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Optimal Pricing and Ordering Policy under Permissible Delay in Payments

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Abstract

This study develops an inventory model to determine an optimal cycle time and optimal total annual profit for non-deteriorating items under permissible delay in payments. Mathematical models have been derived for obtaining the optimal cycle time and optimal price, so that the annual total profit is maximized. This paper also develops the model by considering particular cases (A) and (B) respectively. We obtain price and lot size simultaneously when supplier offers a permissible delay in payments. The demand rate is assumed to be a function of price and time. Finally, a numerical example is given to illustrate the proposed model.

Key words: Pricing, Inventory, Permissible delay, Non- deterioration, Finance, Quantity

1. Introduction

The traditional economic order quantity (EOQ) model assumes that the retailer must be paid for the items as soon as the items were received. But it may not be true in general. In practice the supplier offers the retailer a period (called delay period or trade credit period) for setting the account. Before the end of this period, the retailer can sell the goods and accumulate revenue and earn interest. An interest is charged if the retailer unable to settle the account by the end of the credit period. Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the end of the delay period allowed by the supplier. During the past few years, many articles dealing with various inventory models under permissible delay have appeared in various research journals.

In past decade, mathematical ideas have been used in different area for controlling inventory. The important concerns of the management are to decide when and how much to order or to manufacture, so that total cost associated with the inventory system should be minimum. Deterioration cannot be ignored in business management. Deterioration refers to damage, change, decay, spoilage obsolescence and loss of original value in the item those results in the decreasing usefulness from the original one. The certain products such as medicine, vegetable, blood, gasoline and radioactive chemicals decrease under deterioration during their normal storage period. As a result, the loss due to deterioration cannot be ignored for determining optimal inventory policy. To accumulate more practical features of the real inventory system, the deteriorating inventory models have been continuously modified. Number of researchers has been discussed inventory models for non- deteriorating items. However, there are certain substances in which deterioration play the main role and commodities cannot be stored for a long time. Non deteriorating items like, wheat, rice, some types of dry fruits, etc.

Teng et al. (2004) developed a model on optimal pricing and ordering policy under permissible delay in payments, in which deterioration rate is constant and demand rate is a function of price. In this paper Tenj et al.(2004) obtained optimal cycle time and optimal total annual profit. This paper is the extension of Teng et al. (2004) in which deterioration rate is zero and demand rate is a function of price and time. Teng (2002) in his paper discussed on the economic order quantity under condition of permissible delay in payments for non-deteriorating items. Goyal (2985) developed an EOQ model under conditions of permissible delay in payments. He ignored the difference between the selling price and the purchase cost, and concluded that the economic replenishment interval and order quantity increases marginally under permissible delay in payments. Dave (1985) corrected Goyal's model by assuming the fact that the selling price is necessarily higher than its purchase price. Aggarwal and Jaggi (1995) then extended Goyal's model for deteriorating items. Jamal *et al.*(1997) further generalized the model to allow for shortages and deterioration. Liao et al. (2002) developed an inventory model for stock- dependent demand rate when a delay in payment in permissible.

Huang (2003) implicitly assume that the inventory level is depleted by customer's demand only. This assumption is valid for non- deteriorating or non- perishable inventory items. Mahata and Mahata (2009) modified Huang (2003) model by developing an inventory model for deteriorating items under condition of permissible delay in payments. Chung (1998) presented the discounted cash flow (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. Hwang and Shinn (1997) extended Goyal's (1985) model to consider the deterministic inventory model with a constant deterioration rate. Manisha Pal and S.K. Ghosh (2006) developed an inventory model with shortage and quantity dependent permissible delay in payment for non- deteriorating items.

In this paper we establish an appropriate model for a retailer to determine its optimal price and lot size simultaneously when the supplier offer a permissible delay in payments. In this paper the deterministic inventory model with time –dependent demand pattern is developed for non- deteriorating items in which inventory is depleted only by demand. The paper is organized as follows: In section 2 assumptions and notations are mentioned. In section 3, the mathematical model is formulated. In section 4 the optimal replenishment time for given price is mentioned in which we considered two particular cases viz; case (A) and case (B) respectively. In section 5 optimal prices is obtained. In next section numerical example is cited to validate the proposed model followed by concluding remark and future research is detailed in the last section.

