3-Stage Specially Structured Flow Shop Scheduling to Minimize the Rental Cost Including Transportation Time, Job Weightage and Job Block Criteria

Deepak Gupta[1] Shashi Bala[2*] Payal Singla[3] Sameer Sharma[4]
1. Department of Mathematics, M.M.University,Mullana,Ambala,India
2. Department of Mathematics, M.P.College for women, Mandi Dabwali,
3. Department of Mathematics, Baba Farid College of Engineering & Technology, Bathinda, Punjab, India
4. Department of Mathematics, D.A.V. College, Jalandhar, Punjab, India
* E-mail of the corresponding author: shashigarg97@gmail.com

Abstract
This article describe the development of a new heuristic algorithm which guarantees an optimal solution for specially structured flow shop scheduling problem with n-jobs,3- machines, to minimize the rental cost under specified rental policy with transportation time, job weightage and job block criteria. Further the processing times are not merely random but bear a well defined relationship to one another. Most of literature emphasized on minimization of idle time/ make span. But minimization of make span may not always lead to minimize rental cost of machines. Objective of this work is to minimize the rental cost of machines under a specified rental policy irrespective of make span.
Keywords: rental policy, Job weightage, job block, transportation time, utilization time.

1. Introduction

Scheduling can be defined as the allocation of resources over a period of time to perform a collection of tasks. The goal is to specify a schedule that specify when and on which machine each job is to be executed. A variety of approaches have been developed to solve the problem of scheduling. Scheduling problems are common occurrence in our daily life e.g. ordering of jobs for processing in a manufacturing plant, programs to be run in a sequence at a computer center etc. Such problems exist whenever there is an alternative choice in which a number of jobs can be done. Now-a-days, the decision makers for the manufacturing plant have interest to find a way to successfully manage resources in order to produce products in the most efficient way. They need to design a production schedule to minimize the flow time of a product. The number of possible schedules in a flow shop scheduling problem involving n-jobs and m-machines is $(n!)^m$. The optimal solution for the problem is to find the optimal or near optimal sequence of jobs on each machine in order to minimize the total elapsed time. Majority of research in scheduling assumes transportation time (loading time, moving time and unloading time) from one machine to another as negligible or included in processing time .But in some real life situations transportation time has great impact on the performance measure, separate consideration is needed. The basic concept of equivalent job for a job block has been introduced by Maggu and Dass (1977). All the scheduling models beginning from Johnson's work in 1954 upto 1980, there is no reference of job weightage in the literature. The weights of a job show its relative priority over some other jobs in a scheduling model. Higher the weight of a job has, the more important it becomes for processing in comparison with other jobs in the operating schedule. The scheduling problems with weight arise when inventory costs for jobs are involved. Further the scheduling problem which does not involve "weight" of job is called "simple or unweighted scheduling problem", whereas the scheduling problem involving "weight" of jobs is referred to as "weighted scheduling problem".

The first research concerned to the flow shop scheduling problem was proposed by Johnson (1954). Johnson described an algorithm to minimize make span for the n-jobs 2- stage flow shop scheduling problem. Smith (1970) considered minimization of mean flow time for n jobs,m-machines. Gupta, J.N.D. (1975) gave an algorithm to find the optimal schedule for specially structured flow shop scheduling. Gupta (2012]) studied specially structured two stage flow shop problem to minimize the rental cost of the machines under pre-defined rental policy in which the probabilities have been associated with processing time. Maggu & Das (1977) consider a two machine flow shop problem with job block citeria. Ghanshiam(1978) studied three machine sequencing problem with equivalent jobs for job blocks for n-jobs. Anup(2002) discussed two stage flow shop scheduling with ordered job block. Heydari (2003)dealt with a flow shop scheduling problem where n jobs are processed in two disjoint job blocks in a string consists of one job block in which order of jobs is fixed and other job block in which order of jobs is arbitrary. Miyazaki.(1980) minimized weighted mean rental cost in two stage flow shop scheduling in which the processing time are associated with probabilities including job block. Chen and Lee (2001)studied the transportation machine scheduling. Khodadadi (2011) developed a new heuristic for three machine flow shop scheduling . Maggu and Dass (1981), studied the concept of transportation time and equivalent job for a job block. Singh,T.P.(1994) gave heuristic approach to solve two stage flow shop scheduling problem with random processing and transportation time under group restriction on jobs. Gupta (2012) studied 3-Stage specially structured flow shop scheduling to minimize the rental cost including job weightage. Bagga, P.C. (1969) solved sequencing problem in a rental situation.

