Profit Maximization In A Product Mix Company Using Linear Programming

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Abstract

This paper demonstrates the use of linear programming methods as applicable in the manufacturing industry. Data were collected as extracts from the records of KASMO Industry Limited, Osogbo, Nigeria, on four types of sales packages adopted for selling her medicated soap product which include 1 tablet per pack, 3 tablets per pack, 12 tablets per pack and 120 tablets per pack. Information on selling price per pack and on the cost of five basic raw materials used as well as the quantity of each of the raw material held in stock per month for the production of soap tablets were available in the records of the company. Based on the costs of raw materials, the maximum profit that would accrue to the company given the product mix was determined. The results showed that the company would attain optimal monthly profit level of about ₦271,296 if she concentrates mainly on the unit sales (one tablet per pack) of her medicated soap product, ignoring other forms of sales packages. By this, her sales turn-over per month would be about 18,900 soap tablets. The analysis was carried out with R statistical package (www.cran.org) using the library "`lpSolve`".

Key Words: Linear programming, Simplex Method, Objective function, Product mix, R software.

JEL Classification: C40, C44, C61

1. Introduction

Linear programming (LP) can be defined as a mathematical technique for determining the best allocation of a firm's limited resources to achieve optimum goal. It is also a mathematical technique used in Operation Research (OR) or Management Sciences to solve specific types of problems such as allocation, transportation and assignment problems that permits a choice or choices between alternative courses of action (Yahya, 2004). Linear programming is a term that covers a whole range of mathematical techniques that is aimed at optimizing performance in terms of combinations of resources (Lucey, 1996).

Linear Programming being the most prominent OR technique, it is designed for models with linear objective and constraint functions. A LP model can be designed and solved to determine the best course of action as in a product mix subject to the available constraints.

Generally, the objective function may be of maximization of profit (which is the focus of this paper) or minimization of costs or labor hours. Moreover, the model also consists of certain structural constraints which are set of conditions that the optimal solution should justify. Examples of the structural constraints include the raw material constraints, production time constraint, and skilled labour constraints to mention a few. An optimum solution is a solution that fulfills both the constraints of the problem and the set objective to be met.

The term "linear", as stated by Akingbade (1996), implies proportionality, which means that the elements in a situation are so connected that they appear as straight line when graphed. While the "programming" indicates the solution method which can be carried out by an iterative process in which a researcher advances from one solution to better solution until a final solution is reached which cannot be improved upon. This final solution is termed the optimal solution of the LP problem.

This work demonstrates the pragmatic use of linear programming methods in a manufacturing company in Nigeria –

KASMO Industry Limited. The problem addressed here was to determine the product mix (combination of sales package) to be adopted by the company for selling her medicated soap product at which the optimal profit level would be attained.

2. Materials and Methods

The dataset used for this work was collected as extracts from the records of KASMO Industry Limited, Osogbo, Osun State, Nigeria on her main product line (medicated soap) and four different types of sales packages adopted for selling her soap product in 2010. These four types of sales packages include sales in 1 soap tablet (unit sales), sales in 3 soap tablets (3 tablets per pack), sales in dozen soap tablets (12 tablets per pack), and sales in carton soap tablets (120 tablets per pack). The marketing strategy of the company is to ensure a relative reduction in the selling price per unit of soap tablet as the number of units in each pack increases. This was designed to encourage wholesales purchase of the soap product by the users.

Data on the five basic raw materials used for the manufacturing of KASMO medicated soaps were available in the records of the company. These raw materials are Caustic soda, Palm Kernel Oil (PKO), Colourant, Perfume and Disinfectant. Information on the quantity of each raw material held in stock per month, probably due to space or financial constraint, was obtained. Also, information on the quantity mix of these basic five raw materials and their costs for effective manufacturing of the soap product was equally available and obtained from the company's records. Finally, data were collected on the quoted selling prices of the four sales packages of soap adopted by the company. However, information regarding labour cost, sales and marketing expenses and other related overheads were not obtainable, and as such, their effects were ignored in the analysis. Therefore, the only cost element considered for soap manufacturing in this paper is the cost of raw materials.

In a nutshell, the main focus of this work is to determine the quantity of each of the four sales packages that will maximize the profit of the company given the aforementioned raw materials constraints.

The analysis was carried out using linear programming techniques. The linear programming problem developed here is a mathematical program in which the objective function is linear in the unknown variables and the constraints have linear equation or linear inequality or both. The general form of a linear programming problem is stated in matrix form as shown below:

Objective function: Maximize $Z = \mathbf{CX}$ (1)

Linear Constraints: Subject to $\mathbf{AX = b}$ (2)

Non-negativity condition: $\mathbf{X} \geq 0$ (3)

The **C** in (1) is a row vector of m-dimension representing the objective function coefficients, **X** is a $m \times 1$ column vector of the decision variables of the LP model, **A** is an $m \times k$ matrix of coefficients, and **b** is an $k \times 1$ column vector of values in the right hand side of the constraint equations in (2).

Based on the data analysed in this work, the number of decision variables is four setting $m = 4$ in (1). The four decision variables X_1, X_2, X_3 and X_4 in vector **X** in the objective function represent the four types of sales packages of soaps adopted by the company with X_1 represents sales in 1 tablet (unit sales), X_2 represents sales in 3 tablets per pack, X_3 represents sales in dozen (12 tablets per pack) while X_4 represents sales of soaps in carton (120 tablets per pack).

Also, since the company uses five different raw materials for the manufacturing of her medicated soaps, therefore, there are five linear constraints for the LP model, setting $k = 5$ in (2). The whole analysis was performed using R statistical package (www.cran.org). A self contributed library package in R, the '`lpSolve`' that implements the LP was adapted in this study.

3. Data Analysis

The data collected from KASMO Industry Limited (KIL) Osogbo, Nigeria on her main product line – medicated soap were analysed to determine the best sales package that would yield maximum profit to the company.

Table 1 in Appendix A presents the five raw materials used for the production of soaps at KIL, Osogbo. The maximum quantity of each raw material held in stock for monthly production is also reported in the table.

The combinations of the quantities of these five basic raw materials (raw material mix) for soap production per boiling (in litres) are presented in Table 2 in Appendix A. This information is used to determine the production cost (in terms of raw materials) per tablet of medicated soap produced by the company.

Not only that, the company's quoted selling price per package of soap as indexed by the four decision variables X1, X2, X3 and X4 are presented in Table 3 in Appendix A. This information was later used to determine the marginal profit derived from each sale's package by the company as presented in Table 4 in Appendix A.

All the information provided in Tables 1 through 4 was used to form the linear programming model of the maximization type for the data as stated below.

Maximize the objective function (profits):

$$Z = 14.36X_1 + 33.08X_2 + 112.32X_3 + 1023.20X_4$$

Subject to (raw materials constraints):

Caustic Soda: $0.88995X_1 + 2.66984X_2 + 10.67937X_3 + 106.793650X_4 \leq 16820$

Palm Kernel Oil: $2.14815X_1 + 6.44444X_2 + 25.77778X_3 + 257.77778X_4 \leq 40600$

Colourant: $0.00153X_1 + 0.00460X_2 + 0.01841X_3 + 0.18413X_4 \leq 29$

Perfume: $0.00614X_1 + 0.01841X_2 + 0.07365X_3 + 0.73651X_4 \leq 116$

Disinfectant: $0.02762X_1 + 0.08286X_2 + 0.33143X_3 + 3.31429X_4 \leq 522$

$X_1, X_2, X_3, X_4 \geq 0$

In order to represent the above LP model in canonical form, five slack variables Xi (i = 5, 6, 7, 8, 9) were introduced into the model. This changed the inequalities signs in the constraint aspect of the model to equality signs. A slack variable will account for the unused quantity of raw material (if any) it represents at end of the production. As a result, the above LP model becomes that of

Maximize $Z = 14.36X_1 + 33.08X_2 + 112.32X_3 + 1023.20X_4$

Subject to (raw materials constraints):

Caustic Soda: $0.88995X_1 + 2.66984X_2 + 10.67937X_3 + 106.793650X_4 + X_5 = 16820$

Palm Kernel Oil: $2.14815X_1 + 6.44444X_2 + 25.77778X_3 + 257.77778X_4 + X_6 = 40600$

Colourant: $0.00153X_1 + 0.00460X_2 + 0.01841X_3 + 0.18413X_4 + X_7 = 29$

Perfume: $0.00614X_1 + 0.01841X_2 + 0.07365X_3 + 0.73651X_4 + X_8 = 116$

Disinfectant: $0.02762X_1 + 0.08286X_2 + 0.33143X_3 + 3.31429X_4 + X_9 = 522$

with non-negativity constraint that $X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9 \geq 0$.

For analysis proper, the Simplex method proposed by George B. Dantzig in 1947 as published in Dantzig(1963) was adopted to solve the above LP problem. The Simplex method has been found to be more efficient and convenient for computer software implementation in many instances (Yahya, 2004). The initial tableau of the above LP model as used by Simplex algorithm is given by Table 5 in Appendix A. The R statistical package was employed to fit the LP model using the 'lpSolve' library. The R codes used to fit the model are given in Appendix B. The results obtained from this analysis are presented in the next section.

4. Results

Results from the analysis carried out on the LP model in the previous section using Simplex method estimated the value of the objective function to be 271,296.4. The contributions of the four decision variables X1, X2, X3 and X4 into the objective function are 18893, 0, 0 and 0 respectively. This simply shows that only X1 variable contributed meaningfully to improve the value of the objective function of the LP model.

It is observed from the result's output in the appendix that the optimal solution to the linear programming model fitted here was attained at the first iteration. This simply shows the goodness of the structure of the data collected for analysis.

5. Discussions and Conclusion

The appropriateness of the linear programming methods for optimal resource allocation in industry has been demonstrated in this work. This is evident from the results obtained from the profit maximization type of the LP model fitted to the data collected on soap manufacturing from KASMO Industry Limited Osogbo, Nigeria.

From the results of the LP model in Section 3 as reported in Section 4, it is desirable for the KIL company to concentrate on the unit sales (1 soap tablet per pack) of her soap products. By this, total sales of about 18,893 tablets would be sold by the company per month. This would fetch the company an optimal profit of about N271,296.4 per month based on the costs of raw materials only. A simple division of the value of the objective function, N271,296.4 by 18,893 soap tablets sold based on 1 tablet per pack strategy (as shown in Table 3) yields a profit of N14.36 per soap tablet. This value agreed perfectly with the profit expected by the company on the sales of a soap tablet as contained in Table 4.

The results of the LP model fitted to the data collected from KIL are only based on the cost of raw materials used for soap production. Therefore, it is quite instructive to remark that if information on other elements of cost of production such as labour and overhead costs is available and incorporated into the LP model formulation and analysis, the results reported here might be remarkably different. Nonetheless, findings from this work could still serve as useful guides to the management of KIL in the formulation of production and marketing strategies for their soap product.

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Appendix A

Table 1: Types and quantity of raw materials used for the production of medicated soaps tablets per month

Raw Materials	Quantity held for production per month (in kg)
Caustic Soda	16,820
Palm Kernel Oil	40,600
Colourant	29
Perfume	116
Disinfectant	522

Table 2: Quantities of the five raw materials (raw material mix) used for soap production per boiling

Raw Materials	Quantity used (raw material mix) per boiling (in liters)
Caustic Soda	290
Palm Kernel Oil (PKO)	700
Colourant	0.5
Perfume	2
Disinfectant	9

Table 3: The selling price per pack of soap tablet(s) and the unit selling price in each pack of soaps

Soap Package	Selling Price per pack (₦)	Unit selling price per soap tablet (₦)
X_1 (1 tablet per pack)	30.00	30.00
X_2 (3 tablets per pack)	80.00	26.67
X_3 (12 tablets per pack)	300.00	25.00
X_4 (24 tablets per pack)	2900.00	24.17

Table 4: The production cost, selling price and profit earned per pack of soap tablets

Soap Package	Production Cost per pack of soap (₦)	Selling Price per pack (₦)	Profit (₦)
X_1	15.64	30.00	14.36
X_2	46.92	80.00	33.08
X_3	187.68	300.00	112.32
X_4	1876.80	2900.00	1023.20

Table 5: Initial tableau of the linear programming model for soap data

Slack Variable	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	Quantity
X_5	0.88995	2.66984	10.79365	106.79365	1	0	0	0	0	16820
X_6	2.14815	6.44444	25.77778	257.77778	0	1	0	0	0	40600
X_7	0.00153	0.00460	0.01841	0.18413	0	0	1	0	0	29
X_8	0.00614	0.01841	0.07365	0.73651	0	0	0	1	0	116
X_9	0.02762	0.08286	0.33143	3.31429	0	0	0	0	1	522
Z	-14.36	-33.08	-112.32	-1023.20	0	0	0	0	0	0

Appendix B

R codes employed to fit the linear programming model for soap data.

```
## Linear Programming Implementation in R for soap data ##
########################################################################

      ## Load the required library ##
      library(lpSolve)

      KASMO <- lp(objective.in = c(14.36, 33.08, 112.32, 1023.20),
                 const.mat = matrix(c(0.88995, 2.66984, 10.67937,
                 106.79365, 2.14815, 6.44444, 25.77778, 257.77778,
                 0.00153, 0.00460, 0.01841, 0.18413, 0.00614, 0.01841,
                 0.07365, 0.73651, 0.02762, 0.08286,0.33143, 3.31429),
                 nrow= 5, byrow = T), const.rhs = c(16820, 40600,
                 29, 116, 522), const.dir = rep("<=", 5), direction =
                 "max")
      KASMO

      ## Print the quantity of each of the decision variables that   ##
      ## contributed to the value of the objective function (profit) ##
      KASMO$solution

       [1] 18892.51     0.00     0.00     0.00

      ## Print the value of the objective function ##
      KASMO$objval
       [1] 271296.4

      ## Determine the profit per soap tablet ##
      KASMO$objval/KASMO$solution
       [1] 14.36   Inf   Inf   Inf
########################################################################
```