

Residential Real Estate for Financing and Investments

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Abstract

Interest in real estate is increasing dramatically in recent years as the various ways of investing in and financing real estate continue to evolve. There are myriad of alternative ways of making equity investments in real estate, ranging from purchasing properties like an apartment building, either as an individual or in a partnership. Investments can also be made in mortgages, which is used in real estate ranging from making a mortgage loan (as a lender) or purchasing a publicly traded mortgage-backed security (residential or commercial). This paper calculates the asset value of mortgages real estate over the period of time with the help of Mathematica and computational power is used as evidenced through symbolic, numerical and graphical output. We present mathematical modeling of residential real estate financing and investment. It is helpful for the persons who just want to better understanding the real estate for their own personal investment and take financing decisions.

Key Words: Interest, mortgage, CAM, CPM

1. Introduction

Mathematics of finance is the most important topic in the economic world and real estate is one of the most important subjects in finance (Zima.P. and Robert L.B., 2005). It is the branch of applied mathematics concerned with the financial market (John B. C. 2001).

The term real in real estate comes from the term realty (Creedy, J. and Wall N.F. 1979), which has, for centuries, meant land and things permanently attached (the latter would include immovable things such as buildings and other structures). All other items not considered realty have been designated as personality, which include all movable things (e.g. automobiles, shares of stock, bank accounts, patents). The term estate has evolved to mean "all that a person owns," including both realty and personality. Hence, the portion of a person's estate that consists of realty has come to be known as real estate. However, in current business practice, although the term realty is sometimes used, we generally use the term real estate to mean land and all things permanently attached (William B. B. and Jeffrey D. F., 2005).

We concentrate briefly in this section on real estate as a real asset. Real assets, such as gold and real estate, take the form of useful things, for example, jewelry and homes (Charles J. J., 2002). They also take the form of investments that change in value due to changes in supply and demand. Financial assets, such as stocks and bonds, only take the form of investments. This type of asset originates and prospers on the basis of the efforts of humans making productive use of real assets (Seldin & Maury, 1980).

Real estate, the real asset, provides the space humans need and costs money to physically produce. It exists because humans put their money and effort into transforming vacant land into useful space. The wealth of real estate represents all previous contributions humans made to improve land and the prevailing demand supply conditions in the asset market that affect current wealth levels (Wofford, L. E, 1983). During each period, new contributions, investments, add to the level of wealth.

The benefits of real estate investments may be classified as follows: cash flow, tax shelter, and proceeds of sale. Cash flow is the difference between the sum of cash received and the sum of cash disbursed during a period, usually a year. It is not income. It is not earning. It is not profit. It is just the difference between the cash that comes in and the cash goes out. The tax shelter benefits are the savings derived from preferential income tax treatment of the earnings from the property (Boaz G. and Suzanne G., 2006). The proceeds of sale are the monies that the investor receives when the property is sold and settled and income taxes are paid.

In this paper, we try to present mathematical modeling of residential real estate financing and investment. To do this, we discuss; various types of annuity, fixed and adjustable mortgage loans also compare graphically three types of fixed rate mortgage loans. Here we basically use mathematical formula and programming language to solve the real estate problem.

2. Fixed Interest Rates Mortgage Loans

This section deals with various approaches to pricing and structuring fixed interest rate mortgage loans. By pricing a loan, we refer to the rate of interest, fees, and other terms that lenders offer and that borrowers are willing to accept when mortgage loans are made. Another major objective of this section is to illustrate techniques for determining the yield to the lender and the actual to the borrower when various provisions exist in loan agreements.

2.1 Determinants of Mortgage Interest Rates

- Changing economy conditions have forced the real state finance industry to go through an important evolution. These changing conditions now require lenders and borrowers to have a better understanding of the sources of funds used for lending and the nature of how risk, economic growth, and inflation affect the availability and the cost of mortgage funds. When considering the determinants of interest rates on mortgage loans used to finance single-family residences, we must consider the demand and supply of mortgage funds. The market rate of interest on mortgage loans is established by what borrowers are willing to pay for the use of funds over a specified period of time and what lenders are willing to accept in the way of compensation for the use of such funds.

