

Measures of Risk on Variability with Application in Stochastic Activity Networks

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Abstract

We propose a simple measure of variability of some of the more commonly used distribution functions in the class of New-Better-than-Used in Expectation (NBUE). The measure result in a ranking of the distributions, and the methodology used is applicable to other distributions in NBUE class beside the one treated here. An application to stochastic activity networks is given to illustrate the idea and the applicability of the proposed measure.

Keywords: Alternative Risk measure, Portfolio, Coefficient of Variation, Skewness, Project management, Stochastic activity networks.

1. Introduction

An important issue that looms high in the management of real life project is that of risk under conditions of uncertainty. "Risk" is may be used as a synonymous with "Variability", which is best estimated by the variance of the completion time of 'milestone events', including the project completion time, or the project cost, or the resources consumed in its execution or might be R&D see, e.g., (Elamaghraby, 2005; Wang &Hwang, 2007; Durbach & Stewart, 2009). On the other hands, Liu & Liu (2002) defined the expected value and variance for measuring the portfolio return and the risk, respectively. Therefore, one of the most basic measure of risk is the standard deviation or variance. However, other measures of risk may be considered as well see, e.g., (Yitzhaki, 1982; Shailt & Yitzhaki, 1989; Konno & Yamazaki, 1991; Kijima & Ohnishi, 1993; Duffie & Pan, 1997; Levy, 1998; Ringuest et al., 2000; Leshno & Levy, 2002; Tang, 2006; Neiger et al., 2009). Graves & Ringuest (2009) discussed the well known six decision making methods: 1. Mean-Variance, 2. Mean-Semivariance, 3. Mean-Critical Probability, 4. Stochastic Dominance, 5. Almost Stochastic Dominance, and 6. Mean-Gini. They showed the strength and limitations of each technique, and the relationships between the various methodologies. On the other hand, one would prefer a portfolio return with larger degree of asymmetry when the mean and variance are same. A concept of skewness is introduced by Li et al. (2010) to measure the asymmetry of fuzzy portfolio return. The squared coefficient of variation $[CV\,](=variance\,/\,mean)$ is used to characterize the variability of a distribution see, e.g., (Nuyens & Wierman, 2008). Distributions with $CV^2 > 1$ are said to have high variability and distributions with $CV^2 < 1$ are said to have low variability.

Most of the above literature discussed and compared measures of risk in portfolio decision making. In this paper, we propose a measure that assists in comparing variances, hence variability. This measure of risk is used to rank many proposed distribution functions and applying it in project management. Besides, it can be applied in portfolio decision making when different distributions are used. The concept of skewness as well as the squared coefficient of variation are also considered. An application to stochastic activity networks (SANs) is given to illustrate the effectiveness of the proposed measure.

Many probability distributions are discussed in literature as representative of a project activity times. To the best of our knowledge, all suggested distributions fall in the class of 'New Better than Used in Expectation (NBUE)' distributions. For the sake of self-containment we state the distributions that are most frequently discussed in treatises on stochastic activity networks, their mean (denoted by $E(.)$ or μ), variance (denoted by $Var(.)$ or σ^2) and skewness (denoted by $\gamma_1 = \mu_3 / \sigma^3$, where μ_3 is the third central moment and σ is the standard deviation).

- *The Generalized Beta distribution* with probability density function

$$f(x) = \frac{1}{(b-a)^{k_1+k_2-1} B(k_1,k_2)} (x-a)^{k_1-1} (b-x)^{k_2-1}, \quad a \le x \le b, \tag{1}$$

where $B(k_1,k_2)$ is the complete Beta function. $B(k_1,k_2)=\Gamma(k_1)\Gamma(k_2)/\Gamma(k_1+k_2)$, and $\Gamma(k)=(k-1)!$ if k is a positive integer, and $\Gamma(k)=\int_0^\infty t^{k-1}e^{-t}dt$ if $k>0$. The mean, the variance and skewness are given by

$$E(X)=a+(b-a)\frac{k_1}{k_1+k_2}, \qquad Var(X)=(b-a)^2\frac{k_1k_2}{(k_1+k_2)^2(k_1+k_2+1)},$$
$$\gamma_1=\frac{2(k_2-k_1)\sqrt{(1+k_1+k_2)}}{\sqrt{(k_1k_2)}(2+k_1+k_2)}. \tag{2}$$

The beta distribution with $k_1=2$ and $k_2=3$ is discussed by Golenko (1989).