In this paper we presents a specially structured flow shop scheduling model to minimize the utilization time of the machines and hence their rental cost under specified rental policy with transportation time, job block and job weightage. Most of the work emphasize on minimization of make span. Here we have discussed the algorithm which shows that minimization of make span does not always lead to minimize rental cost of machines.

2. Practical Situation

Manufacturing industries are the backbone in the economic structure of a nation, as they contribute to increasing G.D.P. / G.N.P. and providing employment. Productivity can be maximized, if the available resources are utilized in an optimized manner. Optimized utilization of resources can only be possible if there is a proper scheduling system making scheduling a highly important aspect of a manufacturing system. The practical situation may be taken in a paper mill, sugar factory and oil refinery etc. where various qualities of paper, sugar and oil are produced with relative importance i.e. weight in jobs, hence weightage of jobs is significant. Transportation becomes significant when the machines on which jobs are to be processed are placed at different places. Due to unavailability of funds in real life in his starting career one has to be taken the machines on rent. To start a fitness centre many machines like tread mill, laser hair removal equipment, ski cabin, cardiovascular, stretches, free weights, elliptical, cycles, rowers, plate loaded & benches, multi station, one does not want to invest huge money by buying all the machines, instead he prefer to take the machines on rent. By renting one can stay financially afloat more easily and still manage to procure the best high technology for customers.

3. Notations

S	: Sequence of jobs 1, 2, 3… n
S_k	: Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, ------
M_j	: Machine j, i= 1, 2, 3
a_{ij}	: Processing time of i^{th} job on machine M_j.
$t_{ij}(S_k)$	: Completion time of i^{th} job of sequence S_k on machine M_j
$I_{ij}(S_k)$	: Idle time of machine M_j for job i in the sequence S_k.
$T_{i,j\rightarrow k}$	:Transportation time of i^{th} job from j^{th} machine to k^{th} machine.
$U_j(S_k)$	: Utilization time for which machine M_j is required
$R(S_k)$	: Total rental cost for the sequence S_k of all machine.
C_j	: Rent cost per unit time of machine M_j.
w_i	: Weight of i^{th} jobs.
β	: Equivalent job for job-block (k, *m)*

4. Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:

$t_{ij} = \max(t_{i-1,j}, t_{i,j-1} + T_{i,j-1\rightarrow j}) + a_{ij}$ for $j \geq 2$.

5. Rental Policy

The machines will be taken on rent as and when they are required as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs and 2nd machine will be taken on rent at time when 1st job is completed on the 1st machine and transported to 2nd machine and 3rd machine will be taken on rent when 1st job is completed on 2nd machine and transported to 3rd machine.

6. Problem Formulation

Let some jobs i(1,2,……..,n) are to be processed on three machines M_j (j= 1,2,3) under the specified rental policy (P). Let a_{ij} be the processing time of i^{th} job on j^{th} machine and $T_{ij\rightarrow k}$ be the transportation time of i^{th} job from j^{th} machine to k^{th} machine. w_i be the weight of i^{th} job. Let β be equivalent job for job block (k,m). Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of all the machines. The mathematical model of the problem in matrix form can be tabulated as in table1.

Mathematically, the problem is stated as:
Minimize

$$R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$$

Subject to constraint: Rental Policy (P)
Our objective is to minimize rental cost of machines while minimizing the utilization time.

7. Assumptions

- Jobs are independent to each other. Let n jobs be processed through three machines M_1, M_2, M_3 in order $M_1M_2M_3$.
- Machine break down is not considered.
- Pre-emption is not allowed.
- Either $\max(a_{i2} + t_{i1\to2}) \leq \min(a_{i1} + t_{i1\to2})$
  $\max(a_{i2} + t_{i2\to3}) \leq \min(a_{i3} + t_{i2\to3})$
  Or both.