The Real Rate of Interest: Underlying Considerations

- When discussing market interest rates on mortgages, we should keep in mind these interest rates are based on a number of considerations. One fundamental Relationship that is common to investments requiring use of funds in the economy is that they earn at least the real rate of interest. This is the minimum rate of interest that must be earned by savers to induce them to divert the use of resources (funds) from present consumption to future consumption.

Interest Rates Risk and Inflation Expectations

- The real rate of interest, a concern that all investors have when making investment decisions is how inflation will investment returns. The rate of inflation is of particular importance to investors and lenders making or purchasing loans made at fixed rates of interest over long periods of time. The uncertainty about what interest rate to change, when a loan is made can be referred to as interest rate risk. For example, anticipated inflation may have been 6 percent at the time our Tk.100, 000 loan was made, but if actual inflation turns out to be 8 percent, this means the interest

rate that should have been changed. We say that the anticipated rate of inflation at the time loan was made was 6 percent.

Prepayment Risk

- The risk that the loan will be prepaid when the interest rate falls below the loan contract rate is referred to as prepayment risk.

Other Risks

- There are additional risks that lenders and investors consider that may vary by type of loan or investment. For example, the liquidity or marketability of loans and investments will also affect the size of the premium that must be earned. Securities that can be easily sold and resold in well-established markets will require lower premiums than those that are more difficult to sell. This is called liquidity risk.

A Summary of Factors Important in Mortgage Pricing

- The interest rate charged on a particular mortgage loan will depend on the real interest rate, anticipated inflation, interest rate risk, prepayment risk and other risk. These relationships can be summarized in general as follows:

$$i = r + p + f$$

2.2 The Constant Amortization Mortgage Loan (CAM)

- Amortization means the process of loan repayment over time. Indeed, a first effort to accomplish this was referred to as the constant amortization mortgage (CAM) loan. Payments on CAMs were determined first by computing a constant amount of each monthly payment to be applied to principle. Interest was then computed on the monthly loan balance and added to the monthly amount of amortization. The total monthly payment was determined by adding the constant amount of monthly amortization to interest on the outstanding loan balance.

Example

For a debt of Tk.1, 000,000 with interest at 9% compounded monthly for 12 years. Show constant amortization schedule and graph of different items

Output

Month	Opening Bal.	Interest	Amortization	Monthly payment	Ending Balance
1	1000000	7500.	6944.44	14444.4	993056.
2	993056.	7447.92	6944.44	14392.4	986111.
3	986111.	7395.83	6944.44	14340.3	979167.
4	979167.	7343.75	6944.44	14288.2	972222.
5	972222.	7291.67	6944.44	14236.1	965278.
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
140	34722.2	260.417	6944.44	7204.86	27777.8
141	27777.8	208.333	6944.44	7152.78	20833.3
142	20833.3	156.25	6944.44	7100.69	13888.9
143	13888.9	104.167	6944.44	7048.61	6944.44
144	6944.44	52.0833	6944.44	6996.53	0

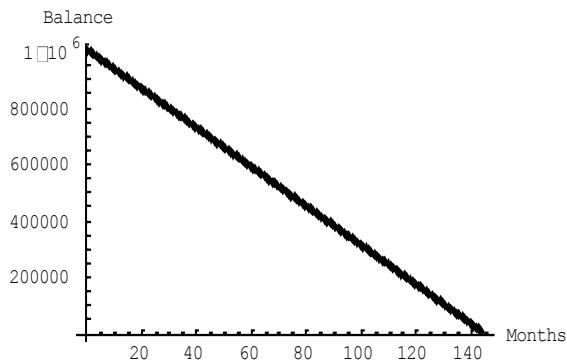


Fig 1: Outstanding Loan Balance

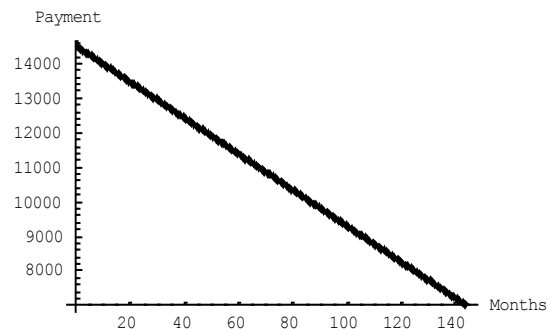


Fig 2: Loan Payment Patterns

Comments

- In a growing economy, borrowers could partially repay the loan over time.
- Monthly payment is decreasing.
- In an economy with long periods, a borrower income was more likely to increase but the first payment is vary high comparing with the last payment which is not an easy task to effort for borrower.