- *The Beta-PERT distribution,* so designated because it was the distribution initially proposed by Malcolm et al. (1959) in their seminal paper on PERT (Program Evaluation and Review Technique). The proposed mean and variance of activity time distribution are given by

$$E(X)=\frac{a+4m+b}{6} \quad and \quad Var(X)=(\frac{b-a}{6})^2, \tag{3}$$

where a, m, and b are; respectively, the 'optimistic', 'most likely' and 'pessimistic' times of the activity. Equation (3) holds when $k_1=k_2=4$, or $k_1=3+\sqrt{2}$ and $k_2=3-\sqrt{2}$, or $k_1=3-\sqrt{2}$ and $k_2=3+\sqrt{2}$ in (2) above. The latter two special cases satisfy $k_1+k_2=6$, see (Elmaghraby, 1977, p236). The skewness will be $0, \frac{-1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$ in the above three cases respectively.

- *The Uniform distribution*, defined on the range $x\in[a,b]$ is discussed by Kleindorfer (1971), with probability density function

$$f(x)=\frac{1}{b-a}, \qquad a\le x\le b,$$

with mean, variance and skewness are given by

$$E(X)=\frac{a+b}{2}, \quad Var(X)=\frac{(b-a)^2}{12} \quad and \quad \gamma_1=0. \tag{4}$$

- *The Triangular distribution* discussed by Williams (1992) and Johson (1997), with probability density function

$$f(x)=\begin{cases}\frac{2(x-a)}{(m-a)(b-a)}, & a\le x\le m\\ \frac{2(b-x)}{(b-a)(b-m)}, & m<x\le b\end{cases}$$

where the range of the random variable (r.v.) is defined on the interval $[a,b]$, $m\in[a,b]$ and m stands for the mode. The mean, variance and skewness are given by

$$E(X)=\frac{a+m+b}{3}, \qquad Var(X)=\frac{1}{18}(a^2+b^2+m^2-ab-am-bm)$$

and

$$\gamma_1=\frac{\sqrt{2}(a+b-2m)(2a-b-m)(a-2b+m)}{5(a^2+b^2+m^2-ab-am-bm)}. \tag{5}$$

- *The exponential distribution* with parameter λ is discussed by many authors (Kambrowski ,1985 and Kulkarni & Adlakha, 1986) with probability density function

$$f(x)=\lambda e^{-\lambda x}, \qquad x\ge 0, \qquad \lambda>0,$$

with mean, variance and skewness are given by

$$E(X)=\frac{1}{\lambda}, \quad Var(X)=\frac{1}{\lambda^2} \quad and \quad \gamma_1=2. \tag{6}$$

- *The Erlang distribution,* discussed by Bendell et al. (1995) and their work is generalized by Abdelkader (2003) with probability density function

$$f(x) = \frac{\lambda^k}{\Gamma(k)} e^{-\lambda x} x^{k-1}, \qquad x \geq 0, \lambda, \ k > 0. \tag{7}$$

The mean, variance and skewness are given by

$$E(X) = \frac{k}{\lambda}, \qquad Var(X) = \frac{k}{\lambda^2} \qquad and \qquad \gamma_1 = \frac{2}{\sqrt{k}}. \tag{8}$$

The remainder of this paper is organized as follows. The main definitions and preliminaries are given in Section 2. Section 3 presents a measures of variability to rank the suggested distribution functions. Besides two other measures which can be used in some special applications. Section 4 is devoted to introduce an application to stochastic activity networks. Finally, concluding remarks are drawn in Section 5.

2. Definitions and Preliminaries

We begin by introducing some required definitions. The following definitions and proposition are presented in Ross (1983).

Definition 1. A nonnegative r.v. X with finite mean μ and probability distribution function F is said to be NBUE (New Better than Used in Expectation) if, for all $x > 0$

$$\int_x^\infty \frac{\overline{F}(y)}{\overline{F}(x)} dy \leq \mu,$$

where $\overline{F}(x) = 1 - F(x) > 0$.