8. Algorithm

The algorithm to minimize the rental cost is as follows:

Step 1: Convert the problem into two machines problem. Let G and H be fictitious machines having Gi and Hi as their processing times :

$G_i = a_{i1} + t_i + a_{i2} + g_i$
$H_i = t_i + a_{i2} + g_i + a_{i3}$

Step 2: If $\min(G_i, H_i) = G_i$

Then $G_i' = \frac{G_i + w_i}{w_i}$, $H_i' = \frac{H_i}{w_i}$

If $\min(G_i, H_i) = H_i$

Then $G_i' = \frac{G_i}{W_i}$, $H_i' = \frac{H_i + w_i}{w_i}$

Step 3: Take equivalent job $\beta = (k,m)$ and calculate processing time G_β' and H_β' on the guide lines of Maggu & Dass (1977) as follows:

$G_\beta' = G_k' + G_m' - \min(G_m', H_m)$
$H_\beta' = H_k' + H_m' - \min(G_m', H_m')$

Step 4: Define a new reduced problem with processing time as in Step 2 & Step 3.

Step 5: Apply Johnson's (1954) technique and obtain the optimal schedule of given jobs. Let the sequence be S_1.

Step 6: Obtain other sequences by putting 2^{nd}, 3^{rd},…..n^{th} jobs of sequence S_1 in the 1^{st} position and all other jobs of S_1 in same order. Let these sequences be S_2, S_3…S_{n-1}.

Step 7: Compute $\sum A_{i1}$, $U_2(S_k)$, $U_3(S_k)$ and

$$R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$$

For all possible sequences S_k (k = 1, 2, …, n)

Step 8: Find min $R(S_k)$; k= 1,2,….,n. let it be minimum for the sequence S_p, then sequence S_p will be the optimal sequence with rental cost $R(S_p)$.

9. Numerical Illustration

Consider 5 jobs and 3 machines flow shop problem in which processing times including transportation time and job weightage are given in the Table- 2.Let β be equivalent job for job block (3,5). The rental cost per units time for machines M_1, M_2 and M_3 are 4 units, 5 units and 2 units respectively. Our objective is to obtain optimal a sequence of jobs with minimum possible rental cost of the machines.

Jobs	Machine M_1	$T_{i1\to2}$	Machine M_2	$T_{i2\to3}$	Machine M_3	Weight
I	a_{i2}	$t_{i1\to2}$	a_{i1}	$t_{i2\to3}$	a_{i3}	w_i
1	40	2	20	1	45	2
2	60	1	15	2	65	1
3	55	3	25	3	50	3
4	35	4	10	5	40	2
5	70	3	30	4	80	1

Table - 2

Solution: Check conditions:
Here max $(a_{i2} + t_{i1\to2}) \leq$ min $(a_{i1} + t_{i1\to2})$ satisfies. Therefore, as per Step 1 the processing time for two fictitious machines G and H are shown in table 3

As per Step 2: The new reduced problem with weighted flow times G_i' & H_i', is as in table - 4
As per step 3: The Processing time for the equivalent job on fictitious machines is shown in table - 5

As per Step 5: Obtaining the sequence with minimum makespan is
$\mathbf{S_1}$: 4 – 1 – 2 – 3 – 5.
As per Step 6: Other feasible sequences which may corresponds to minimum rental cost are :

$\mathbf{S_2}$ = 1 – 4 – 2 – 3 – 5
$\mathbf{S_3}$ = 2 – 4 – 1 – 3 – 5
$\mathbf{S_4}$ = 3 – 5 – 4 – 1 – 2

