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Statistical Models and Parametric Methods to Estimate the Reliability and Hazard Rate Functions of Weibull Distribution

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Abstract
This article aimed to explain the statistical models and parametric methods in order to estimate and evaluate the reliability and hazard rate functions of Weibull distribution to productivity machines of the Textile department-national establishment of the Jordanian textiles in the industrial king Abdullah II City. Also, the article explained the maximum likelihood estimation method to estimate the two parameters of the Weibull distribution. To achieve the study objectives, we depend on the machines work times for the period (1/11/2013 – 31/1/2014).
Keywords: Weibull Distribution, Reliability Function, Hazard Rate, Parametric Methods, Maximum Likelihood Estimation.

1. Introduction
One of the most appealing classical techniques used to fit the statistical models and parametric methods of data as well as providing estimates for the parameters of a model by the Maximum Likelihood Estimation (MLE) method. Lawless, (2003) used the statistical models and methods for lifetime data, and discussed a variety of situations where Weibull distribution has been used to analyze the reliability function. The application of the Weibull distribution in modeling and analyzing reliability function has also been described extensively by (Abed-Ali, et al. 2009) estimated the scale and shape parameters of Weibull distribution by maximum likelihood estimation method. The maximum likelihood estimation method is usual frequents approach in the estimation of parameters for parametric reliability data. The first obtained the estimates under maximum likelihood by making use of the Newton- Raphson maximum likelihood estimation method. Also, Touama & Basha, (2014) used the maximum likelihood estimation method to estimate the parameter of the Exponential distribution, in order to estimate the reliability of the productivity machines and it's availability of AL-qastal factory for producing juice and carbonated soft drinks in Jordan.

2. Methodology
2.1. The Study Problem
The study problem could be explained that, there is no exist a case of modernization to evaluate the reliability and hazard rate functions in National establishment of the Jordanian textiles, and the establishment is lack of the operation scientific approach use to estimate the reliability and hazard rate of the textile department productivity machines.

2.2. The Study Objects
The study objects are given as follows:
a. To identify the statistical models and parametric methods, in which used to estimate the reliability of the productivity machines and the hazard rate function.
b. To explain the maximum likelihood estimation (MLE) method, in which used to estimate the scale (α) and shape (β) parameters.
c. To measure the differences between the work times of the textile department productivity machines.
d. To offer some conclusions and recommendations to the decision making in the National establishment of the Jordanian textiles in the industrial king Abdullah II city.

2.3. The Study Hypothesis
The study hypothesis given in a null form (H_0) as follows:
H_0: There are no statistically significant differences at the significant level ($\alpha = 0.05$), between the work times of the textile department productivity machines.

3. The Theoretical Part
3.1. Weibull Distribution
Is the most useful distribution in reliability analysis by adjustment the distribution parameters that enable us to work fit for the distributions that express to the times or (ages). The first of the physical world discovered by

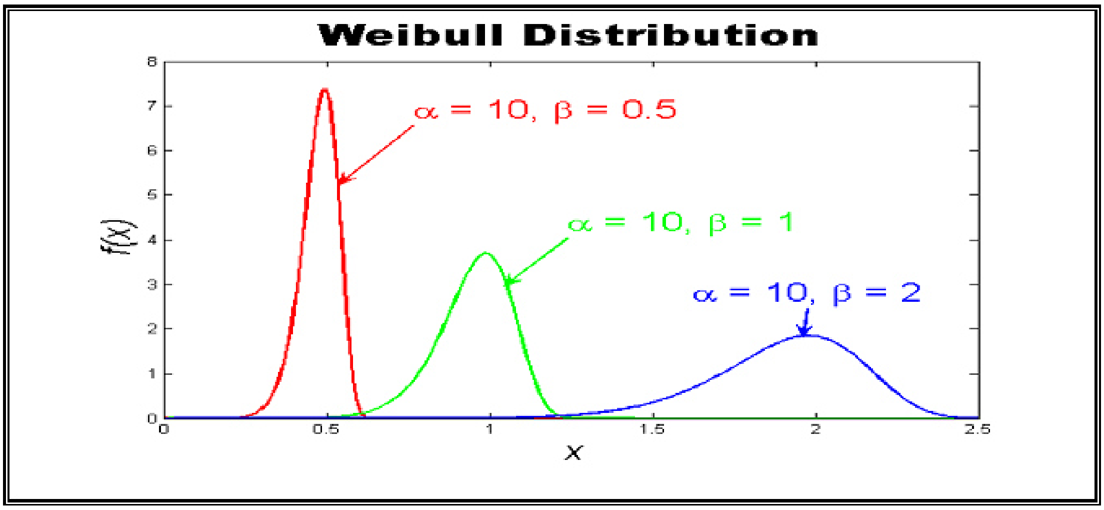
Waloddi Weibull a (Swedish scientist)(1939) and was named the distribution in his name, and use it in reliability analyzes successfully (1951) through the research published about the failures of seven models were characterized by difficulty describe the behavior of data circulating in the distributions (Abernethy, 2004). It is appropriate to say that Weibull distribution describes in a comprehensive manner all the stages of the machine Lifetime, it describes the phenomena of decreasing and increasing the failure rate, in addition to the phenomenon of stability. Weibull distribution is defined by two parameters (α, β), where the probability density function (PDF) for distribution expressed by the following equation (Ebeling, 1997), (Abernethy, 2004):