A random variable is said to be NBUE if its distribution function has that property

Definition 2. If X and Y are non-negative r.v.'s such that $E(X) = E(Y)$, then the r.v. X is said to be more variable than Y, written $X \geq_v Y$, if and only if $E[h(X)] \geq E[h(Y)]$ for all increasing and convex functions h.

Definition 3. If the r.v.'s X and Y have the distributions F and G, respectively, then X is stochastically larger than Y, written $X \geq_{st} Y$, if

$$\overline{F}(a) \geq \overline{G}(a), \qquad \text{for all } a$$

where $\overline{G}(x) = 1 - G(x)$.

Proposition 1. If $F(x)$ is an NBUE distribution having mean μ, then

$$F \leq_v \exp(\mu),$$

Where $\exp(\mu)$ is the exponential distribution with mean μ.

The above proposition states that the exponential distribution has maximal variability in the class of NBUE distributions.

The following Theorem is introduced by Li (2002).

Theorem 1. Suppose X is NBUE with finite mean μ, then for all $k = 1, 2, \ldots$

$$(i)\ E(X^k) \leq k\,\mu\, E(X^{k-1}), \qquad (ii)\ E(X^k) \leq k!\,\mu^k. \tag{9}$$

3. Main Results

In this section we introduce the proposed measure in Proposition 2 and its properties are summarized in corollaries 1 and 2. The variability relationships among the proposed distribution are introduced in Theorems 2 and 3. Two alternative measures of variability are also discussed.

Proposition 2. For any NBUE probability distribution function $F(x)$ with $E(X) = \mu$ and $Var(X) = \sigma^2$, then

$$\Delta(F) \triangleq \mu^2 - \sigma^2 \geq 0 \qquad (10)$$

where $\triangleq$ stands for the definition of $\Delta(F)$.

Proof**.** Set $k = 2$ in part (ii) of Theorem 1, to get

$$E(X^2) \leq 2\mu^2 \qquad (11)$$

which is equivalent to

$$\Delta(F) = \mu^2 - (E(X^2) - \mu^2) \geq 0. \qquad (12)$$

This completes the proof.

Corollary 1. A nonnegative r.v. X has a maximal variability in the class of NBUE if $\Delta(F) = 0$.

This result follows from the fact that the exponential distribution has its $\mu^2 = \sigma^2$. It also immediately follows that for all other distributions in the class NBUE $\Delta(F) > 0$.

Corollary 2. The most variability occurs when the value of $\Delta(F) = 0$, and for all the class of NBUE the value of $\Delta(F) > 0$.

The following Theorems summarize the variability relationships among the following distributions: the exponential, Erlang, Uniform, Triangular, Beta and Beta(PERT). We denote the random variables corresponding to the above distributions respectively by $X_{\exp}, X_E, X_U, X_T, X_{Beta}$ and X_{PERT}.

Theorem 2. If all the distributions stated above are symmetric about their means, then

$$Var(X_U) \geq Var(X_T) \geq Var(X_{Beta(3,3)}) \geq Var(X_{PERT}). \qquad (13)$$

Proof. Under the assumption that all distributions are symmetric about their mean implies the four distributions have the same mean. This, in turn, implies that for the Triangular and PERT the mode $m = (a+b)/2$, and that for the Beta we must have $k_1 = k_2 = 3$ or 4. It is sufficient to show, by Corollaries 1 and 2, that the variances of the corresponding r.v.'s bear the same inequalities to each other. It is easy to demonstrate that since all means $= (a+b)/2$, by substitution in (2)-(5),

$$Var(X_U) = \frac{(a-b)^2}{12}, \quad Var(X_T) = \frac{(a-b)^2}{24}, \quad Var(X_{Beta(3,3)}) = \frac{(a-b)^2}{28},$$
$$Var(X_{Beta(4,4)}) = Var(X_{PERT}) = \frac{(a-b)^2}{36}, \qquad (14)$$

and the ranking of (13) is validated.

For other values of k_1 and k_2, we discuss the following two cases:

- (i) For $k_1 = k_2 = 2$ or 1, the inequality (13) changes to
  $Var(X_U) \geq Var(X_{Beta}) \geq Var(X_T) \geq Var(X_{PERT}).$
- (ii) For $k_1 = k_2 = 5$, the inequality (13) changes to

$$Var(X_U) \geq Var(X_T) \geq Var(X_{PERT}) \geq Var(X_{Beta}),$$

and hence the proof.