2. Assumptions and Notations

The following assumptions are being made to develop the mathematical model

- The demand for the item is a downward sloping function of the price and variable time t.
- Shortage is not allowed.
- Time horizon is infinite.

In addition the following notations are also used throughout the manuscript

H: The unit holding cost per year excluding interest charges

c: The unit purchasing cost, with $c<p$

p: The selling price per unit

l_d: The interest earned per dollar per year

I_c: The interest charged per rupee in stocks per year by the supplier

m: The period of permissible delay in setting account; that is, the trade credit period

s: The ordering cost per order

Q: The order quantity

I(t):The level of inventory at time t, $0 \leq t \leq T$

T: The replenishment time interval

D: The annual demand, as a decreasing function of price and time, we set $D(p, t) = \alpha p^{-\beta} t$, where

$\alpha > 0$ and $\beta > 1$, ($a = \alpha p^{-\beta}$)

Z (T, p): The total annual profit

The total annual profit consists of (a) the sales revenue, (b) cost of placing orders, (c) cost of purchasing, (d) cost of carrying inventory (excluding interest charges), (e) cost of interest payable for items unsold after the permissible delay m (note that this cost occurs only if T > m), and (f) interest earned from sales revenue during the permissible period.

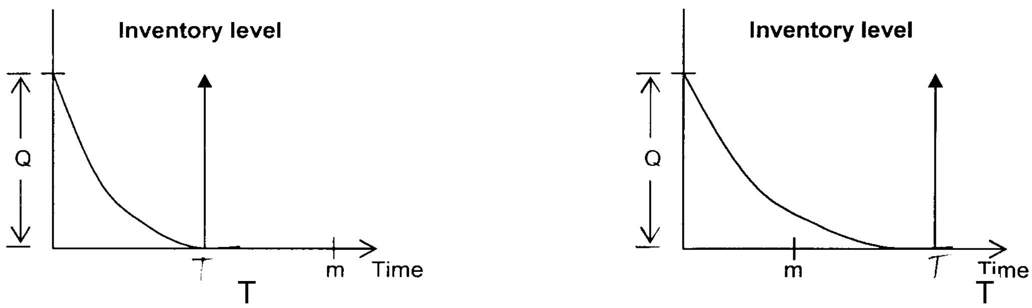
3. Mathematical Formulation

The level of inventory I(t) gradually decreases mainly to meet demands. Hence the variation of inventory with respect to time can be determined by the following differential equations:

$$\frac{dI(t)}{dt} = -D(p, t),\ 0 \le t \le T \tag{1}$$

$$\frac{dI(t)}{dt} = -at,\ 0 \le t \le T, \qquad [\text{where, } a = \alpha p^{-\beta}] \tag{2}$$

With boundary condition I(T) = 0. We have the following two possible cases based on the values of T and m. These two cases are given graphically in Fig. 1.



Case 1: T ≤ m   Case 2 : T ≥ m

Fig. 1: Graphical representation of two inventory systems

Case 1: T ≤ m

In this case, the customer sells $\frac{aT^2}{2}$ units in total by the end of the replenishment cycle time T, and has $\frac{caT^2}{2}$ to pay the supplier in full by the end of the credit period m. Consequently, there is no interest payable. However, the interest earned per year is

$$\frac{pI_d}{T}\left[\int_0^T at^2 dt + (m-T)\int_0^T at\,dt\right] = \frac{pI_d aT}{2}\left(m - \frac{T}{3}\right) \tag{3}$$

The total annual profit $Z_1(T, p)$ is

$Z_1(T, p)$= Sales revenue – Cost of placing order– Cost of purchasing – Cost of carrying inventory + interest earned per year.

$$Z_1(T, p) = \frac{paT}{2} - \frac{s}{T} - \frac{caT}{2} - \frac{haT^2}{3} + \frac{pI_d aT}{2}\left(m - \frac{T}{3}\right) \tag{4}$$

Case 2: T ≥ m

The buyer sells $\frac{am^2}{2}$ unit in total by the end of the permissible delay m and has $\frac{caT^2}{2}$ pay the supplier. The items in stock are charged at interest rate I_c by the supplier starting at time m. Therefore the buyer gradually reduces the amount of financed loan from the supplier due to constant sales and revenue received. As a result, the interest payable per year is

$$\frac{cI_c \int_m^T I(t)dt}{T} = \frac{cI_c}{T}\int_m^T \frac{a(T^2 - t^2)}{2}dt = \frac{cI_c a}{6T}(2T^3 - 3mT^2 + m^3) \quad (5)$$