2.3 Constant Payment Mortgage Loan (CPM)

- The most common loan payment pattern used in real estate finance from the post depression era to the present, and one, which is still very prevalent today, is the fully amortizing, constant payment mortgage. This loan payment pattern is used extensively in financing single-family residences and in long-term mortgage lending on income producing properties such as multifamily apartment complexes and the shopping centers. This payment pattern means simply that a level, or constant, monthly payment is calculated on an original loan amount at a fixed rate of interest for a given term. Like the CAM, payment includes interest and some repayment of principle. At the end of the term of the CPM loan, the original loan amount, the principal, is completely repaid, or fully amortized, and the lender has earned a fixed rate of interest on the monthly loan balance. However, the amount of amortization varies each month with the CPM loan.

Governing equation :
$$R = A / a_{n|i} = A \frac{i}{1 - (1 + i)^{-n}}$$

Example

For a debt of Tk.1, 000,000 with interest at 9% compounded monthly for 12 years. Show constant payment mortgage schedule and graph of different items.

Output

Months	Beg Loan Bal.	Monthly pay	Interest	Principle Rep	Ending Loan Bal.
1	1000000	11380.3	7500.	3880.31	996120.
2	996120.	11380.3	7470.9	3909.41	992210.
3	992210.	11380.3	7441.58	3938.73	988272.
4	988272.	11380.3	7412.04	3968.27	984303.
5	984303.	11380.3	7382.27	3998.03	980305.

·	·	·	·	·	·
·	·	·	·	·	·
139	66524.7	11380.3	498.935	10881.4	55643.3
140	55643.3	11380.3	417.325	10963.	44680.3
141	44680.3	11380.3	335.103	11045.2	33635.1
142	33635.1	11380.3	252.264	11128.	22507.1
143	22507.1	11380.3	168.803	11211.5	11295.6
144	11295.6	11380.3	84.7169	11295.6	0

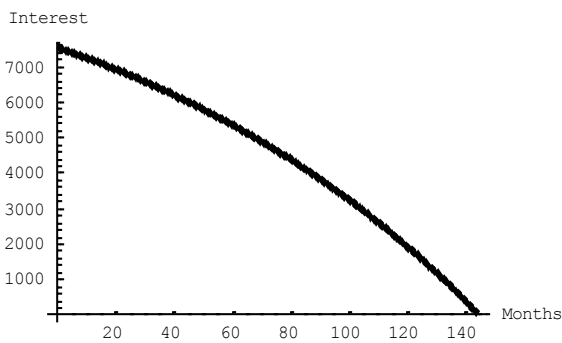


Fig 3: Interest Payment Patterns

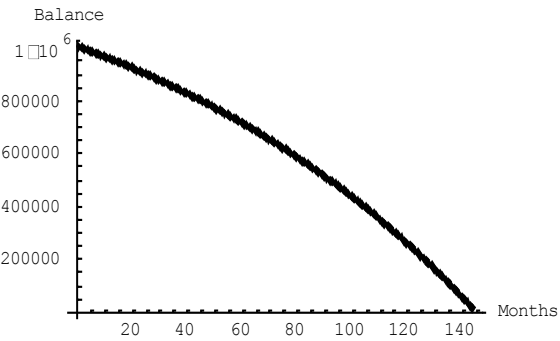


Fig 4: Outstanding Loan Balance

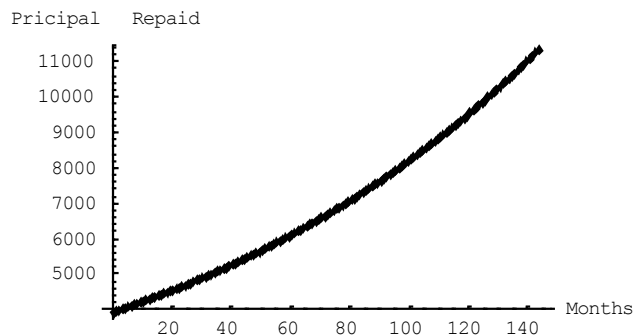


Fig 5: Principle Repaid Patterns

Comments

- The value of outstanding principal is decreasing.
- The value of interest rate is decreasing.
- The value of principal repaid is increasing.