Remark 1. According to the notation of Proposition 2, we can rewrite the statement of Theorem 2 as

$$\Delta(F_U) \geq \Delta(F_T) \geq \Delta(F_{Beta}) \geq \Delta(F_{PERT}),$$

where

$$\Delta(F_U) = \mu^2 - Var(X_U), \quad \Delta(F_T) = \mu^2 - Var(X_T), \quad \Delta(F_{Beta}) = \mu^2 - Var(X_{Beta}),$$

and

$$\Delta(F_{PERT}) = \mu^2 - Var(X_{PERT}).$$

The following Theorem will be discussed, under the same assumption of Theorem 2, for the Erlang distribution when $k = 2,3,4$ and 5. We exclude $k = 1$, since the Erlang distribution in this case is reduced to the exponential distribution.

Theorem 3. If $E(X_U) = E(X_E) = E(X_T) = \mu$,

$$Var(X_{\exp}) \geq Var(X_E) \geq Var(X_U) \geq Var(X_T). \tag{15}$$

Proof. According to the Proposition 1 and Corollary 1 the exponential distribution has a maximal variability in the class of NBUE distributions; hence the inequality '$\leq$' is true for any other distributions. This establishes the left-most inequality. Let

$$\Delta(F_U) = \mu^2 - Var(X_U), \quad \Delta(F_E) = \mu^2 - Var(X_E) \quad and \quad \Delta(F_T) = \mu^2 - Var(X_T), \tag{16}$$

be the measures of variability of the Uniform, Erlang and Triangular distributions; respectively. As indicated in the proof of Theorem 2, equality of the means implies that the mode m of the Triangular distribution is exactly mid-way between a and b, i.e., $m = (a+b)/2$. To prove the theorem it is required to show that

$$\Delta(F_E) \leq \Delta(F_U) \leq \Delta(F_T), \tag{17}$$

where

$$\Delta(F_U) = \frac{(a+b)^2}{4} - \frac{(b-a)^2}{12}, \tag{18}$$

$$\Delta(F_E) = \frac{k^2}{\lambda^2} - \frac{1}{k}\frac{k^2}{\lambda^2} = \frac{(a+b)^2}{4} - \frac{(a+b)^2}{4k}, \tag{19}$$

$$\Delta(F_T) = \frac{(a+b)^2}{4} - \frac{(b-a)^2}{24}. \tag{20}$$

It is easy to see that the inequality (17) is hold when $k = 2$ and 3.

For $k = 4$, the inequality (17) is true when

$$ab \geq \frac{(b-a)^2}{12} \quad or \quad ab \geq Var(X_U) \quad and \quad a,b > 0. \tag{21}$$

For $k = 5$, the inequality (17) is true when

$$ab \geq 2\frac{(b-a)^2}{12} \quad or \quad ab \geq 2Var(X_U) \quad and \quad a,b > 0. \tag{22}$$

The proof is completed.

Corollary 3. The above two Theorems showed that

For $k_1 = k_2 = 2$

$$Var(X_{\exp}) \geq Var(X_E) \geq Var(X_U) \geq Var(X_{Beta(2,2)}) \geq Var(X_T) \geq Var(X_{PERT}).$$

For $k_1 = k_2 = 3$

$$Var(X_{\exp}) \geq Var(X_E) \geq Var(X_U) \geq Var(X_T) \geq Var(X_{Beta(3,3)}) \geq Var(X_{PERT}). \quad (23)$$

For $k_1 = k_2 = 5$

$$Var(X_{\exp}) \geq Var(X_E) \geq Var(X_U) \geq Var(X_T) \geq Var(X_{PERT}) \geq Var(X_{Beta(5,5)}).$$

Remark 2. Haha (2008) showed that the uniform distribution has more variability and the above Theorems showed that the variability of the uniform distribution is less than the exponential and Erlang distributions.

3.1 Alternative Measures of Variability

Besides the measures mentioned in Graves & Ringuest (2009), another two measures are also proposed for comparing between distributions.

The First measure

The following Lemma and the consequence Proposition are presented in Ross (1983).