During the permissible delay period, the buyer sells product and deposits the revenue into an account that earns I_d per dollar per year. Therefore, the interest earned per year is

$$\frac{pI_d \int_0^m at^2\, dt}{T} = \frac{pI_d am^3}{3T} \quad (6)$$

Hence the total annual profit $Z_2(T,p)$ is

$$Z_2(T, p) = \frac{paT}{2} - \frac{s}{T} - \frac{caT}{2} - \frac{haT^2}{3} + \frac{cI_c a}{6}\left(2T^2 - 3mT + \frac{m^3}{T}\right) + \frac{pI_d am^3}{3T} \quad (7)$$

Note that there are many different ways to calculate the interest payable as well as interest earned. For simplicity, we use Goyal's approach throughout this paper.

Hence the total annual profit Z(T,p) is written as

$$Z(T, p) = \begin{cases} Z_1(T, p) \text{ for } T \le m \\ Z_2(T, p) \text{ for } T \ge m \end{cases}$$

Although $Z_1(m,p) = Z_2(m,p)$, Z(T,p) is a continuous function of T either in (0, m) or in (m, ∞), but not in both.

4. Determination of the optimal replenishment time for given price

Differentiating (10) partially with respect to T, we get

$$\frac{\partial Z_1(T, p)}{\partial T} = \frac{ap}{2} + \frac{s}{T^2} - \frac{ca}{2} - \frac{2haT}{3} + \frac{pI_d am}{2} - \frac{pI_d aT}{3} \quad (8)$$

and $$\frac{\partial^2 Z_1(T, p)}{\partial T^2} = -\left(\frac{2s}{T^3} + \frac{2ah}{3} + \frac{pI_d a}{3}\right) < 0 \quad (9)$$

Again differentiating (13) partially with respect to T, we get

$$\frac{\partial Z_2(T, p)}{\partial T} = \frac{ap}{2} + \frac{s}{T^2} - \frac{ca}{2} - \frac{2haT}{3} - \frac{2cI_c aT}{3} + \frac{cI_c am}{2} + \frac{(cI_c - 2pI_d)am^3}{6T^2} \quad (10)$$

and $$\frac{\partial^2 Z_2(T,p)}{\partial T^2} = -\left[\frac{2s}{T^3} + \frac{a}{3}\left\{2h + 2cI_c + (cI_c - 2pI_d)m^3\right\}\right] < 0 \qquad (11)$$

For a fixed p, $Z_1(T,p)$ is strictly concave function of T. Thus there exists a unique value of T, which maximizes $Z_1(T,p)$. Also for a fixed p, $Z_2(T,p)$ is a concave function of T. Thus there exists a unique value of T_2 which maximizes $Z_2(T,p)$. $T^*=T_1$, is obtained by solving $\frac{\partial Z_1(T,p)}{\partial T} = 0$, i.e.

$$2(2h + pI_d)aT^3 - 3a(p - c + pI_dm)T^2 - 6s = 0 \qquad (12)$$

For example, let h = 0.65/unit/year, I_c = 0.09/\$/year, I_d = 0.06/\$/year, c = \$9.0 per unit, p = \$ 10 per unit, m = 2.0 year, s = 50, $\alpha=10^5$, $\beta=2$. Equation (18) becomes $38T^3 - 66T^2 - 3 = 0$, by trial, we get $T_1 = 1.76226$ year (approximately). At $T^* = T_1$, $Z_1(T,p)$ gives the optimal value (maximum value). And optimal (maximum) value of $Z_1(T,p)$ = \$ 926.6859114 (approximately).

Similarly T_2 is obtained by solving $\frac{\partial Z_2(T,p)}{\partial T} = 0$, we get

$$4a(h + cI_c)T^3 - 3a(p - c + cI_cm)T^2 - \{6s + (cI_c - 2pI_d)am^3\} = 0 \qquad (13)$$

For example, let h = \$0.60 / unit / year, I_c = 0.09/\$/ year, I_d = 0.03/\$/year, c = \$8.0 per unit, p = \$10 per unit, m = 2.0 year, s=200, $\alpha=10^5$, $\beta=2$. From (21), we get, $66T^3-129T^2 - 27 = 0$, by trial we get $T^* = T_2$ = 2.05173 year (approximately). And optimal (maximum) value of $Z_2(T,p)$ = \$ 1503.202202 (approximately).