$$f(t_i ;\alpha, \beta) = (\beta/\alpha)\, (t / \alpha)^{\beta-1} \exp\text{-}(t / \alpha)^{\beta} \qquad , t \geq 0$$

Whereas:

α : Scale parameter.

β : Shape parameter.



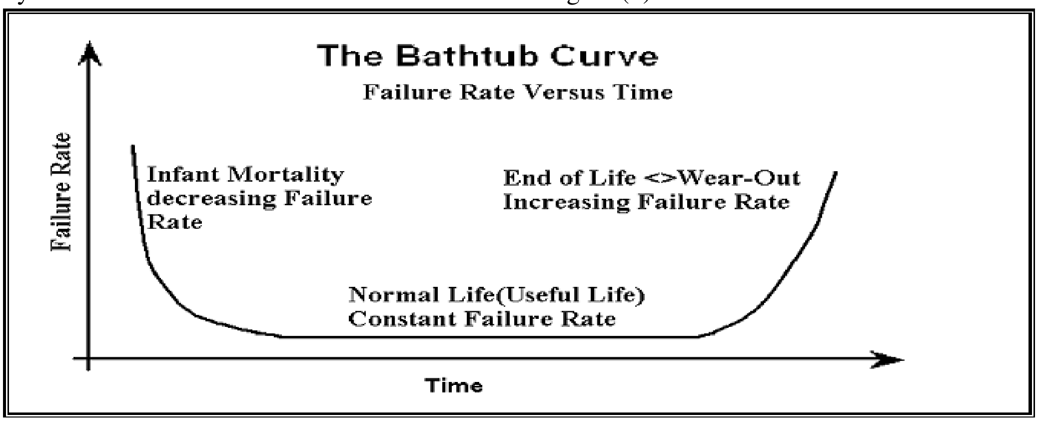
Source: Sarhan, A.M. and Zaindin, M., (2009).

Figure 1: Graphical representation of Weibull distribution density function different values of (α & β).

3.2. The Hazard Rate Curve

Knowing the hazard rate (failure rate) is very important for the machine is used to denote the change that gets them through their lifetime. The relationship between the failure rate and time represented by (Bathtub Curve), as shown in Figure (1), and the diagram shows three phases of the curve. Where is divided into three stages in terms of time, each stage reflects the nature of the failures and the probability distribution and failure rate appointees, and the time here represents the machine operational age (Lewis, 1997).

The failure rate function h(t) is expressed by the following equation, and it seems that the function is not a constant shape along the horizontal extent which is similar to the (Bathtub Curve) diagram or the diagram of the machine lifetime curve, therefore, the distribution represents all stages of the machine lifetime. Accordingly, the beginning of the curve is decreasing, after that to be constant, as in the Exponential distribution and finally be increased as in the Normal distribution. The Figure (2) illustrates that as follows:



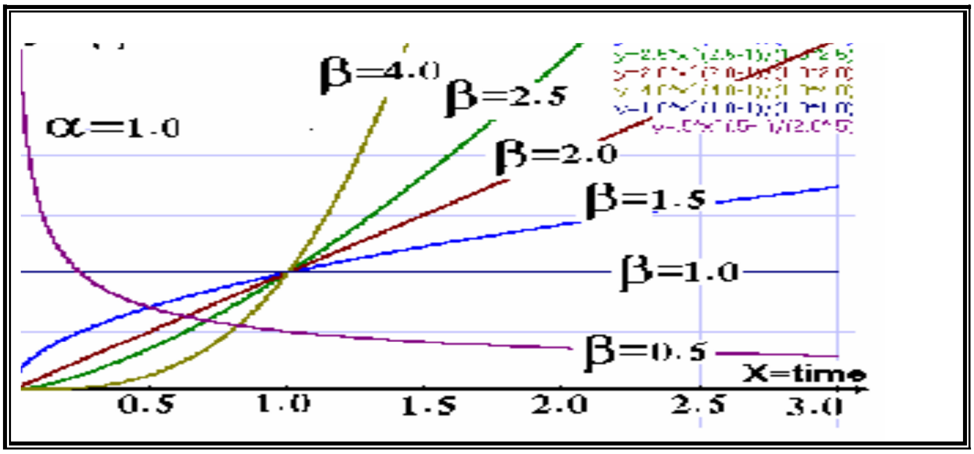
Source: Lewis, E. E., (1997).

Figure 2: Failure rate diagram (Bathtub Curve)

3.3. The Value of (β) and Weibull Analysis (Weibayes)

The determination of the shape and type of the distribution, which is behavioral by the equipment failure data for specified distribution, and follow the steps listed above lends some difficulty to extract the results, and the method can be applied more ease and flexibility to reach at the same desired aim. And by using derived from the analysis of the Weibull distribution, called Weibayes (Abernethy, 2004).

The Weibull distribution addition to describing as one of unknown failure models, it can be used for quick and easy rule for goodness of fit for the behavior of the data for a particular distribution, through the analysis of the value a parameter (β) for Weibull distribution . And Figure (3) illustrates that as follows:



Source: Ebeling, C. E., (1997), Abed-Ali, et al, (2009).

Figure 3: The effect of (β) on the shape of failure rate curve

From the previous figure (2), can be distinguished by the value of some forms (β) as follows (Ebeling, 1997), (Abed-Ali, et al, 2009):

0 <β <1: Decreasing failure rate (DFR), and is expressed in the first stage.
β = 1: The distribution approach to exponential distribution, and the failure rate constant, and is expressed in the second stage.
1 < β <2: Special case, increasing failure rate (IFR), and to be as Concave.
β = 2: Special case, and represents the Rayleigh distribution of the electronic devices.
β > 2: Increasing failure rate (IFR), and to be as (Convex), and is expressed in the third stage.
3 < β <4: The distribution approach to Normal distribution, and increasing failure rate (IFR), and is expressed in the third stage.

The use of Weibayes have extended to include the maintenance works by (Minimize) the cost to a minimum, in order to get the best replacement time has talked preventive maintenance (PM). And the adoption of Weibayes and the use of value (β), can see which parts of the machine needs to be done preventive maintenance, and which does not require for this maintenance schedule, but only corrective maintenance (Lewis, 1997), (Barringer, 2004).

3.4. Maximum Likelihood Estimation (MLE)

There are many ways to estimate the parameters (α, β) based on a set of **n** random work times ($t_1, t_2,\ldots,t_n$) from Weibull distribution with two parameters α and β. The probability density function (PDF) of Weibull distribution is given as follows:

$$f(t_i ;\alpha, \beta) = (\beta/\alpha)\,(t / \alpha)^{\beta-1} \exp-(t / \alpha)^{\beta} \qquad , t \geq 0 \qquad ,\ldots(1)$$

The maximum likelihood estimation is generally the most versatile and popular method. Although MLE in the Weibull case requires numerical methods and a computer, that is no longer an issue in today's computing environment.