Lemma 1. Let F and G be continuous distribution functions. If X has distribution F, then the r.v. $G^{-1}(F(X))$ has distribution G.

Proposition 3. Let F and G are distributions such that $\bar{F}(a) \geq \bar{G}(a)$, then there exist r.v.'s X and Y having distributions F and G respectively such that

$$P(X \leq Y) = 1.$$

This means that, if $X \geq_{st} Y$, then there exist r.v.'s X_1 and Y_1 having the same distributions of X and Y such that X_1 is at least as larger as Y_1 with probability one.

The Second measure

This measure is due to Mood et al. (1974) page 74, which characterizes the distribution by the variance.

Suppose that $f_1(x)$ and $f_2(x)$ are two probability density functions with the same mean μ such that

$$\int_{\mu-c}^{\mu+c} (f_1(x) - f_2(x))dx \geq 0, \quad (24)$$

for every value of c, then

$$\sigma_1^2 < \sigma_2^2,$$

which means that, the variance σ_1^2 in the first density is smaller than the variance σ_2^2 in the second density. The converse of inequality (24) is not true. That is, if $\sigma_1^2 < \sigma_2^2$, we can not conclude that the corresponding densities satisfy the inequality (24) for all values of c.

4. Applications in SANs

This section introduces an application to stochastic activity networks by applying Theorems 2 and 3. The following worked example demonstrates the comparison between the variances of the proposed distribution which are discussed in the paper.

Example The network in Fig.1 is taken from Elmaghraby (1977 p 314). The activity times and tasks description of ship-boiler repair are shown in Table 1. The three durations attached to each activity are the optimistic "a", most likely "m", and pessimistic "b" estimates, obtained by asking the experts. We modified the most likely time to be $(=(a+b)/2)$ to agree with the equality of the means for all distributions. The mean project completion time for all distributions is equal to 47.75. Table 2 shows the squared coefficient of variation (SCV) of each critical activity. The SCV for the exponential distribution is equal to 1 and the SCV for Erlang distribution when $k=2,3,4$ and 5 are $0.5, 0.333, 0.25$ and 0.2, respectively. Table 3 summarizes the variances, skewness and the sum squared coefficient of variation (SSCV) obtained from the proposed distributions. The SSCV for the exponential distribution is equal to 9 and for the Erlang distribution when $k=2,3,4$ and 5 are equal to $4.5, 3, 2.25$ and 2, respectively.

Remark 3. In Table 2, we note that the SCV for the activities 15 and 18 are greater than the SCV of Erlang distribution when $k=4$ and 5. The reason for that is both activities violate the equations (21)

and (22), that is, $a>0$.

Remark 4. From Table 3 we note that

$$Var(X_{\exp}) \geq Var(X_E) \geq Var(X_U) \geq Var(X_T) \geq Var(X_{Beta(3,3)}) \geq Var(X_{PERT}), \tag{25}$$

$$Var(X_{\exp}) \geq Var(X_E) \geq Var(X_U) \geq Var(X_{Beta(2,2)}) \geq Var(X_T) \geq Var(X_{PERT}), \tag{26}$$

$$Var(X_{\exp}) \geq Var(X_E) \geq Var(X_U) \geq Var(X_T) \geq Var(X_{PERT}) \geq Var(X_{Beta(5,5)}). \tag{27}$$

Remark 5. The coefficient of skewness which is shown in Table 3 ranking the exponential and Erlang distributions for different values of k. The zero skewness for the other distributions came from the symmetry assumption of those distributions.

In making decisions about which project to include in a real industrial cases, one should evaluate the risk. Many R&D and IT projects fail in either the marketing stage or the implementation stage. The results of this paper including the above example provide an authoritative source to engineers or managers who need to choose the appropriate distribution for evaluating risk in their decision making. In all distributions discussed in this paper we compute the variance, the skewness and the squared coefficient of variation. Based on these measures the engineers or managers would select an appropriate portfolio.