(i) Particular case (A). If $c = p(1 + I_dm)$, from equation (8) we obtain

$$T = T_1 = \left\{\frac{3s}{a(2h + pI_d)}\right\}^{1/3} \qquad (14)$$

To ensure $T_1 \le m$, we substitute (14) into inequality $T_1 \le m$ and obtain that if only if,

$$3s \le a(2h + pI_d)m^3,\ T_1 \le m \text{ for } c = p(1 + I_dm) \qquad (15)$$

(ii) **Particular case (B).** If $p = c(1 - I_cm)$, from equation (13), we obtain,

$$T = T_2 = \left\{\frac{6s + (cI_c - 2pI_d)am^3}{4a(h + cI_c)}\right\}^{1/3} \qquad (16)$$

To ensure $T_2 \ge m$, we substitute (16) into inequality $T_2 \ge m$ and obtain that if and only if,

$$3s \ge a(2h + pI_d + {}^3/_2\, cI_c)m^3,\ T_2 \ge m, \text{ for } p = c(1 - I_cm) \qquad (17)$$

In classical EOQ model, the supplier must be paid for the items as soon as the customer receives them. It is a special case of (2) with m = 0, as a result,

$$T^* = \left\{\frac{3s}{2a(h + cI_c)}\right\}^{1/3} \qquad (18)$$

Z (T,p) is a continuous function of T either in (o, m) or in (m, ∞) but not in (o, ∞). We know from Theorem 1 below that Z(T,p) is not continuous in (0, ∞), but continuous in (o, m) and (m, ∞). For example choose c, p and I_d such that $c = p(1+I_dm)$, for this let c = \$6 per unit, p = \$5 per unit, I_d = 0.06/\$/year, m = $^5/_3$ year, s = 200, $\alpha = 10^6$, $\beta = 4.0$ and h = \$ 0.065/unit/year. We obtain Theorem 1 below that $3s \le a(2h + pI_d)m^3$ = 740.741 i.e. $Z(T,p) = Z_1(T,p)$ and optimal T^* = 1.55362 < m as shown in Fig. 2. For an example of case 2 (i.e. $Z(T,p) = Z_2(T,p)$). Choose c, p and I_c such that $p = c(1 - I_cm)$, let p = \$5 per unit, c = \$6 per unit, I_d = 0.06/\$/year, I_c = 0.1/\$/year, $\alpha = 10^6$, $\beta = 4.0$, s = 400, h = \$0.65/unit/year and m = $^5/_3$ year. Then we obtain from Theorem 1 that $3s \ge a(2h + pI_d + {}^3/_2\, cI_c)m^3$ = 1157.74, Z (T,p) = $Z_2(T,p)$ and the optimal T^* = 1.686865 > m, as shown in Figure 3.

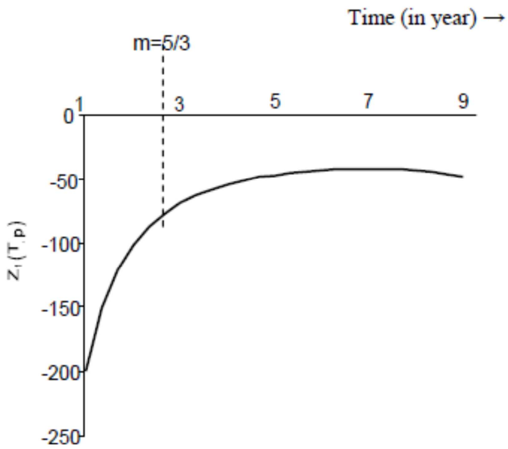


Fig.2.Case1.Particular case(A) when,

$c = p\,(1 + I_d m)$, $T \le m$

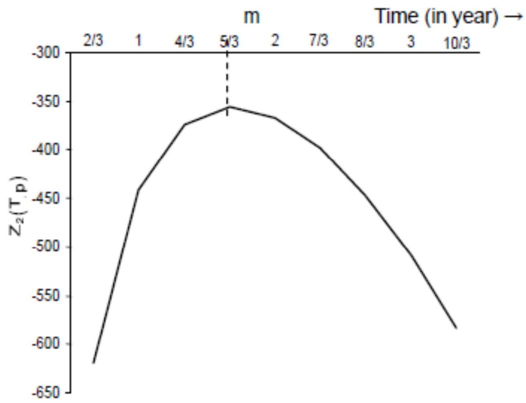


Fig.3. Case 2. Particular case (B), when

$p = c\,(1 - I_c m)$ & $T \ge m$

From (16), the optimal EOQ for case 1 (i.e. $T_1 \le m$) for $c = (1 + I_d m)$

$$Q^*(T_1) = \frac{a}{2}\left\{\frac{3s}{a(2h + pI_d)}\right\}^{2/3}, \quad \text{for } c = p\,(1 + I_d m) \tag{19}$$

From (16) into (1), we obtain

$$Z_1(p) = -\tfrac{1}{2}\,\{9as^2\,(2h + pI_d)\}^{1/3}$$
(20)

Again, the optimal EOQ for case 2 (i.e. $T_2 \ge m$) for $p = c\,(1 - I_c m)$

$$Q^*(T_2) = \frac{a}{2}\left\{\frac{6s + (cI_c - 2pI_d)am^3}{4a(h + cI_c)}\right\}^{2/3}, \quad \text{for } p = c\,(1 - I_c m) \tag{21}$$

Substituting (16) into (7), we obtain

$$Z_2(p) = -\tfrac{1}{4}\,\{4a\,(h + cI_c)\}^{1/3}\,\{6s + (cI_c - 2pI_d)am^3\}^{2/3} \tag{22}$$

From (18), the classical optimal EOQ is

$$Q^* = \frac{aT^{*2}}{2} = \frac{a}{2}\left\{\frac{3s}{2a(h + cI_c)}\right\}^{2/3} \tag{23}$$

By comparing (15) and (17), we have the following results:

Theorem 1: If

(i) $3s \le a\,(2h + pI_d)m^3$, for $c = p(1 + I_d m)$, then $T^* = T_1$

(ii) $3s \ge a\,(2h + pI_d + \tfrac{3}{2}\,cI_c)m^3$, for $p = c\,(1 - I_c m)$, then $T^* = T_2$

(iii) $3s = a\,(2h + pI_d + \tfrac{3}{2}\,cI_c)m^3$, for $p = c\,(1 - I_c m)$, then $T^* = m$.

Proof: It immediately follows from (15) and (17).

Similarly, from (19), (21) and (23), we have the following theorem :

Theorem 2: If

(i) $cI_c > 2\,pI_d$, for $c = p\,(1 + I_d m)$, then $Q^*(T_2)$ and $Q^*(T_1) > Q^*$

(ii) $cI_c < 2\,pI_d$, for $p = c\,(1 - I_c m)$, then $Q^*(T_2)$ and $Q^*(T_1) < Q^*$

(iii) $cI_c = 2\,pI_d$, for $p = c\,(1 - I_c m)$, then $Q^*(T_2) = Q^*$ and $Q^*(T_1) > Q^*$

Proof: It is obvious from (19), (21) and (23).

Note: Theorem 1 and 2 given above are obtained by particular cases (A) and (B).

5. Determination of the Optimal Price

Taking the first derivative of $(2h + pI_d + {}^3/_2\,cI_c)\,a(p)m^3$ with respect to p, we obtain

$$I_d\,a(p)m^3 + (2h + pI_d + {}^3/_2\,cI_c)\left(-\frac{\beta}{p}a(p)\right)m^3$$

$$= m^3\,\{(2h - {}^3/_2\,cI_c)\,a'(p) - I_d\,(\beta - 1)\,a(p)\} < 0 \qquad (24)$$

Hence $(2h + pI_d + {}^3/_2\,cI_c)\,a(p)m^3$ is a strictly decreasing function of p.

Using the fact in (17) , we set p_0 such that

$3s = a(p_0)\,(2h + p_0 I_d + {}^3/_2\,cI_c)\,m^3$

Therefore

$$Z(p) = \begin{cases} Z_1(p) = Z_1(T_1(p),p), & \text{for } p \le p_0 \\ Z_2(p) = Z_2(T_2(p),p), & \text{for } p \ge p_0 \end{cases}$$

To obtain the optimal price taking the first derivative of (20) with respect to p and setting the result to be zero, we have

$$\frac{dZ_1(p)}{dp} = \frac{(3s)^{2/3}}{6}a^{1/3}\left(\frac{\beta g_1^{1/3}}{p} - g_1^{-2/3}I_d\right) = 0 \qquad (25)$$

Where, $g_1 = (2h + pI_d)$

Next, we need to check the second order condition for concavity. That is

$$\frac{d^2Z_1(p)}{dp^2} = -\frac{(3s)^{2/3}\,a^{1/3}\,g_1^{-5/3}}{18}\left\{\frac{\beta(\beta+3)}{p^2}g_1^2 - 2\beta\,g_1 I_d - 2I_d^2\right\} < 0 \qquad (26)$$

From (22) we obtain the first order condition for $Z_2(p)$ as

$$\frac{dZ_2(p)}{dp} = \frac{4^{-2/3}\,(h + cI_c)^{1/3}\,a^{1/3}}{3p}\left[\beta g_2^{2/3} + 2am^3 g_2^{-1/3}\{-2(\beta-1)pI_d + \beta cI_c\}\right] = 0 \qquad (27)$$

Where, $g_2 = cI_c - 2pI_d$.

The second order condition for concavity is

$$\frac{d^2 Z_2(p)}{dp^2} = \frac{-g(\beta+3)a^{1/3}}{6p^2}\left\{\beta g_2^{2/3} + 2am^3 g_2^{-1/3}(2\beta\, pI_d + 2pI_d + \beta cI_c)\right\}$$
$$\frac{-ga^{1/3}}{p^2}\left[\frac{2}{3}\beta g_2^{-1/3} a(\beta\, cI_c - 2\beta\, pI_d + 2pI_d) + 2am^3\left\{\beta g_2^{-1/3} + \frac{1}{3}am^3 g_2^{-4/3}(-\beta cI_c + 2\beta\, pI_d - 2pI_d)\right\}\right.$$
$$\left.\left\{-2\beta\, pI_d + 2pI_d + \beta cI_c\right\} + 4am^3 g_2^{-1/3} p(\beta-1)I_d\right] < 0$$

Where, $g = \dfrac{4^{-2/3}(h + c\, I_c)^{1/3}}{3}$.

Based on the above discussion we develop the following algorithm:

Algorithm

Step 1. Determine p_0 on solving equation (17).

Step 2.If there exist p_1 such that $p_1 < p_0$, and $p_1$satisfies both the first order condition as in (25)and the second order condition for concavity as in (26),then we find $T_1(p_1)$ by (14), and $Z_1(T_1(p_1), p_1)$ by (20).

Step 3. If there exists a p_2 such that $p_2 > p_0$, and p_2 satisfies both the first order condition as in (27)and the second order condition for concavity as in (28), then calculate $T_2(p_2)$ by (16), and $Z_2(T_2(p_2), p_2)$,by (22).

Step 4 If $Z_1(T_1(p_1), p_1) > Z_2(T_2(p_2), p_2)$, then optimal total annual profit is $Z^*(T(p^*), p^*) = Z_1(T_1(p_1), p_1)$ otherwise optimal total annual profit is $Z^*(T(p^*), p^*) = Z_2(T_2(p_2), p_2)$.

6. Numerical Examples

Example 1. For generality, we use the following example in which $cI_c < 2p^*I_d$.Given h= .5/unit/year, I_c =0.09/\$/year, I_d= 0.06/\$/year, c= \$ 4.5 year, s= \$ 200/per order α= 100000, and, $\beta = 2$. We obtain the computational results for various values of m as shown in Table 1.

Table 1.

Optimal solution for different Trade credit period 'm'.

M (days)	p_0	p*	T*	Q	Z*
10	7.048268	p_1 = 4.554794	T_1 = 0.460667	511.4542	- 651.2292
20	7.73048	p_1 =4.609589	T_1 = 0.463956	506.5230	- 646.6136
30	8.042800	p_1 =4.664384	T_1 =0.467224	501.6857	-642.0898
40	8.217605	p_1 =4.719178	T_1 =0.470474	496.9452	-637.6551
50	8.329943	p_1 =4.773973	T_1 =0.473704	492.2936	-633.3064
60	8.408180	p_1 =4.828767	T_1 =0.476916	487.7320	-629.0413
70	8.465783	p_1 =4883562	T_1 =0.480110	483.2569	-624.8572
80	8.509957	p_1 =4.938356	T_1 =0.483285	478.8636	-620.7517
90	8.544907	p_1 =4.993151	T_1 =0.486442	474.5508	-616.7224
100	8.573246	p_1 =5.047945	T_1 =0.489582	470.3181	-612.7672
110	8.596687	p_1 =5.102740	T_1 =0.492705	466.1622	-608.8839
120	8.616398	p_1 =5.157534	T_1 =0.495810	462.0791	-605.0705
130	8.633205	p_1 =5.212329	T_1 =0.498898	458.0697	-601.3249

140	8.647704	p_1 =5.267123	T_1 =0.501970	454.1283	-597.6453
150	8.660341	p_1 =5.321918	T_1 =0.505025	450.2559	-594.0298
160	8.671452	p_1 =5.376712	T_1 =0.508064	446.4505	-590.4767
170	8.681298	p_1 =5.431507	T_1 =0.511087	442.7096	-586.9843
180	8.690084	p_1 =5.486301	T_1 =0.514094	439.0316	-583.5510
190	8.69797	p_1= 5.541096	T_1 =0.517085	435.4141	-580.1751
200	8.70509	p_1 =5.595890	T_1 =0.520061	431.8573	-576.8552
210	8.711551	p_1= 5.650685	T_1 =0.523022	428.3588	-573.5899
220	8.717438	p_1= 5.705479	T_1 =0.525967	424.9157	-570.3775
230	8.722826	p_1= 5.760274	T_1 =0.528898	421.5291	-567.2169
240	8.727775	p_1= 5.815068	T_1 =0.531814	418.1961	-564.1068
250	8.732336	p_1= 5.869863	T_1 =0.534716	414.9162	-561.0458
260	8.736555	p_1= 5.924658	T_1 =0.537603	411.6867	-558.0327
270	8.740467	p_1= 5.979452	T_1 =0.540476	408.5075	-555.0664
280	8.744105	p_1= 6.034246	T_1 =0.543335	405.3772	-552.1457
290	8.747467	p_1= 6.089041	T_1 =0.546180	402.2942	-549.2694
300	8.750667	p_1= 6.143836	T_1 =0.549012	399.2588	-546.4366
310	8.753636	p_1= 6.198630	T_1 =0.551829	396.2667	-543.6462
320	8.756423	p_1= 6.253425	T_1 =0.55463	393.3210	-540.8972
330	8.759044	p_1= 6.308219	T_1 =0.557425	390.4176	-538.1886
340	8.761513	p_1= 6.363014	T_1 =0.560204	387.5580	-535.5195
350	8.763843	p_1= 6.417808	T_1 =0.562969	384.7384	-532.8890
360	8.766046	p_1= 6.472602	T_1 =0.565722	381.9604	-530.2962
380	8.770109	p_1= 6.582192	T_1 =0.571189	376.5205	-525.2202
400	8.774437	p_1= 6.691781	T_1 =0.576607	371.2328	-520.2852
420	8.777089	p_1= 6.801370	T_1 =0.581976	366.0896	-515.4848
440	8.780109	p_1= 6.910959	T_1 =0.587298	361.0858	-510.8135
460	8.782869	p_1= 7.020548	T_1 =0.592574	356.2158	-506.2658
480	8.785402	p_1= 7.130137	T_1 =0.597804	351.4729	-501.8365
500	8.787735	p_1= 7.239726	T_1 =0.602990	346.8534	-497.5207
520	8.789891	p_1= 7.349315	T_1 =0.608132	342.3513	-493.3140
540	8.791888	p_1= 7.458904	T_1 =0.613231	337.9622	-489.2120
560	8.797785	p_1= 7.568493	T_1 =0.618288	333.682	-485.2106
580	8.795473	p_1= 7.678082	T_1 =0.623304	329.5068	-481.3060
600	8.797088	p_1= 7.787671	T_1 =0.628280	325.4328	-477.4943
620	8.798600	p_1= 7.897260	T_1 =0.633216	321.4555	-473.772
640	8.800018	p_1= 8.006844	T_1 =0.638113	317.5718	-470.1364
660	8.801351	p_1= 8.116438	T_1 =0.642972	313.7785	-466.5836

680	8.802606	p_1= 8.226027	T_1 =0.647793	310.0718	-463.1110
700	8.803790	p_1= 8.335616	T_1 =0.652577	306.4489	-459.7157
720	8.804908	p_1= 8.445205	T_1 =0.657325	302.9074	-456.3949
740	8.805966	p_1= 8.554794	T_1 =0.662038	299.4448	-453.1462
760	8.806970	p_1= 8.66438	T_1 =0.666716	296.0577	-449.9669
780	8.807922	p_1= 8.773973	T_1 =0.671359	292.7434	-446.8548
800	8.808827	p_1= 8.883562	T_1 =0.675968	289.4996	-443.8076
820	8.809688	p_1= 8.993151	T_1 =0.680545	286.325	-440.8233
860	8.811291	p_1= 9.212329	T_1 =0.689600	280.1726	-435.0349
900	8.812753	p_1= 9.431507	T_1 =0.698528	274.2682	-429.4744
940	8.814090	p_1= 9.650685	T_1 =0.707334	268.5980	-424.1276
980	8.815319	p_1= 9.869863	T_1 =0.716021	263.1475	-418.9818
1020	8.816453	p_1=9.0890411	T_1 =0.724594	257.9050	-414.0252
1060	8.817501	p_1= 10.308219	T_1 =0.733054	252.8568	-409.2469
1100	8.818473	p_1= 10.527397	T_1 =0.741406	247.9934	-404.6367
1140	8.819377	p_1= 10.746575	T_1 =0.749652	243.3040	-400.1855
1180	8.820221	p_1= 10.965753	T_1 =0.757797	238.7805	-395.8846
1220	8.821009	p_1=11.184932	T_1 =0.765841	234.4124	-391.7260
1260	8.821747	p_1= 11.404110	T_1 =0.773789	230.1931	-387.7020
1300	8.822441	p_1= 11.623288	T_1 =0.781643	226.1147	-383.8068

Q*= 407.8278

Table 1 reveals that (a) a higher value of trade credit period ‘m’ causes a higher value of Z* and higher values of p* and T*. (b) a higher value of ‘m’ causes a lower value of Q*(T). From equation (23) the classical EOQ, Q*= 407.8278 which confirms the result in part (b) of Theorem 2 (i.e. $Q^*(T_1) < Q^*$, if $cI_c < 2pI_d$),which is applicable only for credit period 280 days or more than 280 days. Less than 280 days credit period Theorem 2 contradicts the hypothesis .From the above example we are unable to obtain any value of p_2 which is greater than or equal to p_0.Hence we consider only T_1*, p_1*, $Q^*(T_1)$ and $Z(T_1^*)$ only to compare the result. The special cases (A) and (B) are applicable for limited range, limited value of credit periods for managerial point of view.

7. Conclusion and Future Research

In this paper, we developed an appropriate pricing and lot sizing model for a retailer when the supplier provides permissible delay in payments. We establish the necessary and sufficient conditions for the unique optimal replenishments interval by taking particular cases i.e. case (A) and case (B). Next we derive the first and second order conditions for finding the optimal price. We establish Theorem 1, which provides us to obtain the optimal replenishment interval by taking particular case (A) and case (B). We also obtained Theorem 2 on these particular cases, we also verified case $T_1 \leq m$, for $c= p(1+I_d m)$ and $T_2 \geq m$, for $p= (1-I_c m)$.On these particular cases (A) and (B), the total annual profit is negative which gives us contradictory results. On particular cases (A) and (B) we obtained total annual loss (due to negative sigh of $Z_1(T, p)$ and $Z_2(T,p)$, while Fig 2 and Fig.3 proves the theoretical results (curve is concave in both the cases). Numerical example is given to illustrate the model.

The model proposed in this paper can be extended in several ways. For instance, we may extend the model by considering time dependent deterioration rate. Also we could consider the demand as a function if quantity. We could generalize the model to allow for shortage, quantity, discounts and inflation rates etc.

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