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On Structural Breaks and Nonstationary Fractional Intergration in Time Series

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Abstract

The growth of an economy is determined largely by the growth of its Gross Domestic Product (GDP) over time. However, GDP and some economic series are characterized by nonstationarity, structural breaks and outliers. Many attempts have been made to analyze these economic series assuming unit root process even in the presence of changes in the mean level without considering possible fractional integration. This paper aims at examining the structural breaks and nonstationarity in the GDP series of some selected African countries with a view to determining the influence of structural breaks on the level of stationarity of these series. These series are found to be nonstationary with some evidence of long memory. They were found to experience one or more breaks over the years and this may be due to instability in the government and economic policies in the selected African countries. The measure of relative efficiency shows that autoregressive fractional integrated moving average (ARFIMA) models is better than the corresponding autoregressive integrated moving average (ARIMA) models for the series considered in this study.

Keywords: fractional integration, gross domestic product, structural breaks

1. Introduction

Economic growth for many countries is majorly determined by the country's Gross Domestic Product (GDP). Among African countries South Africa is rated as the richest country because of her highest value of GDP each year. For this reason, it is sensible to study the pattern in which this is realized over the years bearing in mind that the series are usually nonstationary. Most researches in economic time series have concentrated on the behavior of other economic measures and model are fitted to the series but fewer articles have considered GDP.

Economic and financial time series often display properties such as breaks, heteroscedasticity, missing values, outliers, nonlinearity just to mention a few. Of much importance in time series is the structural break or mean shift which affect the level of stationarity in the series. Quite a number of articles have shown that break in structure of the series may cause a stationary series $I(0)$ to be fractionally integrated (Granger and Hyung, 2004; Ohanissian et al., 2008). In the context of nonstationary series, there are fewer articles to show the effect of breaks in the series. Chivillon (2004) in the discussion paper on "A comparison of multi-step GDP forecasts for South Africa" reviewed that structural breaks and unit root occurred in South African's GDP over the last thirty years. Also, Romero-Ávila and De Olavide (2009) considered unit root hypothesis for per capita real GDP series in 46 African countries with data spreading from 1950 to 2001 and found multiple structural breaks. Structural breaks is examined for export, import and GDP in Ethiopia using annual macroeconomic time series from 1974 to 2009 and the study shows that the economy has suffered from structural change in the sample periods 1992, 1993 and 2003 (Allaro *et* al., 2011). Aly and Strazicich (2011) considered the GDP of the North African countries and observed one or two structural breaks except for Morocco where break was not observed.

This study seeks to investigate the stability (stationarity) and/or change in the mean level (structural breaks) over time. We also investigate the nature and type of nonstationarity that may have been brought about as a result of structural breaks in each series.

2. The GDP in African Countries

The World's record in 2005 shows that South Africa was the richest country among African countries with GDP of $456.7 billion. This figure was followed by Egypt, Algeria, Morroco and Nigeria with GDP of $295.2, 196, 128.3, 114.8 and $71 billion leaving Nigeria as the fifth in the ranking. The sixth to 10th countries were Sudan, Tunisia, Ethiopia, Ghana and Congo the Republic (http://www.joinafrica.com/Country_Rankings/gdp_africa.htm).

Similar account reported in World Economic Outlook Database of International Monetary Fund (IMF, 2009) shows that South Africa still maintained her position as the first in 2008 with GDP of $276.8 billion, followed by Nigeria ($207.1 billion), Egypt ($162.6 billion), Algeria ($159.7 billion) and Libya ($89.9 billion). The next countries in the ranking are Morroco, Angola, Sudan, Tunisia and Kenya. IMF (2011) presented the 2010 historical GDP data with similar report on GDP with South Africa having $524.0 billion of GDP, followed by Egypt ($497.8 billion), Nigeria ($377.9 billion), Algeria ($251.1 billion) and Morroco ($151.4 billion). Angola, Sudan, Tunisia, Libya and Ethiopia were in the sixth to 10th wealth position in Africa.

Comparative analysis of the country's wealth in 2005, 2008 and 2010 shows that Nigeria moved from the fifth (2005) to second position in 2008 and later dropped to third position in 2010. This swerve in wealth of a country as determined by the GDP may be due to some government policies and political factors and therefore, there is need to study the pattern in which these series are realized over the years.

Change in government policies and political instability may cause a series to experience a sharp break and these tend to alter the distributional pattern of the series. As part of econometric modelling, we introduce structural breaks in form of mean shift in this work in order to examine possible breaks in the series and econometric time series models are also applied to establish our claim on nonstationarity fractional integration of GDP series.

3. Methodology

The augmented Dickey Fuller (ADF) unit root test is used to establish nonstationarity in the GDP series of each country. Once the unit root is insignificant, we estimate the fractional difference parameter. This is achieved by applying the method used in Shittu and Yaya (2010) which suggest differencing the nonstationary series of order d_0 as many number of times to attain stationarity. Then, the fractional difference parameter is estimated from the resulting stationary series. We then apply "differencing and adding back" method of Velasco (2005) to estimate the nonstationary fractional difference parameter, d_0. That is, assuming the time series X_t and taking the unit difference of the series *n* number of times and this gives the unit difference order as u. We then applied semi-parametric estimation approach of described in Geweke and Porter-Hudak (1983) to estimate the stationary fractional difference parameter assumed to be $-0.5 < d < 0.5$. The estimate of nonstationary fractional difference parameter is then estimated based on $d_0 = d + u$ (see Shittu and Yaya, 2010).