5. Conclusion

In this paper, a simple measure is used to rank the distribution functions. The proposed distributions which are discussed are: the exponential, Erlang, Uniform, Triangular, Beta and Beta (PERT) distribution with different values of k_1 and k_2. All suggested distributions fall in the class of NBUE distributions and considered to be symmetric about their mean. The ranking of the distributions are discussed in Theorems 2 and 3. Corollary 3 summarizes the final results. Two alternative measures are stated for comparing the variability between the distributions. The second measure may be used when the graphical representation is considered. The condition, $\sigma_1^2 < \sigma_2^2$, in the third measure is only evident that $f_1(x)$ has more area near the mean than $f_2(x)$, at least for certain intervals about the mean. Finally, an application to stochastic activity networks is introduced. The example network which is shown in the application verified the results of Theorems 1 and 2. On the other hand, in portfolio selection theory (see, e.g. Markowitz, 1952 1959) we can consider the portfolio return as a constant for all proposed distributions and the risk is varying. The issue that may require further investigation is the development of a measure do not restrict the assumption that the distributions are symmetric about the mean. Also, more distributions may be included for future research.

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Table 1. Activity times and Description

Activity	Describtion	Predecessor	Duration
1	Initial Hydrostatic Test	-	1,3,5
2	Remove air registers	-	0,1.5,3
3	Repair under boiler	-	14,24.5,35
4	Remove drum internals	1	0.5,1.75,3
5	Rag for chemical cleaning	1	5,11,17
6	Clean and repair air register assemblies	2	10,15,20
7	Repair drum internals	4	5,11.5,18
8	Remove refractory material	1, 2	4,7,10
9	Repair inner air casing	8	1,7.5,14
10	Exploratory block	4,8	7,12.5,18
11	Retube and poll	10	4,8,12
12	Repair outer air casing	9	1,5.5,10
13	Rebrick	9,11	5,9.5,14
14	Chemically clean	5,11	4,6,8
15	Preliminary hydrostatic test	14	0,7,14
16	Install drum internals	7, 15	1,2,3
17	Install air registers	6,13	0.5,1.75,3
18	Final hydrostatic tests	16	0,1.5,3
19	Install plastic refractory	17, 18	0.5,0.75,1

Table 2. The squared coefficient of variation of the critical activities for the proposed distributions

CA	U	B(2,2)	T	B(3,3)	B(4,4)	B(5,5)
1	0.148	0.088	0.074	0.063	0.049	0.040
8	0.061	0.036	0.030	0.026	0.020	0.016
10	0.064	0.038	0.032	0.027	0.021	0.017
11	0.083	0.050	0.041	0.035	0.027	0.022
14	0.037	0.022	0.018	0.015	0.012	0.010
15	0.333	0.200	0.166	0.142	0.111	0.090
16	0.083	0.050	0.041	0.035	0.027	0.022
18	0.333	0.200	0.166	0.142	0.111	0.090
19	0.037	0.022	0.018	0.015	0.012	0.010

Where, CA denotes to the Critical Activity, U denotes to the Uniform distribution, T denotes to the Triangular distribution and B(.,.) denotes to the Beta distribution for different values.

Table 3. The variance, skewness and SSCV of proposed distributions

Distribution	Variance	Skewness	SSCV
Exponential	370.0625	2	9
Erlang(k=2)	185.03	1.414	4.5
Erlang(k=3)	123.35	1.155	3
Erlang(k=4)	92.515	1	2.25
Erlang(k=5)	74.013	0.8944	1.8
Uniform=Beta(1,1)	38.519	0	1.179
Beta(2, 2)	23.113	0	0.706
Triangular	19.262	0	0.586
Beta(3,3)	16.508	0	0.5
PERT=Beta(4,4)	12.839	0	0.393
Beta(5,5)	10.505	0	0.317

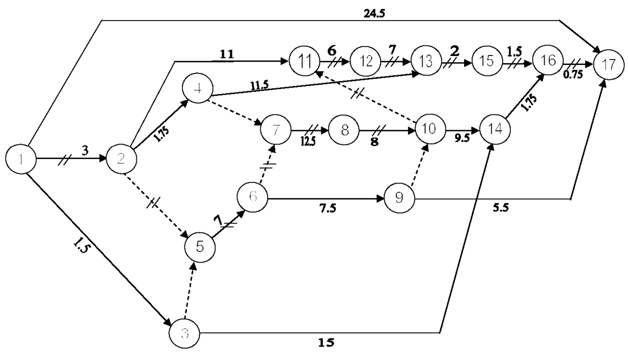

Fig.1 The mean of each activity is indicated on each arc and the critical path is shown in the network