From in – out tables for these sequences, we have:
For $\mathbf{S_1}$: $CT(S_1) = 377$; $U_2(S_1) = 254$; $U_3(S_1) = 323$; $R(S_1) = 2956$
For $\mathbf{S_2}$: $CT(S_2) = 377$; $U_2(S_2) = 251$; $U_3(S_2) = 314$; $R(S_2) = 2923$
For $\mathbf{S_3}$: $CT(S_3) = 377$; $U_2(S_3) = 232$; $U_3(S_3) = 299$; $R(S_3) = 2798$
For $\mathbf{S_4}$: $CT(S_4) = 392$; $U_2(S_4) = 218$; $U_3(S_4) = 306$; $R(S_4) = 2742$
Therefore min $R\{S_K\} = R(S_4) = 2742$ units.
Therefore minimum rental cost is 2742 units and is for the sequence S_4.
Hence the sequence S_4 = 3 – 5 – 4 – 1 – 2 is optimal sequence with minimum rental cost. But total elapsed time / completion time for $\mathbf{S_4}$ is not minimum.

10. Conclusion

The algorithm proposed in this paper to minimize the rental cost of machines given an optimal sequence having minimum rental cost of machines irrespective of total elapsed time. The algorithm proposed by Johnson's (1954) to find an optimal sequence to minimize the makespan/ total elapsed time is not always corresponds to minimum rental cost of machines under a specified rental policy. Further the work can be extended by introducing parameters like, break down interval, set-up etc.

11. References

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Table 1: The mathematical model of the problem

Jobs	Machine M_1	$T_{i1\rightarrow 2}$	Machine M_2	$T_{i2\rightarrow 3}$	Machine M_3	Weight
I	a_{i1}	t_i	a_{i2}	g_i	a_{i3}	w_i
1	a_{11}	t_1	a_{12}	g_1	a_{13}	w_1
2	a_{21}	t_2	a_{22}	g_2	a_{23}	w_2
3	a_{31}	t_3	a_{32}	g_3	a_{32}	w_3
.	.	.	.	.	.	.
.	.	.	.	.	.	.
n	a_{n1}	t_n	a_{n2}	g_n	a_{n3}	w_n

Table 3: The processing time G_i & H_i for two fictitious machines G and H

Jobs	G	H	Weight
i	G_i	H_i	w_i
1	63	68	2
2	78	83	1
3	86	81	3
4	54	59	2
5	107	117	1

Table 4: New reduced problem with weighted flow times G_i' & H_i',

Jobs i	G_i'	H_i'
1	32.5	34
2	79	83
3	28.7	28
4	28	29.5
5	108	117

Table 5: The Processing time for the equivalent job on fictitious machines is:

Jobs i	G_i''	H_i''
1	32.5	34
2	79	83
β	108.7	117
4	28	29.5

Table 6: In-Out table for the optimal sequence S_1 is as follows:

job i	Machine A In - Out	Machine B In - Out	Machine C In - Out
4	0 – 35	39 – 49	54 – 94
1	35 – 75	77 – 97	98 – 143
2	75 – 135	136 – 151	153 – 218
3	135 – 190	193 -218	221 – 271
5	190 -260	263 -293	297 – 377

Table 7**:** In-Out table for the optimal sequence S_2 is as follows:

job i	Machine A In - Out	Machine B In - Out	Machine C In - Out
1	0 – 40	42 – 62	63 – 108
4	40 – 75	79 – 89	108 – 148
2	75 – 135	136 – 151	153 – 218
3	135 – 190	193 -218	221 – 271
5	190 -260	263 -293	297 – 377

Table 8**:**In-Out table for the optimal sequence S_3 is as follows:

job i	Machine A In - Out	Machine B In - Out	Machine C In – Out
2	0 – 60	61 – 76	78 – 143
4	60 – 95	99 – 109	143 – 183
1	95 – 135	137 – 157	183 – 228
3	135 – 190	193 -218	228 – 278
5	190 -260	263 -293	297 – 377

Table 9: The In-Out table for the optimal sequence S_4 is as follows:

job i	Machine A In - Out	Machine B In - Out	Machine C In - Out
3	0 – 55	58 – 83	86 – 136
5	55 – 125	128 – 158	162 – 242
4	125 – 160	164 – 174	242 – 282
1	160 – 200	202 -222	282 – 327
2	200 -260	261 -276	327 – 392