Structural breaks can be visualized in the time plot of the observed series as forms of nonlinearity and outliers. However, this can be viewed more dearly from the plot of the differencing parameter d_0 against the specified time period. The latter method is more objective and in line with agreement of Gil-Alana (2008) and Gil-Alana *et* al. (2011). The papers applied the non-parametric approach of Robinson (1994) and the same will be used in this paper.

$$y_t = \alpha + X_t; \qquad (1 - B)^{d_0} X_t = u_t, \qquad t = 1, 2, ...,$$

(1)

where y_t is the observed time series, α is the intercept, d_0 is the fractional difference parameter and u_t is an $I(0)$ process assumed to be a white noise. When the differencing parameter (d) of a series is stationary fractional, $-0.5 < d < 0.5$ such a series is said to exhibit long memory. The appropriate model for such series is Autoregressive Fractional Moving Average (ARFIMA) model defined as,

$$y_t = (1 - B)^{d_0} X_t$$

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + ... + \theta_q \varepsilon_{t-q}$$

(2)

where $d_0 = u + d$ and $u = 1, 2, \ldots$ depending on the number of the unit differences. The $\{\phi_i, \theta_j\}, i = 1, \ldots, p; j = 1, \ldots, q$ are the parameters in the model and ε_{t-i} are the random process distributed as $\varepsilon_{t-i} \approx N(0,1)$.

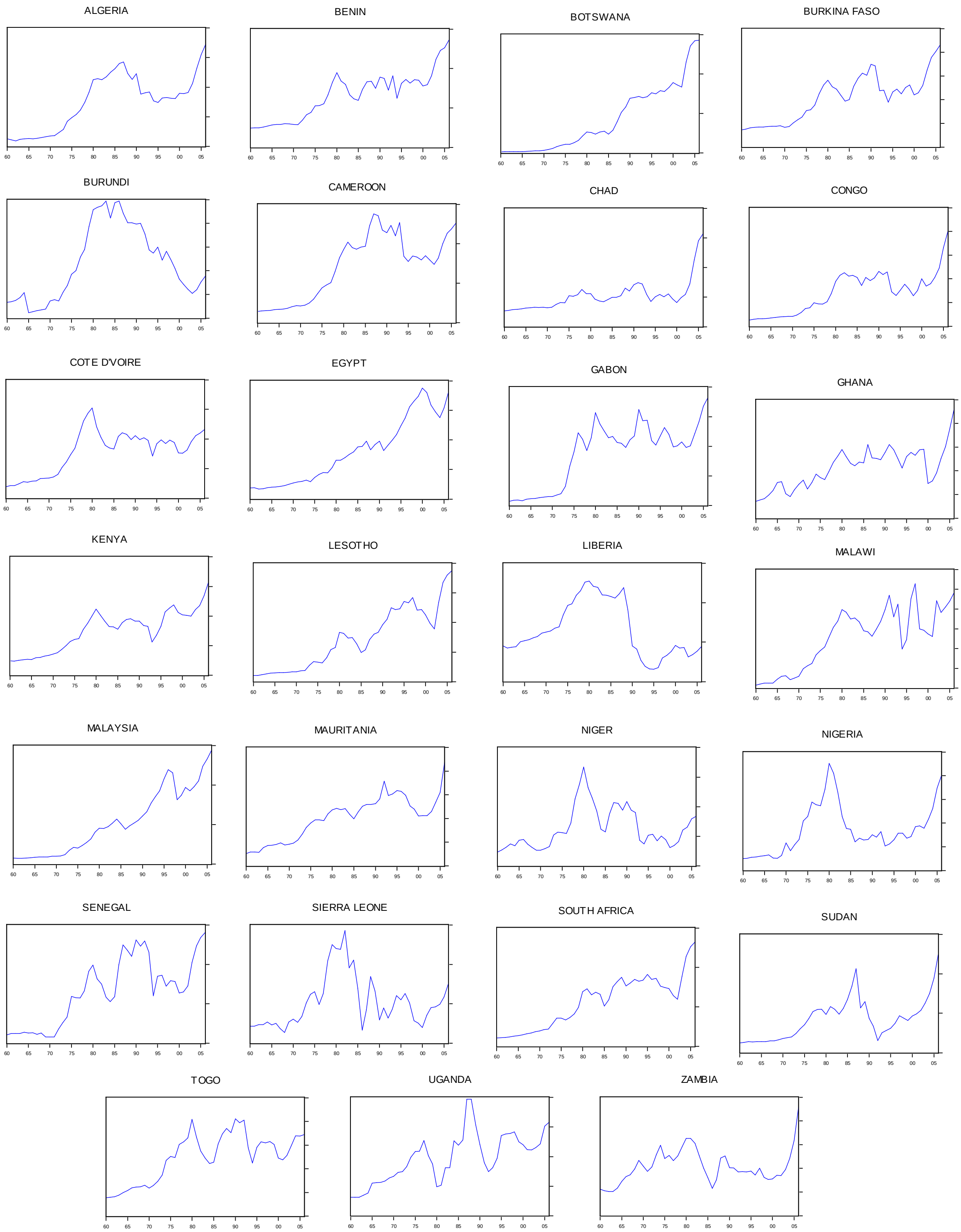
5. Source of Data

The data used in this study were the GDP per capita per person in current US Dollar of 27 African countries from 1960 to 2006. The annual data were sourced from International Monetary Fund (IMF) database. The GDP data is computed from the purchasing power parity (PPP) of countries per capita, that is the value of all final goods and services produced within a country in a given year divided by the average or mid-year population for the same year.

6. Results and Discussion

The time plots of the GDP series for different countries are shown in Figure 1 below. The data used are given in US dollars in order to allow country to country comparison. Various types of movements were noticed in the plots of GDP for these countries. A general upward movements were noticed from 1961 for the next five years, followed by sharp increases from 1966 to 1976 or there about. Thereafter, different types of movements were exhibited by different countries for the rest of the period under study. Most of the countries experienced drops in the GDP which may be due to decrease in the values expressed in the country's local currency. Nigeria for example experienced significant drops between 1980 and 2003.

Figure 1: Time Plots of GDP in African Country with figures given in US Dollars



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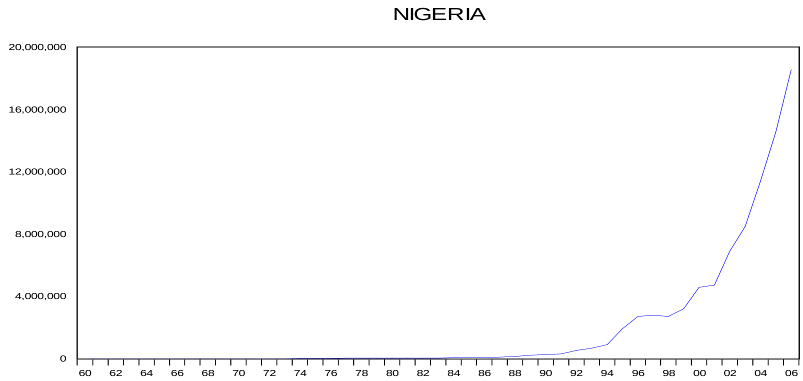
This significant drop experienced in Nigeria is also traced back to the behavior of Naira-US dollar exchange rates in the period under investigation. In that case, the nominal GDP in Naira is given below in Figure 2, and this shows astronomical increase of GDP in the country. Comparison of the two plots of GDP for Nigeria shows that exchange rate has effect on the country's wealth.

Figure 2: Time plot of Nigerian (nominal) GDP in millions of Naira



From Table 1, the ADF unit root test shows that all the series are nonstationary at 5% level of significance. However, all the series attained stationarity after the first difference. The above shows that the GDP series are integrated of order one, $I(1)$. This suggests that the series can be modelled as ARIMA (*p*, *d*, *q*). Shittu and Yaya (2009, 2010) showed that under certain conditions, ARFIMA model may be better than ARIMA model when nonstationarity is established in a series.

Table 1: Unit root tests on GDP series

	Observed Series		First Differenced			Observed Series		First Differenced	
Countries	*ADF*	*Prob.*	*ADF*	*Prob.*	*Countries*	*ADF*	*Prob.*	*ADF*	*Prob.*
Algeria	-0.9247	0.7708	-4.4448	0.0009	Liberia	-1.6164	0.4661	-4.0992	0.0024
Benin	-0.1828	0.9333	-7.3291	0.0000	Malawi	-1.6899	0.4295	-7.4282	0.0000
Botswana	2.1849	0.9999	-4.4063	0.0008	Malaysia	0.9706	0.9956	-4.9800	0.0002
Burkina	-0.2051	0.9303	-5.1443	0.0001	Mauritania	-0.0865	0.9447	-3.5398	0.0113
Burundi	-1.2956	0.6236	-5.1480	0.0001	Niger	-2.5505	0.1108	-4.6275	0.0005
Cameroon	-0.9280	0.7704	-5.9970	0.0000	Nigeria	-0.5198	0.5146	-4.4185	0.0009
Chad	-0.5889	0.8628	-3.5435	0.0111	Senegal	-0.9562	0.7609	-5.6289	0.0000
Congo	0.6859	0.9906	-4.4716	0.0008	Sierra	-2.1146	0.2401	-6.3065	0.0000
Cote	-1.8459	0.3542	-4.6347	0.0005	South	-0.5501	0.8713	-4.6040	0.0005
Egypt	2.5622	1.0000	-4.0914	0.0025	Sudan	-0.1286	0.9399	-6.0260	0.0000
Gabon	-0.8763	0.7869	-5.3685	0.0001	Togo	-1.7109	0.4192	-5.3715	0.0000
Ghana	-0.7451	0.8248	-5.3252	0.0001	Uganda	-2.4599	0.1319	-4.9790	0.0002
Kenya	-0.1854	0.9326	-4.2919	0.0014	Zambia	-1.6277	0.4604	-2.9934	0.0431
Lesotho	0.4566	0.9833	-4.8220	0.0003					

With this in mind, we examined whether or not all the series were actually $I(0)$ or $I(d)$ where *d* is the fractional difference parameter for all the series. The result is shown in Table 3.

Table 3: Estimates of Fractional Difference Parameter

Country	Algeria	Benin	Botswana	Burkina Faso	Burundi	Cameroon	Chad	Congo	Côte D'Ivoire
$\hat{d}$	1.1039	1.0108	1.0801	1.0182	1.1357	1.0785	1.1029	1.0498	1.1158
Country	Egypt	Gabon	Ghana	Kenya	Lesotho	Liberia	Malawi	Malaysia	Mauritania
$\hat{d}$	1.0485	1.0546	1.0361	1.1048	1.0097	1.1258	0.9111	0.9684	1.0465
Country	Niger	Nigeria	Senegal	Sierra Leone	South Africa	Sudan	Togo	Uganda	Zambia
$\hat{d}$	1.0950	1.1265	0.9839	1.0248	1.0505	1.0641	1.0287	0.9996	1.0678

It can be observed that all the series were not exactly of order one, $I(1)$.

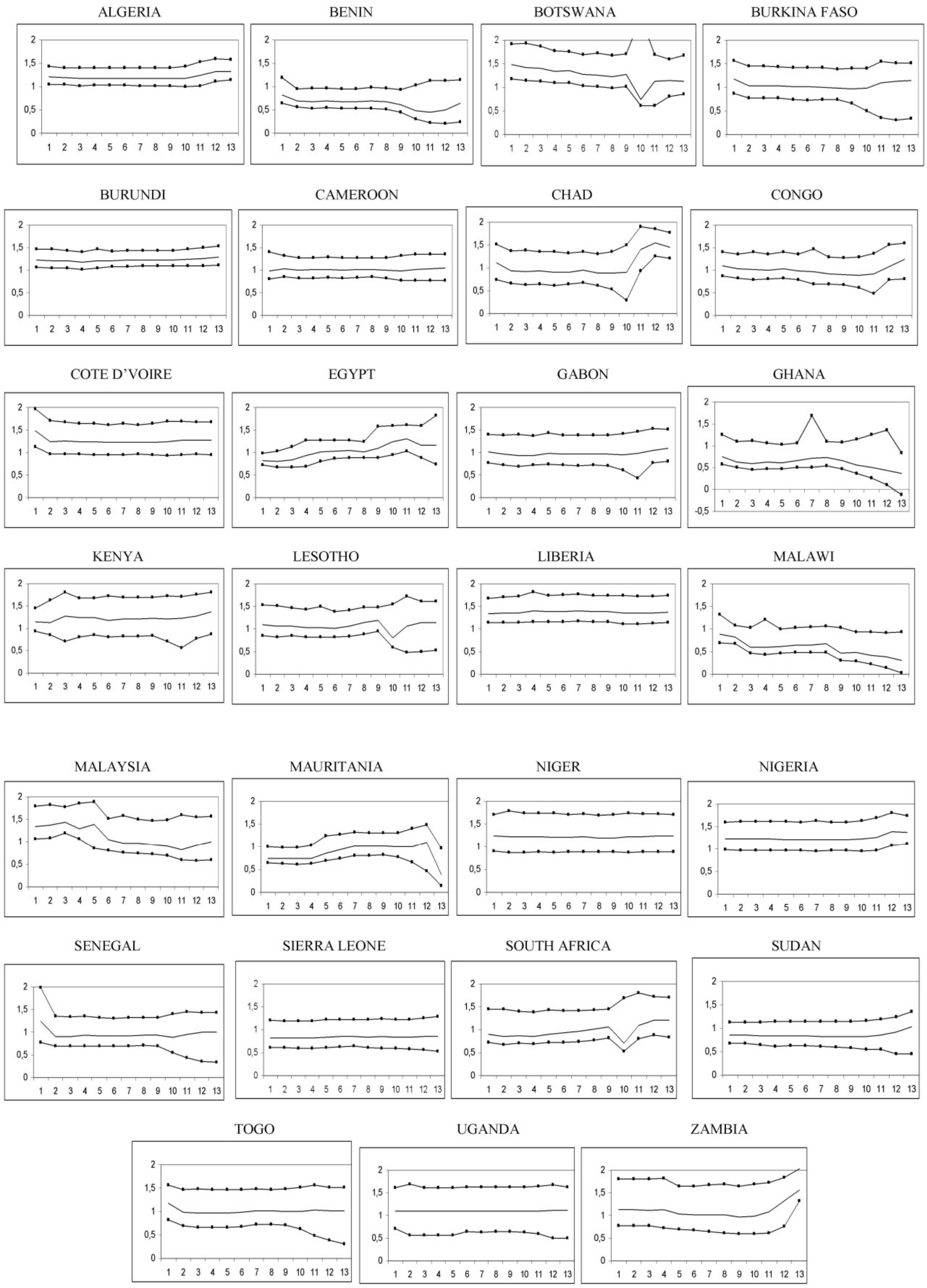
6.1 Investigation of Structural Breaks

The first value corresponds to the estimate of *d* based on the sample with the first 35 observations, that is, from 1960 to 1994, then the following one corresponds to the sample [1961 – 1995], the next. [1962 – 1996] and so on till the last one which corresponds to [1972 – 2006] making 13 blocks of samples i.e. 1960 – 1994, 1961 – 1995, 1962 – 1996, 1963 – 1997, 1964 – 1998, 1965 – 1999, 1966 – 2000, 1967 – 2001, 1968 – 2002, 1969 – 2003, 1970 – 2004, 1971 – 2005, 1972 – 2006. The following figure displays for each country the estimates of differencing parameters d_0 along with the 95% confidence band using the model in (1).

Stable estimates of *d* across the different subsamples are observed in the GDP of Burundi, Cameroon, Gabon, Kenya, Liberia, Niger, Nigeria, Sierra Leone and Uganda. In Benin, Bostwana, Lesotho and South Africa, we notice a decrease in the degree of integration about the 10th estimate [2003]. A slight increase in the estimated value of *d* about the 10^{th} / 11^{th} estimate [2003, 2004] is observed in Algeria, Chad, Congo, Sudan and Zambia. In fact, in the above 10 countries, we observe a sharp increase about the year 2003.

For another group, we observe a slight decrease in the 2^{nd} estimate [1995]. This group include Burkina Faso, Cote de Ivory, Senegal and Togo. For Ghana and Malawi, break is observed in the 8^{th} block [2001].

Figure 3: Plots of estimates of fractional difference parameter *d* across blocks of samples



6.2 Modelling of the Series

To determine the most appropriate model for GDP series in the selected countries in Africa, the ARIMA (*p*, *d*, *q*) and ARFIMA (*p*, *d*, *q*) modelling were carried out on the series with a view to measure the relative efficiency (R.E) of the ARFIMA model over the ARIMA model. The results are shown in Table 4 and 5 below.

Table 4: Estimated Nonstationary ARFIMA Models for the African GDP Series

Algeria	$y_t = (1-B)^{\underset{[0.1181]}{1.2086}} X_t$ $y_t = \underset{[0.1474]}{0.4684} y_{t-3} + \underset{[0.1579]}{0.2607} y_{t-6} - \underset{[0.1504]}{0.4382} y_{t-9} - \underset{[0.1469]}{0.2650} y_{t-13} + \varepsilon_t$ Sk. = -0.5328 Ex. Kurt. = 3.0732 ARCH = 1.8621 [0.1535] σ^2_{ARFIMA} = 24026.4 σ^2_{ARIMA} = 27792 $\sigma^2_{ARFIMA/ARIMA}$ = 0.8645
Benin	$y_t = (1-B)^{\underset{[0.0990]}{1.1494}} X_t$ $y_t = -\underset{[0.1359]}{0.4860} y_{t-4} - \underset{[0.1433]}{0.2694} y_{t-10} + \underset{[0.1755]}{0.3748} y_{t-13} - \underset{[0.2146]}{0.7577} y_{t-14} + \underset{[0.2217]}{0.4421} y_{t-15} + \varepsilon_t$ Sk. = -0.1905 Ex. Kurt. = 0.7008 ARCH = 2.5958 [0.0679] σ^2_{ARFIMA} = 973.858 σ^2_{ARIMA} = 1026.95 $\sigma^2_{ARFIMA/ARIMA}$ = 0.9483
Botswana	$y_t = (1-B)^{\underset{[0.0000]}{1.0000}} X_t$ $y_t = \underset{[0.0000]}{1.6497} + \underset{[0.1150]}{0.6106} y_{t-1} - \underset{[0.1300]}{0.5397} y_{t-2} + \underset{[0.1256]}{0.4146} y_{t-3} - \underset{[0.1831]}{0.3829} y_{t-6} - \underset{[0.2350]}{0.6347} y_{t-12} + \underset{[0.2837]}{1.3092} y_{t-13} - \underset{[0.3086]}{1.0303} y_{t-14} + \underset{[0.2612]}{1.3597} y_{t-15} + \varepsilon_t$ Sk. = -0.3623 Ex. Kurt. = 0.9984 ARCH = 1.4401 [0.2507] σ^2_{ARFIMA} = 25650.7 σ^2_{ARIMA} = 25905.8 $\sigma^2_{ARFIMA/ARIMA}$ = 0.9902
Burkina Faso	$y_t = (1-B)^{\underset{[0.1153]}{1.2786}} X_t$ $y_t = -\underset{[0.1408]}{0.4572} y_{t-4} - \underset{[0.1445]}{0.2769} y_{t-6} - \underset{[0.1504]}{0.3706} y_{t-10} - \underset{[0.1742]}{0.5152} y_{t-14} + \underset{[0.2347]}{0.3825} y_{t-15} + \varepsilon_t$ Sk. = -0.3062 Ex. Kurt. = 2.3121 ARCH = 0.6358 [0.5970] σ^2_{ARFIMA} = 582.259 σ^2_{ARIMA} = 693.206 $\sigma^2_{ARFIMA/ARIMA}$ = 0.8400
Burundi	$y_t = (1-B)^{\underset{[0.3803]}{-1.1904}} X_t$ $y_t = \underset{[0.2951]}{131.79} - \underset{[0.2951]}{1.9205} y_{t-1} - \underset{[0.3333]}{0.9635} y_{t-2} + \underset{[0.1936]}{0.2227} y_{t-6} - \underset{[0.2804]}{0.4155} y_{t-7} + \underset{[0.1662]}{0.2734} y_{t-8} - \underset{[0.1616]}{0.4393} y_{t-13} + \underset{[0.2637]}{0.5748} y_{t-14} - \underset{[0.1381]}{0.1857} y_{t-15} + \varepsilon_t$ Sk. = 0.2042 Ex. Kurt. = 1.7494 ARCH = 0.8209 [0.4914] σ^2_{ARFIMA} = 57436.8 σ^2_{ARIMA} = 59764.3 $\sigma^2_{ARFIMA/ARIMA}$ = 0.9611

Cameroon

$$y_t = (1-B)^{\underset{[0.1088]}{0.8454}} X_t$$

$$y_t = \underset{[750.5]}{2064.90} + \underset{[0.1286]}{0.1797} y_{t-2} + \underset{[0.1387]}{0.2579} y_{t-5} + \underset{[0.1332]}{0.2192} y_{t-6} - \underset{[0.1431]}{0.3466} y_{t-8} - \underset{[0.1407]}{0.2917} y_{t-11} + \underset{[0.1974]}{0.3380} y_{t-13} - \underset{[0.2012]}{0.4426} y_{t-14} + \varepsilon_t$$

Sk. = 0.0245 Ex. Kurt. = 2.3534 ARCH = 0.9534 [0.4266] $\sigma^2_{ARFIMA} = 83.3491$ $\sigma^2_{ARIMA} = 5141.69$ $\sigma^2_{ARFIMA/ARIMA} = 0.0162$

Chad

$$y_t = (1-B)^{\underset{[0.2765]}{1.4056}} X_t$$

$$y_t = \underset{[713.9]}{903.584} + \underset{[0.2666]}{0.3230} y_{t-1} - \underset{[0.1694]}{0.2158} y_{t-9} - \underset{[0.1657]}{0.5773} y_{t-11} + \underset{[0.1820]}{0.6543} y_{t-13} - \underset{[0.1343]}{0.1492} y_{t-15} + \varepsilon_t$$

Sk. = -0.2253 Ex. Kurt. = 0.2498 ARCH = 0.5116 [0.6773] $\sigma^2_{ARFIMA} = 703.893$ $\sigma^2_{ARIMA} = 745.225$ $\sigma^2_{ARFIMA/ARIMA} = 0.9445$

Congo

$$y_t = (1-B)^{\underset{[0.0000]}{0.9999}} X_t$$

$$y_t = \underset{[0.0000]}{2.8E+07} + \underset{[0.1324]}{0.3058} y_{t-1} + \underset{[0.1354]}{0.2137} y_{t-4} + \underset{[0.1316]}{0.3212} y_{t-5} - \underset{[0.1448]}{0.3148} y_{t-6} - \underset{[0.1677]}{0.3848} y_{t-12} - \underset{[0.1635]}{0.5799} y_{t-13} - \underset{[0.2004]}{0.3909} y_{t-14} + \underset{[0.2120]}{0.7431} y_{t-15} + \varepsilon$$

Sk. = 0.7457 Ex. Kurt. = 0.9101 ARCH = 0.4292 [0.7335] $\sigma^2_{ARFIMA} = 9168.88$ $\sigma^2_{ARIMA} = 98822.18$ $\sigma^2_{ARFIMA/ARIMA} = 0.0928$

Cote D'vore

$$y_t = (1-B)^{\underset{[0.2425]}{-0.8550}} X_t$$

$$y_t = \underset{[22.55]}{667.115} + \underset{[0.1900]}{1.7688} y_{t-1} - \underset{[0.1732]}{0.9073} y_{t-2} + \underset{[0.0315]}{0.1078} y_{t-5} + \varepsilon_t$$

Sk. = -0.4447 Ex. Kurt. = 1.1910 ARCH = 1.3816 [0.2640] $\sigma^2_{ARFIMA} = 4366.51$ $\sigma^2_{ARIMA} = 7215.51$ $\sigma^2_{ARFIMA/ARIMA} = 0.6052$

Egypt

$$y_t = (1-B)^{\underset{[0.5550]}{0.0003}} X_t$$

$$y_t = \underset{[702.6]}{9539.39} + \underset{[0.2304]}{0.5071} y_{t-1} + \underset{[0.1469]}{0.6486} y_{t-3} - \underset{[0.1238]}{0.2382} y_{t-4} + \varepsilon_t$$

Sk. = 0.3858 Ex. Kurt. = 0.0884 ARCH = 0.5362 [0.6604] $\sigma^2_{ARFIMA} = 369149$ $\sigma^2_{ARIMA} = 431325$ $\sigma^2_{ARFIMA/ARIMA} = 0.8558$

Gabon

$$y_t = (1-B)^{\underset{[0.2472]}{0.5071}} X_t$$

$$y_t = \underset{[1676]}{240208} + \underset{[0.0814]}{1.1412} y_{t-1} - \underset{[0.1598]}{0.2260} y_{t-4} - \underset{[0.2092]}{0.2446} y_{t-6} + \underset{[0.2745]}{0.2956} y_{t-9} + \underset{[0.2363]}{0.7712} y_{t-10} - \underset{[0.2732]}{0.4994} y_{t-12} - \underset{[0.1856]}{0.2381} y_{t-14} + \varepsilon_t$$

Sk. = -1.3655 Ex. Kurt. = 3.4728 ARCH = 0.1632 [0.6886] $\sigma^2_{ARFIMA} = 2341.07$ $\sigma^2_{ARIMA} = 385109$ $\sigma^2_{ARFIMA/ARIMA} = 0.0061$

Ghana

$$y_t = (1-B)^{\underset{[0.0895]}{1.1947}} X_t$$

$$y_t = -\underset{[0.1521]}{0.3243}\, y_{t-2} - \underset{[0.1470]}{0.3836}\, y_{t-3} - \underset{[0.1549]}{0.4246}\, y_{t-4} - \underset{[0.1530]}{0.4651}\, y_{t-5} - \underset{[0.1591]}{0.3290}\, y_{t-6} - \underset{[0.1746]}{0.6226}\, y_{t-14} + \underset{[0.1906]}{0.3669}\, y_{t-15} + \varepsilon_t$$

Sk. =- 0.5379 Ex. Kurt. = 0.2612 ARCH = 0.4112 [0.7460] σ^2_{ARFIMA} = 870.617 σ^2_{ARIMA} = 988.036 $\sigma^2_{ARFIMA/ARIMA}$ = 0.8812

Kenya

$$y_t = (1-B)^{\underset{[0.0963]}{-0.1079}} X_t$$

$$y_t = \underset{[139.6]}{516.274} + \underset{[0.0886]}{1.1268}\, y_{t-1} - \underset{[0.1502]}{0.5641}\, y_{t-3} - \underset{[0.1242]}{0.3305}\, y_{t-4} + \underset{[0.0541]}{0.0839}\, y_{t-9} - \underset{[0.1340]}{0.8004}\, y_{t-13} + \underset{[0.1263]}{0.7603}\, y_{t-14} + \varepsilon_t$$

Sk. = -0.2812 Ex. Kurt. = 0.9785 ARCH = 0.2226 [0.8800] σ^2_{ARFIMA} = 489.247 σ^2_{ARIMA} = 686.408 $\sigma^2_{ARFIMA/ARIMA}$ = 0.7128

Lesotho

$$y_t = (1-B)^{\underset{[0.1237]}{-1.1697}} X_t$$

$$y_t = \underset{[76.47]}{302.755} + \underset{[0.1107]}{1.8804}\, y_{t-1} - \underset{[0.1292]}{1.0093}\, y_{t-2} + \underset{[0.0752]}{0.2157}\, y_{t-6} - \underset{[0.0931]}{0.1757}\, y_{t-9} + \underset{[0.0871]}{0.2718}\, y_{t-12} - \underset{[0.0471]}{0.2060}\, y_{t-15} + \varepsilon_t$$

Sk. = 0.4575 Ex. Kurt. = 1.0820 ARCH = 1.2264 [0.3156] σ^2_{ARFIMA} = 1027.59 σ^2_{ARIMA} = 1744.13 $\sigma^2_{ARFIMA/ARIMA}$ = 0.5892

Liberia

$$y_t = (1-B)^{\underset{[0.1580]}{-0.7004}} X_t$$

$$y_t = \underset{[3.307]}{293.397} + \underset{[0.1181]}{1.5866}\, y_{t-1} - \underset{[0.0962]}{0.7066}\, y_{t-2} + \underset{[0.1006]}{0.1679}\, y_{t-13} - \underset{[0.1291]}{0.2869}\, y_{t-14} + \varepsilon_t$$

Sk. = -1.2088 Ex. Kurt. = 5.7365 ARCH = 2.2433 [0.0.0000] σ^2_{ARFIMA} = 741.198 σ^2_{ARIMA} = 1243.01 $\sigma^2_{ARFIMA/ARIMA}$ = 0.5963

Malawi

$$y_t = (1-B)^{\underset{[0.2053]}{-0.0249}} X_t$$

$$y_t = \underset{[16.25]}{177.192} + \underset{[0.1804]}{0.5780}\, y_{t-1} - \underset{[0.1230]}{0.2957}\, y_{t-2} + \underset{[0.1282]}{0.3326}\, y_{t-6} - \underset{[0.1301]}{0.3023}\, y_{t-7} + \underset{[0.1548]}{0.6015}\, y_{t-9} - \underset{[0.1757]}{0.7598}\, y_{t-10} + \underset{[0.2911]}{1.2405}\, y_{t-13} - \underset{[0.2630]}{0.9110}\, y_{t-14} + \varepsilon_t$$

Sk. = 0.4675 Ex. Kurt. = 1.3541 ARCH = 1.4290 [0.2531] σ^2_{ARFIMA} = 344.225 σ^2_{ARIMA} = 499.393 $\sigma^2_{ARFIMA/ARIMA}$ = 0.6893

Malaysia

$$y_t = (1-B)^{\underset{[0.0158]}{-0.0399}} X_t$$

$$y_t = -\underset{[354.9]}{16511} + \underset{[0.1340]}{1.1310}\, y_{t-1} - \underset{[0.1953]}{0.5538}\, y_{t-2} + \underset{[0.1410]}{0.3311}\, y_{t-3} - \underset{[0.1447]}{0.4766}\, y_{t-6} + \underset{[0.2190]}{0.5520}\, y_{t-7} - \underset{[0.2273]}{0.4791}\, y_{t-8} + \underset{[0.1521]}{05175}\, y_{t-9} + \varepsilon_t$$

Sk. = 0.4675 Ex. Kurt. = 1.3541 ARCH = 1.4290 [0.2531] σ^2_{ARFIMA} = 344.225 σ^2_{ARIMA} = 70358.4 $\sigma^2_{ARFIMA/ARIMA}$ = 0.0049

Mauritania

$$y_t = (1-B)^{\underset{[0.1078]}{1.2180}} X_t$$

$$y_t = \underset{[0.1549]}{0.2634}\, y_{t-5} - \underset{[0.1466]}{0.4700}\, y_{t-6} - \underset{[0.1524]}{0.3856}\, y_{t-7} - \underset{[0.1620]}{0.4677}\, y_{t-8} - \underset{[0.1684]}{0.2837}\, y_{t-9} - \underset{[0.1693]}{0.4916}\, y_{t-10} + \underset{[0.3031]}{0.9912}\, y_{t-15} + \varepsilon_t$$

	Sk. = 0.0605 Ex. Kurt. = 3.0977 ARCH = 2.6109 [0.0678] $\sigma^2_{ARFIMA} = 1535.41$ $\sigma^2_{ARIMA} = 1673.63$ $\sigma^2_{ARFIMA/ARIMA} = 0.9174$
Niger	$y_t = (1-B)^{\underset{[0.2335]}{-0.6793}} X_t$ $y_t = \underset{[7.295]}{231.022} + \underset{[0.1681]}{1.7400} y_{t-1} - \underset{[0.1628]}{0.8326} y_{t-2} - \underset{[0.1524]}{0.3856} y_{t-7} - \underset{[0.1620]}{0.4677} y_{t-8} - \underset{[0.1684]}{0.2837} y_{t-9} - \underset{[0.1693]}{0.4916} y_{t-10} + \underset{[0.3031]}{0.9912} y_{t-15} + \varepsilon_t$ Sk. = -0.11144 Ex. Kurt. = 0.93369 ARCH = 1.0214 [0.3949] $\sigma^2_{ARFIMA} = 701.416$ $\sigma^2_{ARIMA} = 868.225$ $\sigma^2_{ARFIMA/ARIMA} = 0.8079$
Nigeria	$y_t = (1-B)^{\underset{[0.1520]}{-0.2367}} X_t$ $y_t = \underset{[16.38]}{367.169} + \underset{[0.1724]}{1.35327} y_{t-1} - \underset{[0.1497]}{0.5931} y_{t-2} - \underset{[0.0662]}{0.0570} y_{t-6} - \underset{[0.08571]}{0.1975} y_{t-12} + \underset{[0.1606]}{0.3115} y_{t-14} - \underset{[0.1230]}{0.2621} y_{t-15} + \varepsilon_t$ Sk. = 0.61314 Ex. Kurt. = 0.99332 ARCH = 1.5671 [0.2159] $\sigma^2_{ARFIMA} = 3351.88$ $\sigma^2_{ARIMA} = 4469.75$ $\sigma^2_{ARFIMA/ARIMA} = 0.7499$
Senegal	$y_t = (1-B)^{\underset{[0.2682]}{0.2074}} X_t$ $y_t = \underset{[120.1]}{668.380} + \underset{[0.2472]}{0.8240} y_{t-1} - \underset{[0.1848]}{0.3064} y_{t-2} + \underset{[0.1781]}{0.2723} y_{t-3} - \underset{[0.1426]}{0.2773} y_{t-4} + \underset{[0.1866]}{0.2744} y_{t-13} - \underset{[0.2278]}{0.3983} y_{t-14} + \underset{[0.1253]}{0.2262} y_{t-15} + \varepsilon_t$ Sk. = 0.25406 Ex. Kurt. = 1.5953 ARCH = 0.20417 [0.8927] $\sigma^2_{ARFIMA} = 2534.19$ $\sigma^2_{ARIMA} = 3054.97$ $\sigma^2_{ARFIMA/ARIMA} = 0.8295$
Sierra Leone	$y_t = (1-B)^{\underset{[0.1828]}{-0.5454}} X_t$ $y_t = \underset{[3.800]}{210.744} + \underset{[0.1756]}{1.1793} y_{t-1} - \underset{[0.1393]}{0.3563} y_{t-2} - \underset{[0.1318]}{0.2995} y_{t-8} + \underset{[0.2009]}{0.5308} y_{t-9} - \underset{[0.1407]}{0.3600} y_{t-10} + \underset{[0.1301]}{0.2756} y_{t-14} - \underset{[0.1182]}{0.2918} y_{t-15} + \varepsilon_t$ Sk. = -0.3899 Ex. Kurt. = 1.6657 ARCH = 0.3354 [0.7998] $\sigma^2_{ARFIMA} = 780.841$ $\sigma^2_{ARIMA} = 1119.07$ $\sigma^2_{ARFIMA/ARIMA} = 0.6978$
South Africa	$y_t = (1-B)^{\underset{[0.0008]}{0.0007}} X_t$ $y_t = -\underset{[3255]}{102905} + \underset{[0.1337]}{1.3083} y_{t-1} - \underset{[0.1536]}{0.5221} y_{t-2} - \underset{[0.1895]}{0.3280} y_{t-5} + \underset{[0.2752]}{0.5787} y_{t-6} - \underset{[0.2856]}{0.4823} y_{t-7} + \underset{[0.2789]}{0.6697} y_{t-8} - \underset{[0.2290]}{0.5150} y_{t-9} + \underset{[0.1307]}{0.2907} y_{t-11} + \varepsilon_t$ Sk. =-0.3446 Ex. Kurt. = 0.4664 ARCH = 1.8418 [0.1601] $\sigma^2_{ARFIMA} = 64656$ $\sigma^2_{ARIMA} = 87353$ $\sigma^2_{ARFIMA/ARIMA} = 0.7402$
Sudan	$y_t = (1-B)^{\underset{[0.1525]}{-0.5032}} X_t$ $y_t = \underset{[75.69]}{431.490} + \underset{[0.1003]}{1.3176} y_{t-1} - \underset{[0.1041]}{0.4135} y_{t-3} + \underset{[0.0748]}{0.1427} y_{t-8} - \underset{[0.0806]}{0.2083} y_{t-11} + \underset{[0.0462]}{0.1329} y_{t-15} + \varepsilon_t$ Sk. = -1.0503 Ex. Kurt. = 6.2138 ARCH = 0.76472 [0.5217] $\sigma^2_{ARFIMA} = 6567.49$ $\sigma^2_{ARIMA} = 8511.1$ $\sigma^2_{ARFIMA/ARIMA} = 0.7716$

Togo	$y_t = (1-B)^{\underset{[0.2223]}{0.5463}} X_t$ $y_t = \underset{[161.7]}{23.9437} + \underset{[0.2018]}{0.6282}\, y_{t-1} + \underset{[0.1391]}{0.4044}\, y_{t-12} - \underset{[0.1876]}{0.3241}\, y_{t-13} - \underset{[0.1888]}{0.3058}\, y_{t-14} + \underset{[0.1644]}{0.4967}\, y_{t-15} + \varepsilon_t$ Sk. = -0.12717 Ex. Kurt. = 1.6211 ARCH = 2.3798 [0.0868] $\sigma^2_{ARFIMA} = 986.417$ $\sigma^2_{ARIMA} = 1047.13$ $\sigma^2_{ARFIMA/ARIMA} = 0.9420$
Uganda	$y_t = (1-B)^{\underset{[0.1432]}{0.2332}} X_t$ $y_t = \underset{[16.16]}{237.479} + \underset{[0.1380]}{1.0418}\, y_{t-1} - \underset{[0.1547]}{0.2862}\, y_{t-2} - \underset{[0.1522]}{0.3745}\, y_{t-4} + \underset{[0.2020]}{0.3871}\, y_{t-5} - \underset{[0.1995]}{0.3573}\, y_{t-6} + \underset{[0.1313]}{0.2304}\, y_{t-7} + \varepsilon_t$ Sk. = 0.73710 Ex. Kurt. = 5.5493 ARCH = 1.1942 [0.3266] $\sigma^2_{ARFIMA} = 892.609$ $\sigma^2_{ARIMA} = 1043.65$ $\sigma^2_{ARFIMA/ARIMA} = 0.8553$
Zambia	$y_t = (1-B)^{\underset{[0.0676]}{0.4324}} X_t$ $y_t = \underset{[31.00]}{459.367} + \underset{[0.1554]}{1.3991}\, y_{t-1} - \underset{[0.1623]}{0.6012}\, y_{t-2} - \underset{[0.1037]}{0.1563}\, y_{t-10} + \varepsilon_t$ Sk. = 0.55756 Ex. Kurt. = 2.4524 ARCH = 0.72279 [0.5448] $\sigma^2_{ARFIMA} = 3686.5$ $\sigma^2_{ARIMA} = 4341.06$ $\sigma^2_{ARFIMA/ARIMA} = 0.8492$

The relative efficiency between the ARFIMA (p, d, q) and ARIMA (p, d, q) are given in Table 5.

Table 5: Relative Frequency of ARFIMA model over ARIMA model

Break in 2nd subsample		***Break in 8th subsample***		***Break in 10th/11th subsamples***		***No Break***	
Country	**R.E**	**Country**	**R.E**	**Country**	**R.E**	**Country**	**R.E**
Burkina Faso	0.8400	Ghana	0.8812	Algeria	0.8645	Burundi	0.9611
Cote D'ivoire	0.6052	Malawi	0.6893	Chad	0.9445	Cameroon	00162
Senegal	0.8295			Congo	0.0928		
Togo	0.9420			Sudan	0.7716		
				Zambia	0.8492	Gabon	0.0061
				Benin	0.9483	Kenya	0.7128
				Botswana	0.9902	Liberia	0.5963
				Lesotho	0.5892	Niger	0.8079
				South Africa	0.7402	Nigeria	0.7499
						Sierra Leone	0.6978
						Uganda	0.8553

One would have expected that the R.E for those countries with stable estimates of *d* across the different subsamples would have R.E = 1. This is not so because the series also exhibit long memory in their stationary processes.

5. Conclusion

We have considered the dynamics of GDP of some African countries in this paper using the econometric time series modelling approach. This approach involved studying the property of the series via testing for occasional breaks. The approach proposed in Robinson (1994) was used to examine the breaks in the series and one or more breaks were observed in some countries. Some of these countries are poor and their GDPs tend to rise and fall as expressed in dollars, though the value may rise astronomically as in the case of Nigeria when expressed in Naira.

Finally, we applied the nonstationary ARFIMA models on the 27 series considered in the paper and found that the GDP series are actually nonstationary. The ARFIMA models are found to perform better that the corresponding ARIMA models and these results follow that of Shittu and Yaya (2009, 2010).

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