2.4 Sum -of -Digits Method

- This method, which has a built-in prepayment penalty, was initially used as close approximation to the amortization. At high rates of interest, however, the outstanding principal as determined by the sum of digits method leads to a significant penalty against the borrower.

Example

For a debt of Tk.500, 000 with interest at 15% compounded monthly for 12 years. Show constant payment mortgage schedule and graph of different items.

Output

Months	Beg. Loan Bal.	Monthly payment	Interest	Princ Repaid	Ending Loan Bal.
1	500000	7504.38	8008.71	□504.323	500504.
2	500504.	7504.38	7953.09	□448.707	500953.
3	500953.	7504.38	7897.48	□393.091	501346.
4	501346.	7504.38	7841.86	□337.475	501684.
5	501684.	7504.38	7786.24	□281.859	501965.
.
.
139	43858.4	7504.38	333.696	7170.69	36687.7
140	36687.7	7504.38	278.08	7226.3	29461.4
141	29461.4	7504.38	222.464	7281.92	22179.5
142	22179.5	7504.38	166.848	7337.54	14841.9
143	14841.9	7504.38	111.232	7393.15	7448.77
144	7448.77	7504.38	55.616	7448.77	0

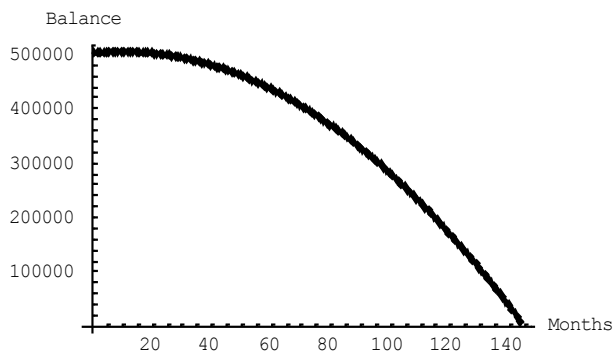


Fig 6: Outstanding Loan Balance

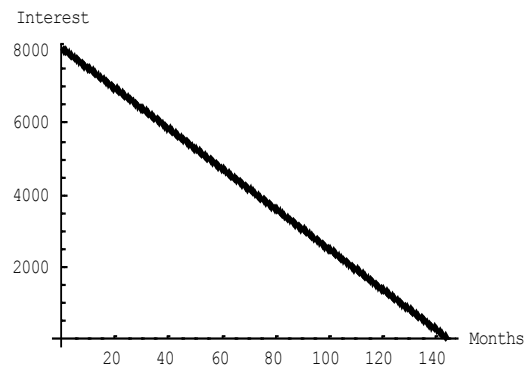


Fig 7: Interest Payment Patterns

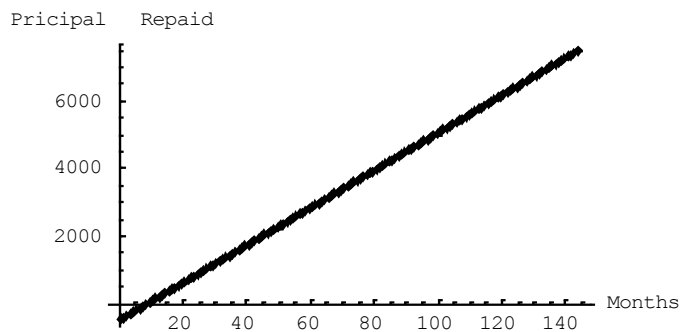


Fig 8: Principle Repaid Patterns

Comments

- Outstanding loan balance is decreasing slowly.
- Outstanding loan balance exceeds the original loan amount.
- The value of interest is decreasing.
- The value of principle repaid is increasing.

2.5 Comparison of Loan Patterns

- For example, a borrower is repaying a debt of Tk.500,000 with monthly payments over 12 years at $j_{12}=15\%$. Compare graphically outstanding loan balance by using a) The amortization method, b) The constant payment mortgage and c) The sum of digit method.

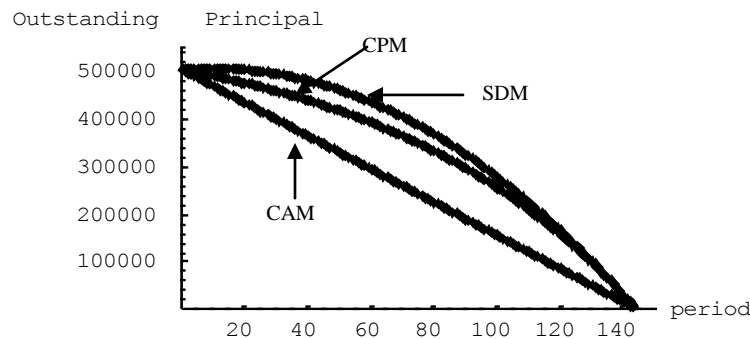


Fig 9: Loan Balance for CPM, Sum-of Digits Method and CAM

Comments

- The outstanding principal balance under the sum of digit method and constant payment mortgage exceed the outstanding principal under the constant amortization mortgage.
- Each outstanding principal under the sum-of-digits methods exceeds the true balance under the amortization method. Hence, if the loan is refinanced before the end of its term, there is a built in a penalty for the borrower.
- If the loan is paid off full-term, the total amount of interest under the sum-of-digits method is the same as under the amortization method, and no penalty is occur.
- The sum-of-digits method loan riskier to the lender than a CPM, because more consideration must be given to future market values of real estate and future borrower income.

3. Conclusion

Real estate is a business, not a profession. Real estate is sometimes inaccurately spoken of as a profession, but it is essentially a business. A profession applies science, art or learning to the use of others, the profit to the professor or person applying it being incidental; whereas a business is engaged in primarily for profit, and the profit is to the one engaging in the business.

The principal divisions of the real estate business are investment, operation and agency. Investment is the employment of capital in the acquisition of real estate or interests therein for permanent ownership or actual use of the person acquiring it. Investment in real estate is generally made for either of two purposes: (a) to derive an income, (b) to hold for resale in expectancy of an increase in value. Investment for income may be for one of two purposes, (1) the derivation of rental—that is, the direct return for the use of real property for definite periods, or (2) the obtaining of income through others upon money lent on the security of real property. Operation is the employment of capital in the acquisition or improvement of real estate or interests therein for commercial operations. Agency is dealing in or with real estate on behalf of others.

The initial object is to study some mathematical formula, assumption and example to discuss different types of interest, annuity and financing residential properties. By using programming Mathematica we introduce some program and their graphs.

In fixed and adjustable variable payments mortgage loans, we discussed various approaches to pricing and structuring fixed and loans with adjustable interest rates mortgage loans. Loans with adjustable interest rates become necessary from time to time, depending on the rate of economic expansion and expected rate of inflation. It has been seen that the price of interest rate on the loan depends on a number of factors, including various types of risk that affected mortgage lenders. In this paper briefly three types of fixed rates mortgage loans are discussed individually; these mortgage loans are constant amortization mortgage (CAM), constant payment mortgage (CPM) and sum-of-digits method. In CAM method, monthly payments and outstanding loan balance are decrease. In CPM loans, outstanding balance and interest payments are also decreased but principal repaid patterns are increased. Sum-of-digits method, outstanding loan balance decrease more slowly; many times outstanding loan balance exceeds the original loan amounts. In an economy experiencing real economic growth with relatively stable prices, increases income and property values would reduce investor default risk associated with CPM, especially sum-of-digits method. Further, as real incomes increased, property values were likely to remain higher than the outstanding loan balance over the term of the loan; hence the collateral value of the real estate relative to more slowly declining mortgage loan balance was considered by investors to be adequate. We will find that the riskiness of the mortgage is also a factor in the risk and expected rate of return for investors in real estate income properties. However the focus of this section has been on residential mortgage, the concepts and calculations are equally important for commercial mortgage.

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