European Journal of Business and Management
ISSN 2222-1905 (Paper) ISSN 2222-2839 (Online)
Vol.9, No.9, 2017 www.iiste.org

Simulation and Hedging Oil Price with Geometric Brownian Motion and Single-Step Binomial Price Model

Chioma N. Nwafor
Glasgow School for Business and Society, Department of Law, Economics, Accountancy and Risk
Glasgow Caledonia University, Cowcaddens Road, Glasgow G4 0BA, Scotland, UK

Azeez A. Oyedele
School of Business and Enterprise, University of the West of Scotland
Paisley Campus, Paisley PA1 2BE, Scotland, UK

Abstract
This paper[1] uses the Geometric Brownian Motion (GBM) to model the behaviour of crude oil price in a Monte Carlo simulation framework. The performance of the GBM method is compared with the naïve strategy using different forecast evaluation techniques. The results from the forecasting accuracy statistics suggest that the GBM outperforms the naïve model and can act as a proxy for modelling movement of oil prices. We also test the empirical viability of using a call option contract to hedge oil price declines. The results from the simulations reveal that the single-step binomial price model can be effective in hedging oil price volatility. The findings from this paper will be of interest to the government of Nigeria that views the price of oil as one of the key variables in the national budget.
JEL Classification Numbers: E64; C22; Q30
Keywords: Oil price volatility; Geometric Brownian Motion; Monte Carlo Simulation; Single-Step Binomial Price Model

1. Introduction

The large fluctuations in crude oil prices in recent years have put significant pressure on the fiscal balances of both oil exporting and importing countries, Husain *et al*., (2015). Oil prices have halved since June 2014, likely bringing an end to a four-year period of high and stable prices. Governments of oil exporting countries generally rely heavily on revenue from oil productions and therefore, tend to suffer financially from oil price decline. Reliable forecasts of the price of oil for oil exporting countries like Nigeria, Angola, Saudi Arabia etc. is of extreme importance. Oil price is a key variable in Nigeria's national budget. For instance, the government of Nigeria recently approved an oil price benchmark of $42.50 per barrel at a production assumption of 2.2 million barrels per day, for revenue calculation in its 2017 budget, Budget and National Planning Office (2017)[2]. This compares with its 2016 budget, which had a price benchmark of $38/barrel at a 2.2 million barrels/day output figure, however, the government struggled to implement the 2016 budget due to amongst other things high volatility of crude oil prices. For instance, in January of 2016, oil prices went as low as below $25 per barrel. Since then, the prices have slowly surged to $40 and then to the present almost $50 per barrel. Given the social, political, and economic cost of volatile oil prices; different oil producing countries have tried to solve the problem of their oil price risk exposure in variety of ways. For instance, some national governments have relied on stabilization and savings funds[3], Landon and Smith (2010) to deal with oil price risk. With regards, to choosing oil benchmarks, most oil-rich developing economies have used the naïve forecasting strategy. However, these methods are not effective since these countries still bear oil price risk.

Apart from the Niger-Delta militancy[4] in the southern part of the country, there may be nothing as worrisome to the Federal Government of Nigeria as the unstable prices of crude oil in the international market.

[1] Acknowledgement: We wish to thank the two anonymous reviewers for their insightful comments and kind considerations.
Memos to: Azeez Abiola Oyedele, School of Business and Enterprise, University of the West of Scotland, Paisley Campus, Paisley PA1 2BE, Scotland, Email: abiolaoyedele@yahoo.com
[2] The 2017 Budget is based on a benchmark crude oil price of US$42.5 per barrel; an oil production estimate of 2.2 million barrels per day; and an average exchange rate of N305 to the US dollar. Based on these assumptions, aggregate revenue available to fund the federal budget is N4.94 trillion. This is 28% higher than 2016 full year projections. Oil is projected to contribute N1.985 trillion of these amounts.
[3] According to Daniel (2001), the use of stabilization and savings funds although have helped countries to manage oil price windfalls and to turn depreciating wealth into productive assets. Their performance has been very weak in reducing the effect of volatile oil prices on government revenues and spending.
[4] In 2016 Nigeria witnessed sporadic attacks of International Oil Companies facilities in the Niger Delta region with huge negative impact on the country's productivity. The affected companies include Shell Petroleum, ExxonMobil and Agip which account for over 90 per cent of the country's crude output. These attacks by militant groups in the Niger Delta had caused substantial losses to the country in terms of accruable crude oil earnings as productivity falls immediately after the attacks.

Despite the efforts to diversify the country's economy, Nigeria government still relies heavily on oil to generate her foreign exchange, and fund her yearly budgets. For an economy dependent on crude oil for 70 per cent of government revenues, more accurate forecasts of the price of oil and the use of market based risk instruments[1] have the potential of improving government budgets. This is because basing the nation's budget on oil price forecasts that could turn out to be very far away from the actual price will give rise to huge budget deficit, and the use of market based risk instruments can allow the government to lock-in the price of their future production or consumption today, against volatility in oil price movements. Using these methods, the revenue stream is made more stable and predictable. Furthermore, some sectors of the economy depend on forecasts of the price of oil for their business budget and planning. For example, airlines rely on oil price forecasts in setting airfares.

A major problem for the managers of oil-rich developing economies like Nigeria is that energy prices are exogenous, they are determined in the international market or North American markets, and are therefore, out of the control of the government of Nigeria, Landon and Smith (2010). Indeed, countries like Nigeria are net price-takers in the international oil market. Consequently, a starting point to managing fluctuations in oil prices to ensure that declines in prices did not lead to an overall budget deficit for the country will be an understanding of a stochastic process to represent the evolution of the price of oil and the design of suitable hedging strategies to mitigate its adverse effects. Thus, our objectives in this paper are two folds; first, is to determine whether the geometric Brownian motion can perform well as a proxy for the movement of oil prices, and second, to test the viability or otherwise of using options contracts in mitigating government revenue fluctuations as a result of decline in oil prices. Our objective is to explore whether this theoretical solution to managing oil price risk might be able to work in practice.

Oil price movements are difficult to predict, largely because of the difficulties in identifying a forecasting model. In addition, oil price time series may display signs of nonlinearity that may not be captured by conventional linear forecasting techniques, often producing unsatisfactory results. Researchers as well as policy analysts have used the Vector Auto Regressive (VAR) models, GARCH and its variants to predict short-to-medium term real oil price. Baumeister and Kilian (2012) and Alquist, Kilian and Vigfussion (2013) in their studies showed that real oil price forecasts using VAR produce more accurate predictions of the future path of real oil prices relative to futures or other models. In this paper, we attempt to forecast the nominal price of international oil benchmarks namely West Texas Intermediate spot price (WTISP), Europe Brent spot price (EBSP) and NYMEX Crude Oil futures contract using Monte Carlo Simulation. We used the Geometric Brownian Motion (GBM) to model the behaviour of crude oil price in the simulation. This heuristic approach allows us to extract more from the data than traditional regression techniques.

The rest of this paper is structured as follows. Section 2 provides a brief review of previous literatures, section 3 presents the description of the data used in this study, including correlation matrix and results of the unit root test. In section 4, we describe the benchmark naïve model and the geometric Brownian model. Section 5 is the discussion of our evaluation approach, measuring forecast unbiasedness and accuracy as well as discussing key findings; section 6 presents the single-step binomial option pricing model while section 7 is the summary and conclusions.

2. Review of literatures on stochastic models and market based mechanism to dealing with oil price risk

Academic literatures on oil price forecasting can be divided into two strands; the first strand of literatures use continuous stochastic models such as the Geometric Brownian Motion (GBM), the Mean Reversion Process (MRP) and the combined process of Mean-Reversion with Jumps (MRPJ). The other strand of literatures is mostly based on regression analysis. For the first strand, early works in this area model the evolution of oil prices as a Geometric Brownian Motion. These studies include, Passock *et al*., (1988); Brennan and Schwartz (1985); and McDonald and Siegel (1985). More recently, Postali and Picchetti (2006), in their study showed that geometric Brownian motion performs well as a proxy for the movement of oil prices and for a state variable to evaluate oil deposits.

Other researchers, such as Schwartz (1997) on the other hand argue that oil price would be correctly modelled as a mean reverting[2] process also known as Ornstein-Uhlenbeck process. Dixit and Pindyck (1994) developed a variant of the Ornstein-Uhlenbeck mean reverting process the so called Geometric Ornstein-Uhlenbeck (GOU) process because the original process allows for negative prices. The current study used GBM because of its tractability, operational easiness and the ability of this model to review all predictions at the same ratio in the event of an unanticipated change in prices. This is not the case with the mean revision models that

[1] Market based risk instruments are one of the best methods or strategies for dealing with commodity price volatility, Daniel (2001). This involves the transfer of oil price risk outside the country to those better able to bear it. Market based hedging instruments include the use of futures and options contracts as well as complex combinations of collars, over-the-counter, amongst others to hedge against adverse oil price movement.

[2] Mean reversion is a theory that suggests that prices and returns eventually return back toward the mean or average.

imposes an upper bound to the expected changes, which is the highest when prices are equidistant from zero and their equilibrium level, Postali and Picchetti (2006).

Next to this strand are literatures that use Vector Auto-Regression (VAR) models with endogenous regressors suggested by Economic theory. For instance, Kilian (2009) in the study of the determinants of real price of oil included a number of global economic aggregates including oil supply and demand, global crude oil inventories amongst others. Alquist *et al*., (2013) found that proxies such as global oil demand measured by global industrial production and the index of global real economic activity developed by Kilian (2009) provide better forecasts than models with U.S GDP. Reichesfeld and Roache (2011) investigated the predictive ability of futures for different commodities. They concluded that energy futures perform better in forecasting future spot prices than non-energy commodity futures. Alquist *et al*., (2013) posit that a VAR model with global oil supply, Kilian's index of real economic activities and crude oil inventories outperforms the futures forecast for short forecast horizons. This study departs from the above-mentioned studies by studying oil price series directly. Our objective is to ascertain whether GBM can represent a suitable approximation for the evolution of oil prices. The results from the simulation are benchmarked against naïve forecasting technique to determine the potential added value of this method as a forecasting tool.

Turning attention to market based mechanism to deal with oil price risk, previous studies on the use of futures and options markets in reducing oil price volatility by sovereign governments suggest that these financial market instruments are effective in hedging price risk at least in the short-run. Countries like Mexico, U.S, and Norway amongst others have frequently relied on the use of financial derivatives to smooth out fluctuations in revenue as a result of oil price movements. Sadorsky (2014) suggests that hedging oil price volatility in emerging markets provide a better means of managing oil price risk. Larson and Varangis (1996) reported that in late 1990 and early 1991, the Mexican government purchased put options with a strike price of US$17 per barrel in order to protect its oil-related revenues from a price decline. Swidler *et al*., (1999) showed that Texas operated on a continuous basis between 1992 and 2000 a hedging instrument to ensure that declines in oil prices did not lead to an overall budget deficit for the two-year duration of each budget period.

Using data form 1990-2001, Daniel (2001) compares a strategy that would involve selling oil each month at the 12-month ahead futures price to a strategy that involves selling the same oil at the spot price when it is received in 12 months. He discovered that the futures price is not as volatile as the spot price, although the price resulted in a slightly lower average price over the sample. Domanski and Heath (2007) also found that futures energy prices may be less volatile than spot prices, at least for futures contracts that are far enough in the future, generally at least 12 months. Blas (2009) reveals that Mexico spent nearly $1.2 billion on purchasing options to hedge 230 million barrels of oil exports in 2010 at $57 a barrel, the second year in a row the nation has done this to protect government spending from price fluctuations. These findings point to the fact that the use of futures contract can be effective in hedging against oil price risk albeit in the short-run.

While futures markets can reduce revenue uncertainty, Landon and Smith (2010), highlighted the shortcomings of this market. According to them, the majority of transactions within the futures markets involve relatively short-term contracts (one or two months), but to have a substantial impact on uncertainty and the volatility of revenues, the government would need to enter into futures contracts that cover at least the current budget year. In addition, authors such as (Larson, Varangis and Yabuki, 1998; Domanski and Heath, 2007; Borensztein *et al*., 2009) have also highlighted the illiquidity of this market at longer maturities which may make the sale of futures contracts more difficult and costly. Also, the price locked in by futures contracts may change from year to year since the contracts are sold in different years. An additional shortcoming of futures markets as a method to smooth revenues is that, while futures contracts remove downside price risk, they also eliminate the potential benefit of energy price increase. A possible solution to overcoming the limitations of futures contracts is the use of options[1] contracts. These contracts provide insurance against price declines by giving the holder of the contract the option, but not the obligation, to sell a commodity at a stated price if the price falls below a particular "strike price". Using options contracts ensure that a government obtains the benefit of a price increase if oil prices rise rather than fall, but is at the same time, protected against a price fall.

Despite the potential benefits of price hedging, many oil producing countries have hesitated to use options markets to reduce the volatility of revenues largely because of lack of technical knowhow or even for political reasons. Daniel (2001), Caballero and Cowan (2007) and Frankel (2010) posit that the political costs of

[1] A number of oil exporting countries like Ecuador, Mexico and Texas, for example – have used option contracts. Blas (2009) reported that in 2008, with the price of oil over US$100, Mexico hedged all of its oil sales for 2009 through the purchase of put options with a strike price of US$70. In December 2009, Mexico announced that it had hedged 230 million barrels of oil, a large proportion of its 2010 oil production, at a strike price of $57. To hedge all its oil sales in 2009, Mexico paid $1.5 billion for put options with a strike price of $70 when the spot price was over $100. In the end, Mexico made a net gain of $5 billion from this strategy (McCallion, 2009). If prices had not fallen below $70, the Mexican government would have spent an extra $1.5 billion without an obvious payoff.

hedging can outweigh the benefits[1]. This study is a first attempt towards testing the viability or otherwise of using options contracts in mitigating government revenue fluctuations as a result of declines in oil price.

3. Data analysis: descriptive statistics, correlation matrix, and Unit root tests

We used daily data from May 20th 1987 to Dec 27th 2016 to forecast the likely future price paths of two international oil benchmarks and oil futures: Europe Brent spot price, EBSP and West Texas Intermediate WTISP and NYMEX futures contract COFC. Geometric Brownian model is used to model the dynamics of the oil price series. These variables were sourced from the U.S Energy Information Agency (EIA). Table I presents the descriptive statistics of the oil proxies. The average of Nigeria prime lending rate is used in the single-step binomial price model as a proxy for the lending rate. The prime lending rate series from 2nd January 2015 to 27 January 2016 was sourced from the Central Bank of Nigeria's database.

Table 1: Summary Statistics

	$EBSP$	$WTISP$	$COFC$
Mean	44.76	43.96	43.93
Median	27.99	29.24	29.05
Minimum	9.10	10.82	10.72
Maximum	143.95	145.31	145.29
Standard Deviation	33.65	30.26	30.31

Note: $EBSP$ denotes Europe Brent Spot Price FOB (Dollars per Barrel), $WTISP$ denotes West Texas Intermediate Spot Price FOB (Dollars per Barrel), $COFC$ denotes Cushing, Oklahoma Crude Oil Futures Contract (Dollars per Barrel). These series are available at U.S. Energy Information Administration.

The descriptive statistics reveal that the average prices of the 3 measures of oil price are quite close: EBSP \$45, WTISP \$44 and COFC \$44. The median prices during the period under review are approximately \$28 for EBSP, \$29 for WTISP and \$29 for COFC. The standard deviations for the three-price series indicate a very high level of volatility. The correlation matrix is reported in table 2.

Table 2: Correlation Matrix

	$EBSP$	$WTISP$	$COFC$
$EBSP$	1.00	0.986	0.982
$WTISP$	0.986	1.00	0.995
$COFC$	0.982	0.995	1.00

Note: Correlation coefficients, using the observations 1987-05-20 to 2016-03-07 5% $critical\ value$ (two-tailed) = 0.0226 for $n = 7514$: EBSP denotes Europe Brent Spot Price FOB (Dollars per Barrel), WTISP denotes West Texas Intermediate Spot Price FOB (Dollars per Barrel), COFC denotes Cushing, Oklahoma Crude Oil Futures Contract (Dollars per Barrel). These series are available at U.S. Energy Information Administration.

The correlation matrix shows that the three measures of crude oil prices are positively and closely related. Next, we performed a unit root test on the growth rate of the variables using the Augmented Dickey Fuller ADF) test. The results from the ADF test reveals that all the price series are stationary at the one percent levels of significance.

Table 3: Unit Root Tests: Augmented Dickey-Fuller test with Constant and Trend Included in The Test Equation

Variables	$Test\ statistic$
$EBSP\_gr$	-20.4309***
$WTISP\_gr$	-34.3024***
$COFC\_gr$	-33.4591***

Note: $EBSP\_gr$ denotes Growth rate of Europe Brent Spot Price FOB (Dollars per Barrel), $WTISP\_gr$ denotes Growth rate of West Texas Intermediate Spot Price FOB (Dollars per Barrel), $COFC\_gr$ denotes Cushing, Oklahoma Crude Oil Futures Contract (Dollars per Barrel). These series are available at U.S. Energy Information Administration. Unit-root null hypothesis:$a = 1$.

The growth rate of the oil price series spanning from May 20th 1987 to December 27th 2016 are used to calculate the expected return, the variance, the historical volatility and the drift parameters that were used in the simulations. We used the data from January 2nd 2015 to 27th December 2016 to perform the simulations.

[1] In the event of a fall in the spot price, any financial gains from hedging instruments may be seen as speculative returns. It is easy to blame the international oil market for budget deficits if a country had not hedged. Landon and Smith (2010) suggest that a government that has purchased an options contract may be blamed if the spot price remains above the strike price for the duration of the contract. This is because the government that purchased the put option would not reap an explicit benefit, but would still bear the cost of purchasing the option.

European Journal of Business and Management www.iiste.org
ISSN 2222-1905 (Paper) ISSN 2222-2839 (Online)
Vol.9, No.9, 2017

4. Geometric Brownian Model (GBM) and naïve strategy

The naïve strategy simply assumes that the most recent period price of an asset is the best predictor of the future price. The model is defined by:

$$f_{t+1} = y_t \tag{1}$$

Where y_t is the actual price at period t and f_{t+1} is the forecast price for the next period. The performance of the naïve strategy is compared with that of the Monte Carlo Simulation method with Geometric Brownian motion[1] using different forecast evaluation techniques.

A Monte Carlo Simulation (MSC) is an attempt to predict the likely future path of a variable of interest many times over. At the end of a Monte Carlo Simulation (MCS), thousands of "random trials" produce a distribution of outcomes that can be analysed. In this paper, we review a basic MCS applied to three measures of oil prices namely; West Texas Intermediate spot price, Europe Brent spot price and NYMEX Crude Oil futures contract. We used the Geometric Brownian Motion (GBM) to model the behaviour of crude oil price. The GBM is a continuous time stochastic process in which the logarithm of a random varying quantity follows Brownian motion which in stochastic terms is a wiener[2] process. GBM is also described as a Markov process. A Markov process has the property that the future is independent of the past, given the present state, this is somewhat consistent with the weak form of the Efficient Market Hypothesis (EMH): past price information is already incorporated and the next price movement is conditionally independent of past price movements.

Historically, Brownian motion and other stochastic processes constructed from it have been used to model stock prices subject to random noise. The GBM started from the seminal work of Bachelier (1900) a century ago and the subsequent work of Black and Scholes some decades after with the geometric Brownian motion, this model has been used to solve problems in numerous fields including Finance, Engineering, and Applied Statistics amongst others. While the Geometric Brownian Motion (GBM) model for stock prices has been used extensively in developed and some emerging markets to model the evolution of stock price levels and their returns, very few studies Paddock *et al.,* (1988), Brennan and Schwartz (1985), McDonald and Siegel (1985) have modelled commodities prices as a Geometric Brownian Motion (GBM). Although the most popular forecasting approaches are based on conventional econometrics model; however, the premise of this study is based on the observation of Brown (1827)[3].

Suppose that $\mathbf{Z} = \{Z_t : t \in (0, \infty)\}$ is standard Brownian motion[4] and that $\mu \in \mathbb{R}$ and $\sigma \in (0, \infty)$. Let

$$X_t = exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma Z_t\right], t \in (0, \infty) \tag{2}$$

The stochastic process $\boldsymbol{X} = \{X_t : t \in (0, \infty)\}$ is geometric Brownian motion with drift parameter $\mu - \frac{\sigma^2}{2}$ and volatility parameter σ. It is important to note that the stochastic process;

$$\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma Z_t : \ t \in (0, \infty)\right\} \tag{3}$$

is a Brownian motion with drift parameter $\mu - {\sigma^2}/{2}$ and scale parameter σ, so geometric Brownian motion is simply the exponential of this process. Indeed, the process is always positive. This is one of the reasons that geometric Brownian motion is used to model financial and other processes that cannot be negative. The geometric Brownian motion $\boldsymbol{X} = \{X_t : t \in (0, \infty)\}$ satisfies the stochastic differential equation (SDE):

$$dX_t = \mu X_t dt + \sigma X_t dZ_t \tag{4}$$

Where Z_t follows a Wiener process, μ is the expected growth rate and σ is the volatility or standard

---

[1] Norbert Wiener in a series of papers from 1918 was the first researcher to model Brownian motion as a mathematical random process. Consequently, the Brownian motion process is also known as the Wiener process.

[2] A Wiener process can be characterised by three properties:
$W_0 = 0$
W_t =is continuous
W_t has independent increments with $W_t - W_s \sim N(0, t-s)\ for\ 0 < s < t$

[3] Brownian motion is the physical phenomenon named after the founder Robert Brown, who discovered the motion in 1827. Brownian motion is the "zig-zagging" motion exhibited by small particles such as a grain of pollen, immersed in a liquid or gas.

[4] A standard Brownian motion is a random process $\boldsymbol{X} = \{X_t : t \in \{0, \infty\}\}$ with state space $\mathbb{R}$ that satisfies the following properties:

a) $X_0 = 0$ (with probability 1).
b) $\boldsymbol{X}$ has stationary increments. That is, for $s, t \in (0, \infty)$ with $s < t$, the distribution of $X_t - X_s$ is the same as the distribution of X_{t-s}.
c) $\boldsymbol{X}$ has independent increments. That is, for $t_1, t_2, \dots, t_n \epsilon (0, \infty)$ with $t_1 < t_2 < \cdots < t_n$, the random variables $X_{t1}, X_{t2} - X_{tn} - X_{tn-1}$ are independent.
d) X_t is normally distributed with mean 0 and variance t for each $t \in (0, \infty)$.
e) With probability 1, $t \to X_t$ is continuous on $(0, \infty)$.

deviation. The parameter $\mu - \sigma^2/2$ in equation (2) determines the asymptotic[1] behaviour of geometric Brownian motion. Asset returns have normal distribution $dlnX_t \sim N(\mu d_t, \sigma d_t)$. In the GB model, the expected growth rate of the asset is constant, given by $E(dX/X) = \mu dt$. The standard deviation σ is also constant, which indicates an increase in expected price volatility as time horizon increases, Postali and Picchetti (2006). That is, as prices increase (or decrease) by more than predicted in a given period, all future price forecasts will also increase (decrease) by an equal amount[2]. Monte Carlo simulations can be used to determine results for dZ_t via random sampling from normal distribution. Monte Carlo simulation provides a powerful tool for simulating possible future price paths of financial assets. This technique is used by professionals and has found applications in Operational Research, Physics, and Finance amongst others.

4.1 Parameter estimation

From the discussion, so far, it is easy to notice that Brownian motion assumes that there are two parts to a random movement, the first is an overall constant driving force called the drift and the second is the random component. Therefore, the rate of the asset's changes in value each day can be broken down into the two parts noted above. To create a Monte Carlo simulator to model possible future oil price outcomes, we estimated the constant drift and the random component using daily data form May 20th 1987 to December 27th 2016 as shown below:

$$ln\left(\frac{X_t}{X_{t-1}}\right) = \alpha + Z_t\sigma \qquad (5)$$

Where $ln\left(\frac{X_t}{X_{t-1}}\right)$ is the periodic continuously compounded daily return, α is the constant drift component and $Z_t\sigma$ is the random shock component. In this paper, X_t represents the three measures of oil prices namely, the Europe Brent Spot crude oil price (EBSP), West Texas Intermediate Spot crude oil price (WTISP), and the NYMEX Futures Crude Oil Contract (COFC). To model the drift, we calculated the expected periodic daily rate of return; defined as the rate with the greatest odds of occurring. In this paper, the drift is computed using the standard Brownian motion formula as:

$$\mu - \frac{\sigma^2}{2} \qquad (6)$$

Equation (6) defines the drift[3] as the average of the historical periodic daily returns eroded by volatility at the rate of half of the variance over time. This is simply the average of the historical periodic daily returns minus half the variance. We calculated the periodic daily returns, the mean, variance and standard deviations for each of the oil price proxies. Using these values, we developed the formula of the Drift (equ 6) plus a random stochastic component. The random part combines with the drift to produce theoretical daily returns that are normally distributed around the drift.

We used two Excel functions $NORMSINV(Percent) = Z\ Score$. This function takes a percentage of the area under a curve and finds the number of standard deviations that is furthest away from the mean; this is known as the $Z\ Score$. The second spreadsheet function that we used is the $RAND$ function. This produces a random number between 0 and 1.When combined with the $NORMSINV(RAND())$ function, the $RAND$ produces random percentages between 0 and 1. The $NORMSINV$ function converts those random percentages to random standard deviations away from the mean, in other words random $Z\ Scores$. Using these functions, we simulated possible forecasted prices[4] using equation (7) and compared them with the actual prices starting from January 2nd 2015 to December 27th 2016.

$$Oil\ Price\ Today = (Previous\ Day's Oil\ Price) * \exp(Drift + Std.Dev * NORMSINV(RAND())$$
(7)

Equation (7) stipulates that the price of crude oil today is equal to yesterday's price multiplied by e^r raised to a power that is normally distributed around the drift. Figures 1 to 3 below show the graphs of the three oil price proxies and the simulated prices.

---

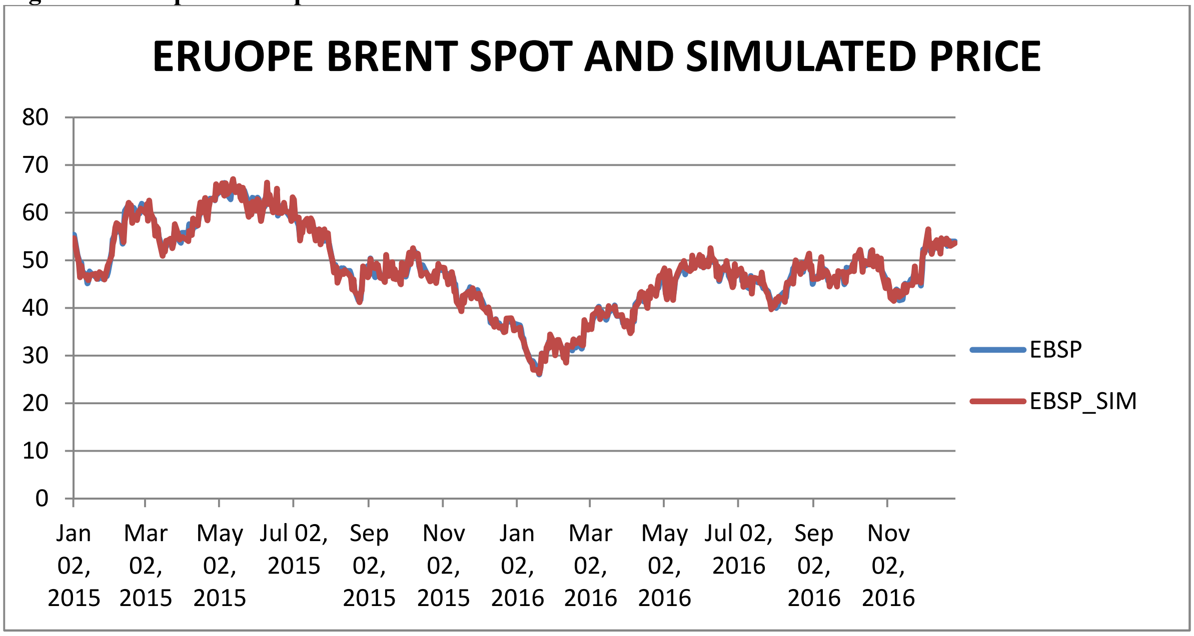
[1] Asymptotic behaviour:
If $\mu > \sigma^2/2$ then $X_t \to \infty$ as $t \to \infty$ with probability 1
If $\mu < \sigma^2/2$ then $X_t \to 0$ as $t \to \infty$ with probability 1
If $\mu = \sigma^2/2$ then X_t has no limit as $t \to \infty$ with probability 1.

[2] According to Postali and Picchetti (2006), "this means that GBM implies a high degree of volatility in predicted prices and embeds a high level of uncertainty". This is similar with the behaviour of spot oil prices.

[3] A second method of calculating the drift component is $Risk\ Free\ Rate - (Variance/2)$. The risk free rate is the rate that an investor can get by investing in a "riskless" asset such as government bonds. The other theory is that the expected rate of change each day or drift should be zero, this is supported by the random walk theory.

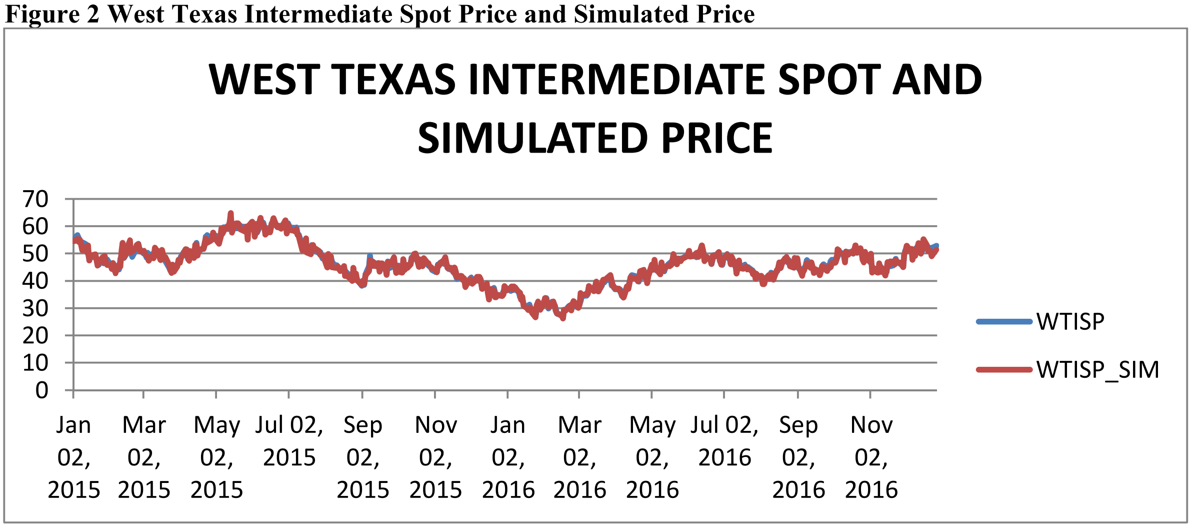
[4] For details of the simulation, please refer to the spread sheet.

Figure 1: Europe Brent Spot Price and Simulated Price



Note: EBSP denotes the Europe Brent Spot Oil Price and EBSP_SIM denotes the simulated Europe Brent Spot Price.

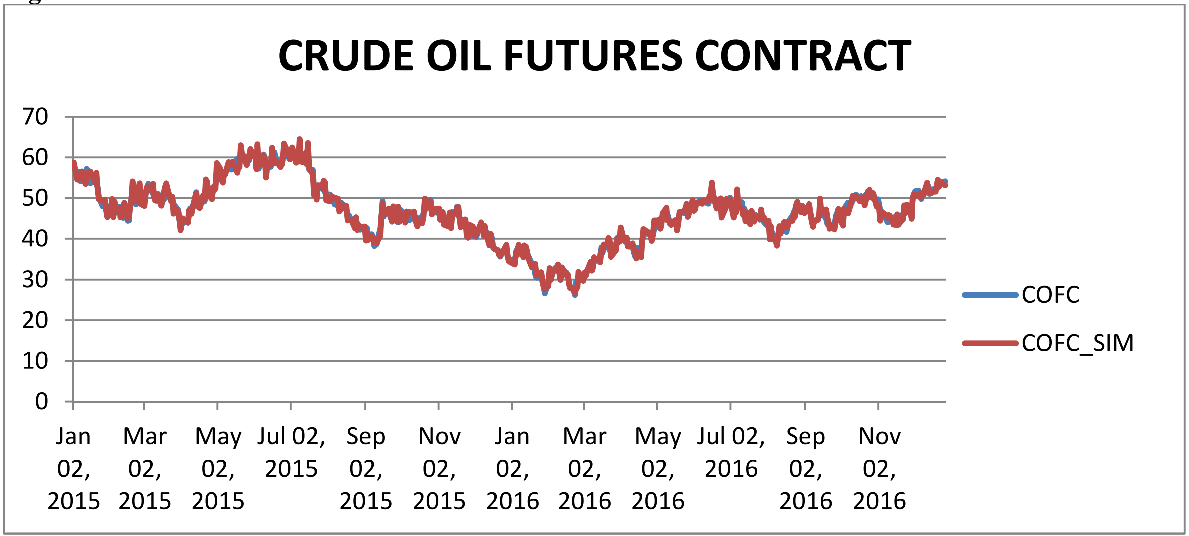
Figure 2 West Texas Intermediate Spot Price and Simulated Price



Note: WITSP denotes the West Texas Intermediate Spot Price and WTISP_SIM denotes the Simulated price of the West Texas Intermediate Spot Price.

European Journal of Business and Management
ISSN 2222-1905 (Paper) ISSN 2222-2839 (Online)
Vol.9, No.9, 2017

Figure 3 Crude Oil Futures Contract and Simulated Price



Note: COFC denotes Crude Oil Futures Contract and COFC_SIM denote the simulated price of the Crude Oil Futures.

The two series (actual and simulated) in all cases have closely tracked each other. In addition, table 4 below is a sample of the forecast prices vs. actual prices for Europe Brent spot price and West Texas Intermediate spot prices. The table shows that the model did an excellent job of tracking the actual prices. If we compare the performance of the nominal Brent price and WTI spot price, the results in table 4 (actual vs. forecasts) shows that the GBM model performs better in tracking movements in Brent spot price than in WTI for the period investigated, and therefore, adds value as a forecasting tool. We reported set of future possible price[1] paths that generated the lowest possible forecast errors based on the forecast evaluation metrics (refer to section 5).

[1] Pressing F9 key changes the random number that the RAND function produces. This recalculates the formula with a new $Z - Score$, creating new sets of possible future prices.

Table 4: A Sample of Forecast Prices vs. Actual Prices for Europe Brent and West Texas Intermediate Spot Price.

Date	Actual EBSP ($ per Barrel)	Forecast EBSP ($ per Barrel)	Actual WTISP ($ per Barrel)	Forecast WTISP ($ per Barrel)
Jan 02, 2015	55	55	55	55
Jan 05, 2015	51	52	57	51
Jan 06, 2015	50	49	56	48
Jan 07, 2015	49	49	55	50
Jan 08, 2015	49	48	53	51
Jan 09, 2015	48	48	54	46
Jan 12, 2015	47	45	53	47
Jan 13, 2015	45	46	53	44
Jan 14, 2015	46	46	50	48
Jan 15, 2015	48	47	48	50
Jan 16, 2015	47	46	49	50
Jan 20, 2015	46	48	49	47
Jan 21, 2015	47	47	48	47
Jan 22, 2015	46	46	46	45
Jan 23, 2015	47	49	46	49
Jan 26, 2015	46	46	48	46
Jan 27, 2015	47	44	46	46
Jan 28, 2015	47	47	48	49
Jan 29, 2015	47	47	47	49
Jan 30, 2015	48	49	48	48
Feb 02, 2015	52	51	46	51
Feb 03, 2015	54	54	45	55
Feb 04, 2015	55	57	45	53
Feb 05, 2015	56	55	46	53
Feb 06, 2015	56	57	44	55
Feb 09, 2015	57	58	44	56
Feb 10, 2015	56	55	48	55
Feb 11, 2015	53	51	49	55
Feb 12, 2015	56	59	53	57
Feb 13, 2015	60	61	48	59
Feb 16, 2015	62	63	50	61
Feb 17, 2015	61	60	52	62
Feb 18, 2015	61	64	53	63
Feb 19, 2015	59	59	50	57
Feb 20, 2015	61	62	49	64
Feb 23, 2015	60	58	51	58
Feb 24, 2015	60	60	53	63
Feb 25, 2015	60	58	54	61
Feb 26, 2015	61	63	52	62
Feb 27, 2015	62	59	51	62

5. Forecast evaluation statistics and interpretation of results

In this section, we will compare the simulated models with the naïve models using eight statistical forecast evaluation measures. The forecast evaluation statistics we considered are: the Mean Error (ME), the Mean Squared Error (MSE), the Mean Absolute Error (MAE), the Mean Percentage Error (MPE), the Mean Absolute Percentage Error ($MAPE$), the Root Mean Squared Error ($RMSE$) and Theil's U-statistics. Theil's U-statistics is presented in both of its specifications, these were labelled U_1 and U_2 ; the more accurate the forecasts, the lower the value of Theil's U, which has a minimum of 0. The Theil's U can be interpreted as the ratio of the RMSE of the proposed forecasting model to the RMSE of the naïve model which simply predicts $x_{t+1} = x_t$ for all t.

The naïve model yields $U_2 = 1$; values less than 1 indicate an improvement relative to the benchmark naïve model and values greater than 1 show a deterioration. Let x_t be the series of interest (crude oil price) at time t and let f_t be the forecast of x_t. The MAE, MSE and $RMSE$ statistics are scale-dependent measures that allow a comparison between the actual and forecast values, the lower the values the better the forecasting accuracy. The $MAPE$ and $Theil - U$ are used to evaluate the forecast errors independent of the scale of the variables. We define the forecast error as $e_t = x_t - f_t$ for $t = 1 \ldots, T$. x_t is the actual change at time t, f_t is the forecast change $t = 1$ to $t = T$ for the forecast period. Given a series of T observations and associated forecasts; we construct eight measures of the overall accuracy of the forecasts. These are defined as follows.

$$ME = \frac{1}{T}\sum_{i=1}^{T} e_t \tag{8}$$

$$MSE = \frac{1}{T}\sum_{t-1}^{T} e_t^2 \tag{9}$$

$$MAE = \frac{I}{T}\sum_{t=1}^{T} |e_t| \tag{10}$$

$$MPE = \frac{1}{T}\sum_{t=1}^{T} 100\frac{e_t}{y_t} \tag{11}$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T} e_t^2} \tag{12}$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T} 100\frac{|e_t|}{y_t} \tag{13}$$

$$U_1 = \frac{\sqrt{\frac{1}{T}\sum_{i=1}^{T}(y_t - f_t)^2}}{\sqrt{\frac{1}{T}\sum_{I=1}^{T} y_t^2 + \sqrt{\frac{1}{T}\sum_{i=1}^{T} f_t^2}}} \tag{14}$$

$$U_2 = \frac{\sqrt{\frac{1}{T}\sum_{i=1}^{T-1}\left(\frac{f_{t+1} - y_{t+1}}{y_t}\right)^2}}{\sqrt{\frac{1}{T}\sum_{i=1}^{T-1}\left(\frac{y_{t+1} - y_t}{y_t}\right)^2}} \tag{15}$$

Table 5 presents the results of the statistical performance measures used to analyse the accuracy of the forecasting techniques.

Table 5: Forecast Evaluation Statistics Geometric Brownian Models (GBM) and Naïve Forecasts (NF)

	***EBSP* GBM**	***EBSP_NF***	***WTISP* GBM**	***WTISP_NF***	***COFC_GBN***	***COFC_NF***
Mean Error	0.03	0.00	0.07	-0.01	0.15	-0.01
Mean Square Error	1.18	1.60	1.11	1.71	1.26	1.72
Mean Absolute Error	0.86	0.95	0.84	1.02	0.88	1.04
Mean Percentage Error	0.04	-0.04	0.08	-0.06	0.13	-0.06
Mean Absolute Percentage Error	1.80	2.04	1.84	2.30	1.88	2.31
Root Mean Square Error	0.86	0.95	0.84	1.02	0.88	1.04
Theil's U1	0.00	0.00	0.00	0.00	0.00	0.00
Theil's U2	0.82	1.00	0.75	1.00	0.79	1.00

Note: EBSP denotes Europe Brent Spot Price FOB (Dollars per Barrel), WTISP denotes West Texas Intermediate Spot Price FOB (Dollars per Barrel), COFC denotes Cushing, Oklahoma Crude Oil Futures Contract (Dollars per Barrel). These series are available at U.S. Energy Information Administration.

Looking at the value for the mean error (ME) for Europe Brent Spot Price ($EBSP$), West Texas Intermediate Spot Price ($WTISP$) and Crude Oil Futures Contract ($COFC$), the results reveal that the naïve forecasts dominate its rival Geometric Brownian model with a zero-value obtained for this metric. Using the ME as the only forecast evaluation technique, one would conclude that there is likely to be no added value using complicated forecasting techniques. However, a low value of the ME may conceal forecasting inaccuracy due to the offsetting effect of large positive and negative forecast errors. The Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE) may overcome the limitations of ME, consequently, we calculated these statistics for the simulated and the naïve models.

On the basis of MSE, RMSE, and MSE, it is clear that the simulated models on all the three measures of oil prices outperformed the naïve models. This implies that the GBM provides more accurate forecasts than the naïve model. These statistics do not take scale of the variables of interest into account, although the scales of the three crude oil series are the same. We calculated three more forecast evaluation statistics: Mean Percentage Error (MPE), Mean Absolute Percentage Error ($MAPE$), and $Theil - U_1$& $Theil - U_2$. These statistics take the scale of the variables of interest into account. Once again, all the four scaled measures show the simulated forecasts to be preferred to the naïve forecasts. Specifically, using the U_1 statistic that is bounded between 0 and 1, with values closer to 0 indicating greater forecasting accuracy, there seems to be no difference between the naïve and GBM models. The calculated U_1 statistic for the three measures of oil price is zero.

Turning attention to U_2 which is generally considered as a superior statistic, Theil (1961), the values for all the three proxies of crude oil price are less than 1. This indicates greater forecasting accuracy than the naïve forecasting method. The U_2 statistic takes the value 1 under the naive forecasting method. The forecasting accuracy statistics seem to suggest that the Geometric Brownian model outperforms the naïve model. It seems plausible to argue that the GB model performs well as a proxy for modelling movements in oil prices. A country like Nigeria that has this exogenous variable as a key input in their budgetary process needs to choose a movement that reflects as likely as possible the dynamics of oil price. Consequently, Nigeria may employ the GBM in forecasting the likely future paths of oil prices because of its important characteristics of allowing closed form solutions to diverse problems on assets evaluation as well as its tractability and parsimony.

A major problem for the managers of Nigeria economy is that they are exposed to large oil price risk which they are not equipped to bear. A simple solution could be to transfer this risk outside the country to those better able to bear it. This can be achieved via oil price risk markets. One way of doing this from an oil producer's perspective will be to sell oil forward[1] another way will be to buy insurance against large price falls. In this way, government will make her revenue stream more stable and predictable. Government could also consider the use of other types of hedging instruments ranging from the simple types such as options[2], and futures contracts[3] to complex combinations of collars, over-the-counter, amongst others. Many oil and indeed commodity producers in developed economies are increasingly using different varieties of hedging instruments to mitigate their exposures to oil price risk; however, the extent of hedging in developing countries; specifically, Nigeria remains limited or in most cases non-existent. In the next section, we show that it is possible to use the single-step binomial option pricing model to simulate theoretical future strike price that will be profitable to the government.

6 Single-step binomial price model

We used statistical and mathematical formulae to derive risk neutral probability under different assumptions. The risk neutral probability is incorporated in the option pricing model to derive the final equation for single-step binomial price model. Hedging price volatility risk requires certain assumptions to define the price dynamics of the underlying asset. One of the key assumptions of single-step binomial price model is that the oil importer requires the use of loan facilities to acquire their products. Oil importers in Nigeria use loan facilities from financial institutions because of the large capital requirement for refined crude oil importation into the country. This assumption makes the single-step binomial price model a realistic model. To determine whether to hold the options position until expiration or to exit the position immediately, one should consider;

I. The effect of the option's volatility on the oil price whether positive or negative
II. The holding period of the option contract

Given the large fluctuations in oil prices, we elect to hold the option contract position to expiration in order to know the worth of the contract. Using Cox *et al.*, (1979) call option's equation,

$$C_o = \max\ [\ 0,\ \ O_T - K] \qquad (16)$$

Where C_o is the oil call, O_T is current or spot oil price, K is the oil strike price. We computed the estimated future value of oil price to reflect the time value of money concept[4]. To calculate the future value concept, we used the continuous compounded interest risk-free rate and time "T" on the spot price of oil. Inserting the above-mentioned parameter into the equation gives:

$$E(O_T) = \ {O_e}^{rT} \qquad (17)$$

Given that oil price could either trade in an upward direction[5] "uO" or in a downward direction "dO" from its initial position; we start the risk neutral probability derivation by graphing the price directions as:

---

[1] Forward contracts are agreements to buy or sell crude oil at a certain time in the future and at a specified price fixed or predetermined by a formula at the time of delivery to the location specified in the contract.

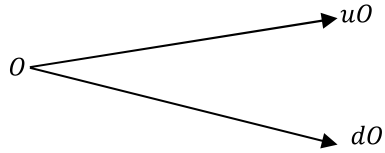
[2] Options are financial instruments that allow the holder the right to buy or sell an underlying asset at a certain price known as exercise or strike price and a specified quantity at a predetermined time, but the holder is under no obligation to do so.

[3] A futures contract allows a buyer to accept and a seller to deliver a given quantity of a particular commodity at a specified place, price and time in the future. It is another form of forward contract that has been standardized for the amount to be delivered or bought.

[4] This time value of money concept uses a discounted asset value formula. The purpose of this concept is to know the original future value of One US dollar ($1) spent today. Literally, the value of $1 spent today on oil is more than $1 spent in the next 1 year on the same oil products.

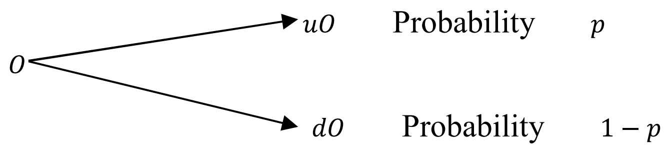
[5] Cox and Ross (1975) referred to the directional movement as a jump process in price. Glasserman (2003) argued that the jump process is an idiosyncratic process that does not affect the whole market, but individual companies.

Figure 4



We start by assigning probability values to the price directions in figure 4 using probability assumptions. The probability of price going in upward direction is "p" while the probability of price moving in downward direction is "$1 - p$" which gives:

Figure 5



The mathematical equation starts with the transformation of figure 5 using "plus" sign to connect the entire figure together to derive:

$$O_T = [\![p(uO) + (1 - p)(dO)]\!] \quad (18)$$

Multiplying the probabilities with price movements yields:

$$[\![puO + dO - pdO]\!] \quad (19)$$

Having derived equation (19), we factor[1] in the interest lending rate "ΨO" into our equation by making the entire equation equal to the interest lending rate. This factorisation gives:

$$[\![puO + dO - pdO]\!] = \Psi O \quad (20)$$

We incorporate the interest lending rate into equation (20) to make the right side of the equation equal zero. This gives:

$$\Psi O - puO + dO - pdO = 0 \quad (21)$$

Collecting like-terms in equation (21) yields:

$$\Psi O - dO = puO - pdO \quad (22)$$

From equation (22), we make "p" the subject of the probability equation and cancel out the "O" from each side of the equation. This helps us derive the actual probability of price moving in upward direction as:

$$p = \frac{\Psi - d}{u - d} \quad (23)$$

Therefore, the probability of price moving in downward direction will be;

$$(1 - p) \quad (24)$$

Using substitution, equation (24) can be rewritten as;

$$(1 - p) = \frac{u - \Psi}{u - d} \quad (25)$$

Equations (23) and (24) are referred to as risk neutral probability equations[2]. Integrating the derived risk neutral probability equations into equation (16) gives:

$$C_s = \{(P \max [\,0,\ O_t - K]) + ((1 - P) \max [\,0,\ O_t - K])\} \quad (26)$$

Equation (26) is the complete single-step binomial price model for call options. We move one step further by calculating the cost of the option contract which is also known as the option premium (i.e. the price of the option) that needs to be paid to the seller of the contract. The purpose of calculating the option premium is to let the option buyer know the price of initial payment he/she needs to pay when entering and agreeing on the contract terms. We calculated the option premium using these three components; the time value of the contract, the intrinsic value and the oil price historical volatility.

$$Premium = Intrinsic\ value + Time\ value + Volatility \quad (27)$$

Where intrinsic value $= O_T - K$, volatility $= (\sqrt{252}) * Historical\ standard\ deviation.$

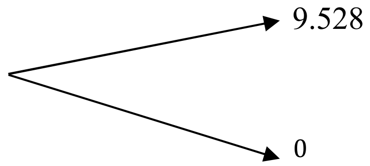
Using historical data on crude oil prices and lending rates, we test the single-step price model in equation (26). To calculate the crude oil strike price (47.95), we used the average price of crude oil from 2nd January 2015 to 27th December 2016. December 27th 2016 crude oil price is used as the spot price in the model. We computed the standard deviation of oil prices as our measure of historical volatility. The average of the

[1] The interest lending rate was factored into our equation because most of the oil importers in Nigeria depend on bank loans for the importation. In addition, without the factorisation of the rate, there will be opportunity for arbitrage in our calculation.

[2] Merton (1990) argued that risk neutrality is a powerful instrument that supports general price equilibrium analysis in a continuous trading environment. In addition, with risk neutral probability, the call option remains an expected discounted option payoff value (Lewis, 2010).

prime lending rate (16.58%) is used in the model as the average lending rate. Using equation 17, we computed the estimated future value of oil spot price as 55.5387 US Dollar. Having derived the estimated future value of crude oil price, we assume that there is a 50% probability that the price of crude oil will go up and a 50% probability that the price will go down. Incorporating the assumed probability into the model and simulating the model yields the result shown in figure 6 below;

Figure 6



Including the average prime-lending rate into equation (23) and equation (25) gives us a probability summation of one (1).

Our mathematical model suggests that using equation (26); the government of Nigeria would be better-off if they sell a call option contract with a strike price of 47.95 US dollar and a maturity period of 1 year. This result seems to suggest that if the future strike price ends-up at a price equal or above 47.95 US dollar, the sell call option contract will generate an intrinsic value (in-the-money) of 4.33014 US dollar based on the simulation. The simulation seems to suggest that the government could insure itself against a revenue decline by purchasing an option to sell oil at a fixed price (the strike price) for a fixed period of time. If the price of oil fell, causing revenues to fall, the government would be able to counterbalance the fall in oil revenues with the profits it would make by exercising the option as long as the spot oil price is below the strike price.

7 Summary and conclusions

This paper examines the use of Geometric Brownian Motion (GBM) to model the stochastic evolution of oil prices and the use of single-step binomial price model that combines both risk neutral probability and option pricing method to hedge against oil price declines. The choice of a stochastic process has important consequences for the forecasting of oil prices. The simulated prices were compared with the naïve benchmark forecasts and the results reveal that the GBM outperforms the naïve strategy in almost all the forecast evaluation statistics. For oil-rich developing country like Nigeria with limited technical know-how, a simple Monte Carlo simulation with Geometric Brownian Motion can be used to model the movement of oil prices. Our results show that the GBM performs well as a proxy for modelling the price of Brent and West Texas Intermediate prices.

The mean reverting process were not considered in this paper because according to Postali and Picchetti (2006) the trend which oil prices revert to is not clear, and this type of stochastic process is subject to a set of complex information set that may not be readily available to everybody, thereby causing significant variability in the forecasted price. Our results especially those reported in table 4; columns 1 and 2: comparison of the actual versus forecasted prices for Europe Brent spot prices reveal that 11 out of the 40 trading days has the simulated prices exactly the same with the actual. This implies that GBM can be used to describe the evolution of oil prices.

In addition, during the remaining 29 days, the forecasted prices missed the actual prices by $\pm\$3$, we can hedge this sort of price differential using option contracts. The beauty of this method is that the forecasters don't need to have a complete understanding of the determinants of oil prices, there are no parameters to be estimated which in most cases will require numerical solutions. All that is required is a choice of a stochastic process to represent the evolution of oil prices. We have shown that the single-step binomial price model that combines both risk neutral probability and option pricing method can be effective in hedging oil price risk in Nigeria, and that the use of Monte Carlo simulation in which we model oil prices as a Geometric Brownian motion can act as a good proxy for the evolution of the prices of oil. Future research should consider other stochastic/time series processes such as Brownian motion with Mean Revision (BMMR), Brownian motion with Mean Revision and Jump Diffusion (BMMRJD), Geometric Brownian motion with Jump Diffusion (GBMJD), Moving Averages, etc., as well as the inclusion of forecast uncertainty-probabilities of future oil prices into the models.

References

Alquist, R., Kilian, L., and Vigfusson, R. J., (2013), "Forecasting the Real Oil Price", in the Handbook of Economic Forecasting, 2, Amsterdam: North Holland.

Baumeister, C. and Kilian, L., (2012), "What Central Bankers Need to Know About Forecasting Oil Prices", CEPR Discussion Papers 9118.

Blas, J., (2009), "Mexico's big gamble pays off", Financial Times, FT.com, 7 September 2009.

Borensztein, E., Jeanne, O., and Sandri, D., (2009), "Macro-hedging for commodity exporters", NBER Working

Paper 15452, October.

Brennan, M.J., Schwartz, E., (1985), "Evaluating Natural Resource Investments", Journal of Business 58 (2), 135 – 157.

Caballero, R. J., and Cowan, K., (2007), "Financial integration without the volatility", Massachusetts Institute of Technology, Department of Economics, Working Paper 08 – 04.

Cox, J. C., Ross, S., and Rubinstein, M., (1979), "Option pricing: a simplified approach", Journal of Financial Economics, Vol. 7, Issue 3, Pp. 229 – 263.

Daniel, J. A., (2001), "Hedging government oil price risk", IMF Working Paper WP/01/185, November.

Davis, M., & Etheridge, A., (2006), "Front Matter. In Louis Bachelier's Theory of Speculation: The Origins of Modern Finance", (pp. I-Iv). Princeton University Press. Retrieved from http://www.jstor.org/stable/j.ctt7scn4.1Copy

Domanski, D., and Heath, A., (2007), "Financial investors and commodity markets", BIS Quarterly Review, March, 53-67.

Dixit, A., and Pindyck, R., (1994), "Investment Under Uncertainty", Princeton University Press, Nova Jersey.

Frankel, J., (2010), "The natural resource curse: A survey", Kennedy School Faculty Research Working Paper 10-005, Harvard University, February.

Husain, A. M., Arezki, R., