098, 2.433) respectively, where all these values are less than the critical value of the test which is (5).

b- One-Sample Kolmogorov-Smirnov Test:

This test was used to verify whether data of the study variables are followed the normal distribution or not, through the following statistical hypothesis:

H₀: The study data are followed the normal distribution.

H₁: The study data aren't followed the normal distribution.

The test was used to verify whether the study data are followed the normal results of One-Sample Kolmogorov-Smirnov Test are shown in table (6) below:

Table 6. The results of One-Sample Kolmogorov-Smirnov Test

The variables	No. of observations	Z _{cal.} value	Sig.
Reliability $R^{\wedge}(t)$	9	1.243	0.091
Failure Rate $h^{\wedge}(t)$	9	0.682	0.741
Availability A(t)	9	0.722	0.674

The tabulated (Z) value is (1.96) at the significance level ($\alpha = 0.05$).

The results in table (6), shows that all the calculated values of (Z) for the variables are less than tabulated (Z) value which is (1.96), and that all the statistical significance values (Sig.) are greater than the significance level ($\alpha = 0.05$). Depend on the above results, the null hypothesis (H₀) (The study data are followed the normal distribution) has been accepted.

Based on the foregoing, and after it has been proven that there is no Multicollinearity between the variables and that the study data are followed the normal distribution, the study hypothesis related to measure the (relationships and differences), will be tested as follows:

5.1. Test the first hypothesis:

H₀₁: There is no statistically significant relationship at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $R^{\wedge}(t)$ for the productive machines and it's availability A(t) and the estimated values of the failure rate function $h^{\wedge}(t)$.

In order to test the true of the first hypothesis, the correlation coefficient (Pearson) has been used, as shown in table (7) below:

Table 7. The Correlations Coefficients (Pearson) and it's Significant

Variables	Reliability $R^{\wedge}(t)$	Failure Rate $h^{\wedge}(t)$	Availability A(t)
Reliability $R^{\wedge}(t)$	1	- 0.768 *	0.795*
	-	(0.016)	(0.010)
Failure Rate $h^{\wedge}(t)$		1	- 0.915 **
		-	(0.001)
Availability A(t)			1
			-

The values between parentheses refer to the significant correlations coefficients.

* Correlation is significant at the ($\alpha = 0.05$) level (2-tailed).

** Correlation is significant at the ($\alpha = 0.01$) level (2-tailed).

The results in table (6), refers to:

- 1- There exist a statistically significant (positive) relationship at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $R^{\wedge}(t)$ for the productivity machines and it's availability A(t).
- 2- There exist a statistically significant (negative) relationship at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $R^{\wedge}(t)$ for the productivity machines and the estimated values of the failure rate function $h^{\wedge}(t)$.
- 3- There exist a statistically significant (negative) relationship at the significance level ($\alpha = 0.05$), between the estimated values of the failure rate function $h^{\wedge}(t)$ for the productivity machines and the availability A(t).

5.2: Test the second hypothesis:

H₀₂: There are no statistically significant differences at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $R^{\wedge}(t)$ for the productivity machines.

To test the differences between the estimative values of the reliability functions $R^{\wedge}(t)$ of the productivity

machines in (Al-Qastal) factory for Producing Juice and Carbonated Soft Drinks which are to be studied, **Friedman Test** was used, and the results of this test shown in table (8) as follows:

Table 8. The results of Friedman Test

Production lines	Mean Rank	N	Chi-Square (χ^2)	df.	Sig.
TBA/19-200 ML	2	3	0.000	2	1.000
TBA/19-250ML	2				
Tetra Pak A1 -150ML	2				

The tabulated (χ^2) value is (5.991) at (df=2), and the level ($\alpha = 0.05$).

By comparing the calculated value of Chi-Square (χ^2) amounting to (0.000) in table (8), with it's tabulated value (5.991) at the significant level ($\alpha = 0.05$), it appeared that the calculated value is less than the tabulated value, therefore the hypothesis (H_{02}) is accepted, which says: "There is no statistically significant differences at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $R^{\wedge}(t)$ for the productivity machines.

6. Conclusions and Recommendations

6.1. Conclusions:

- a-** The results of parameters estimation (λ^{\wedge}) of the machines work times till the occurrence of the damages mentioned in table (1) of all machines to be studies should that the machine of type (Tetra Cardboard Packer 70 / Line 3) is the best depend on its (mean time between failure = 955.5 hrs), and it was of the first rank.
- b-** The results shows that the machine reliability ($R^{\wedge}(t) = 0.999$) type (Tetra Cardboard Packer 70 / Line 3), which is higher of the another machines in the production lines.
- c-** The results refers to the machine failure rate ($h^{\wedge}(t) = 0.0005$) type (Tetra Cardboard Packer 70 / Line 3), which is lower of the another machines in the production lines.
- d-** The results shows that the machine availability ($A(t) = 0.991$) type (Tetra Cardboard Packer 70 / Line 3), which is higher of the another machines in the production lines.
- e-** There exist a statistically significant (positive) relationship at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $R^{\wedge}(t)$ for the productivity machines and it's availability $A(t)$. Also, there exist a statistically significant (negative) relationship at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $R^{\wedge}(t)$ for the productivity machines and the estimated values of the failure rate function $h^{\wedge}(t)$.
- f-** There are no statistically a significant differences at the significance level ($\alpha = 0.05$), between the estimated values of the reliability functions $R^{\wedge}(t)$ for the productivity machines.

6.2. Recommendations

- a- Adoption of an information system for the failures of the productivity machines, maintain a file about each machine or production line and feeds through daily or weekly reports for easy review information on cases of failure of any machine with a view to their adoption in the quantitative analysis of random failures, ensuring prolongation of the useful life of the machine.
- b- To define the machines and equipments specifications which would be purchased or imported from international country origin out matched by the designs and high techniques, therefore which leads to decrease the production costs and increasing the productivity from one side, and lessening the period of the machines destruction from another side .
- c- The study suggestion over production organizations the necessity of running a new special administration for reliability called the (Reliability Administration), as a result of the development nature in the reliability field.

References:

- 1- Aarset, M. V., (1987). How to Identify Bathtub Hazard Rate, *IEEE Trans. Rel.*, Vol. R-36, No. 1, pp. 106–108, Apr.
- 2- Barlow, R. E. and Proschan, F., (1981) . *Statistical Theory of Reliability and Life Testing*, Begin With, Silver Spring, MD .
- 3- Bain, L.J. and Engelhardt, M., (1991). *Statistical Analysis of Reliability and Life-Testing Models, Theory and Methods*, Second Edition, New York, Dekker.
- 4- Bhote, K.A., (2004). *World Class Reliability using MEOST to Make it Happens*, Amacom.
- 5- Eddy, O. N., (2007). *Applied Statistics in Designing Special Organic Mixtures*, Applied Sciences , 9, pp. 78-85.
- 6- Gbogboade A. and Charlotte E. R., (2003). *Evaluating the Reliability and Validity of Three Tools to Assess the Quality of Health on the Internet*, Patient Education and Counseling, Vol. 50(2), pp.151-155 .
- 7- Ghitany, M.E., (2006). *Reliability Properties of Extended Linear Failure Rate Distributions*, Probability in the Engineering and Information Sciences, 21 (2006), 441 - 450.

- 8- Heizer, J. and Render, B., (1996) . *Production, Operations Management Strategic and Tactical Decisions* . Prentice – Hall, Inc .
- 9- Janice M., Michael B., Maria M., Karin O. and Jude S., (2002) . *Verification Strategies for Establishing Reliability and Validity in Qualitative Research* , International Journal of Qualitative Methods, Vol. 1(2).
- 10- Johnson, W. and Christensen, R., (1988). *Modeling Accelerated Failure Time with a Dirichlet Process*. Biometrika, Vol. 75(4).
- 11- John, A., (2005). Earliest Uses of Symbols in Probability and Statistics, Electronic Document, Retrieved March 20.
- 12- Kapur, K. C. and Lamberson, L. R., (1977). *Reliability in Engineering Design*, John Wiley, New York.
- 13- Lawless, J.F., (2003). *Statistical Models and Methods for Life-time Data*, John Wiley and Sons, New York .
- 14- Lai, C.D., Xie, M. and Murthy, D.N., (2001). Bathtub Shaped Failure Rate Distributions, *Handbook in Reliability*, Balakrishnan, N. and Rao, C.R. Eds., Vol. 20, pp. 69–104.
- 15- Lin, C.T., Wu, S.J. and Balakrishnan, N., (2006). *Monte Carlo Methods for Bayesian Inference on the Linear Hazard Rate Distribution*, Communications in Statistics- Theory and Methods, 35, pp. 575-590.
- 16- Meeker, W. and Escobar, L., (1998) . *Statistical Methods for Reliability Data*, John Wiley .
- 17- Michael, A., (1991). *Reliability for Engineers: An Introduction*, Macmilian Education LTD.
- 18- Mood, A. M., Graybill, F. A. and Boes, D. C., (1974) . *Introduction to Theory of Statistics* . McGraw – Hill International Book Company, London .
- 19- Nelson, W., (2004) . *Accelerated Testing, Statistical models, Test Plans and Data Analysis*, John Wiley.
- 20- Pollock and Conroy, (1989). *Estimation and Analysis of Survival Distributions* . Biometrics , Vol. 45(1).
- 21- Shooman, M. L., (1968). *Probabilistic Reliability: An Engineering Approach*. McGraw – Hill International Book Company, New York .
- 22- Sinha, S. K. and Kale, B. K., (1980) . *Life Testing and Reliability Estimation* . John Wiley Eastern, LTD .
- 23- Smith, C. O., (1976). *Introduction to Reliability of Design* . McGraw – Hill, Book Company, New York .
- 24- Touama, H.Y., (2010), *Decisions Making Theory, darsafa for publishing and distribution*, Amman, Jordan.

Appendix (1)

The sum of work times & sum of repair times of the machines, during the period (1/10/2013 – 31/3/2014)

Production Lines	Mmachines	Sum of Work Times / hrs	n	Sum of Repair Times / hrs	m
Line (1) TBA/19-200 ML	Tetra Cardboard Packer 70: M ₁	1850	3	70	2
	Tetra Straw Applicator 21: M ₂	1802	5	118	4
	Tray Shrink 51: M ₃	1905	2	15	1
Line (2) TBA/19-250ML	Tetra Cardboard Packer 70: M ₄	1810	4	110	3
	Tetra Straw Applicator 21: M ₅	1901	3	19	2
	Tray Shrink 51: M ₆	1899	3	21	2
Line (3) Tetra Pak A1 -150ML	Tetra Cardboard Packer 70: M ₇	1911	2	9	1
	Tetra Straw Applicator 21: M ₈	1853	4	67	3
	Tray Shrink 51: M ₉	1876	3	44	2