When ($t_1, t_2,\ldots,t_n$) with density $f_{\alpha,\beta}(t)$, then the maximum likelihood estimation of (α,β) is that value $\alpha\hat{}$, $\beta\hat{}$ which maximizing the likelihood, is given as follows:

$$L(t_1, t_2,\ldots, t_n, \alpha, \beta) = \prod_{i=1}^{n} f_{\alpha,\beta}(t_i) \qquad ,\ldots(2)$$

Over α, β which gives highest local probability to the observed sample ($t_1, t_2,\ldots,t_n$), i.e.:

n

$$L(t_1, t_2,\dots, t_n, \alpha, \beta) = \sup \{\prod_{i=1} (\beta/\alpha)(t/\alpha)^{\beta-1} \exp-(t/\alpha)^{\beta}\} \qquad ,\dots(3)$$

Often such maximizing values α^ & β^ are unique and one can obtain them by solving equation (2) with respect to α & β, as follows:

$$\partial L/\partial \alpha = \partial/\partial \alpha \{\prod_{i=1}^{n} (\beta/\alpha)(t/\alpha)^{\beta-1}\exp-(t/\alpha)^{\beta}\} = 0 \qquad ,\dots(4)$$

$$\partial L/\partial \beta = \partial/\partial \beta \{\prod_{i=1}^{n} (\beta/\alpha)(t/\alpha)^{\beta-1}\exp-(t/\alpha)^{\beta}\} = 0 \qquad ,\dots(5)$$

Since taking derivatives of a product is tedious (product rule) one usually resorts to maximizing the log of the likelihood, i.e.:

$$l(t_1, t_2,\dots, t_n, \alpha, \beta) = \log \{L(t_1, t_2,\dots, t_n, \alpha, \beta)\} = \sum_{i=1}^{n} \log \{f_{\alpha,\beta}(ti)\} \qquad ,\dots(6)$$

It is a lot simpler to deal with the likelihood equations, i.e.:

$$\partial l/\partial \alpha = \sum_{i=1}^{n} \partial/\partial \alpha \ \log \{(\beta/\alpha)(t/\alpha)^{\beta-1}\exp-(t/\alpha)^{\beta}\} \qquad ,\dots(7)$$

$$\partial l/\partial \beta = \sum_{i=1}^{n} \partial/\partial \beta \ \log \{(\beta/\alpha)(t/\alpha)^{\beta-1}\exp-(t/\alpha)^{\beta}\} \qquad ,\dots(8)$$

when solving the above equations (7) and (8), we can get the estimates of the scale (α^) and the shape (β^) parameters, and after we substitutes α^ and β^ in the functions of hazard rate r(t) and reliability R(t), which explained by the following formulas:

$$\hat{r}(t) = (\beta/\alpha)(t/\alpha)^{\beta-1} \qquad ,\dots(9)$$

And the formula of reliability function is given as follows:

$$\hat{R}(t) = \exp-(t/\alpha)^{\beta} \qquad ,\dots(10)$$

4. The Applied Part

4.1. Collection Data

The study data depends upon the work times of the machines in the Textile department-national establishment of the Jordanian textiles in the industrial king Abdullah II City for the period (1/11/2013–31/1/2014). To achieve the study objects, a great number of the Textile department machines and used for performance of the suggested models for production by the design section. The researcher selects five types of machines, for being shared in the establishment production and from different countries origin, which were (Vamatex, Santex, Twisting, Punching, and Sewing).

Thereafter, the actual time for the five machines work to be studied was registered, till the damage breakdown occurrence for the mentioned period, where the machine damage should be repaired and remake ready for work.

4.2. Statistical Analysis of Data

Before starting to estimate the reliability and hazard rate functions, some tests to be made on the study data must be verified, as follows:

a. Shapiro-Wilk Test:

This test was used to verify whether the machines work times data are followed the normal distribution or not, through the following statistical hypothesis:

H_0: Machines work times data are followed the Normal Distribution.

To test the previous hypothesis, was used **Shapiro-Wilk Test,** as shown in Table (1) as follows:

Table 1. The Shapiro -Wilk test results

No.	The Machines	Shapiro-Wilk statistic	Sig.
1	Vamatex	0.818	0.157
2	Santex	0.973	0.683
3	Twisting	0.937	0.514
4	Punching	0.821	0.165
5	Sewing	0.925	0.472

The results in table (1), shows that all the statistical significance (Sig.) values for the machines are higher than the significance level ($\alpha = 0.05$). Based on the results, the null hypothesis (H_0) has been **accepted**, that is mean the (machines work times data are followed the Normal Distribution).

b. Test the Study Hypothesis

H_0: There are no statistically significant differences at the significant level ($\alpha = 0.05$), between the work times of the textile department productivity machines.

To test the above hypothesis, was used **Runs Test,** as shown in Table (2) as follows:

Table 2. The Runs Test results

No.	The Machines	Test Value	No. of Runs	Z values	Sig.
1	Vamatex	175	2	0.000	1.000
2	Santex	62	2	-1.637	0.102
3	Twisting	116	4	0.612	0.540
4	Punching	94	4	0.109	0.913
5	Sewing	86	2	- 0.982	0.326

Critical value of (Z) is (1.96) at the significant level ($\alpha = 0.05$).

The results in Table (2), explained that the calculated values of (Z) for all machines are less than the critical value of (Z) (1.96) at the significant level ($\alpha = 0.05$), as well as that all p-values (Sig.) are greater than the significant level ($\alpha = 0.05$). Based on the results, the null hypothesis (H_0) has been **accepted**, that is mean (there were no statistically significant differences at the significant level ($\alpha = 0.05$), between the work times of the textile department productivity machines).

4.3. Estimation the Scale and Shape Parameters of Weibull Distribution

The results in Table (3), refers to the estimation of the scale and the shape parameters of Weibull distribution, as follows:

Table 3. Estimation the Scale and the Shape Parameters

No.	The Machines	Scale Parameter ($\hat{\alpha}$)	Shape Parameter ($\hat{\beta}$)
1	Vamatex	160.0	1.02
2	Santex	68.6	1.81
3	Twisting	120.0	1.17
4	Punching	96.0	1.25
5	Sewing	94.4	1.32

4.4. Estimation of the Hazard Rate and Reliability Functions

We got the estimators of the Hazard Rate (failure rate) $\hat{r}(t)$ of the machines, and the Reliability function $\hat{R}(t)$ using the relations (9) and (10) respectively, as shown in Table (4) as follows:

Table 4. Estimation of the Hazard Rate and Reliability Functions

No.	The Machines	T_i / hrs	$\hat{r}(t)$	$\hat{R}(t)$	Rank
1	Vamatex	50	0.006	0.737	1
2	Santex	50	0.020	0.569	5
3	Twisting	50	0.008	0.698	2
4	Punching	50	0.012	0.643	4
5	Sewing	50	0.011	0.649	3

The results in Table (4), refers to the machine reliability $\hat{R}(t)$ of type **(Vamatex)** is considered to be the highest and coming in the first rank, comparing thereof with the reliability of the other types of machines, and the reliability machine of type (**Vamatex**) was coupled with the lower hazard rate [$\hat{r}(t)$ = 0.006].

The machine of type **(Twisting)** comes at the second rank with respect of it's reliability, and it is followed by the machine of type **(Sewing)** at the third rank, and the machine of type **(Punching)** comes at the forth rank, finally the machine of type **(Santex)** comes at the fifth rank with higher hazard rate [$\hat{r}(t)$ = 0.02].

5. Conclusions and Recommendations

5.1. Conclusions

a. The results of Shapiro-Wilk test, shows that the all machines work times are followed the Normal Distribution.

b. The results of Runs test, refers to there were no statistically significant differences at the significant level (α = 0.05), between the work times of the textile department productivity machines.

c. The results of the Reliability analysis, shows that the machine reliability of type (Vamatex) is considered to be the highest and coming in the first rank.

d. The results of the Hazard analysis, refers to the machine failure rate of type (Vamatex), is considered to be the lowest, comparing with the failure rate of the other types of machines

5.2. Recommendations

a. Necessarily design an information system for the productivity machines failures, in order to maintain a file about each machine, to document the failures through daily or weekly reports for easy review information on cases of failure about any machine.

b. Interest by the work conditions of the productivity machines through out qualifying technical cadres of high efficiency, and reserve the necessary materials for the production operation duration in the Jordanian textiles establishment.

c. Modernization the productivity machines specifications which would be importation from international countries origin out matched by the high techniques, therefore which leads to decrease the production costs and increasing the productivity machines in the Jordanian textiles establishment.

d. Suggestion a new special administration for the reliability named (Reliability Administration), as a result of the development nature in the reliability analysis field.

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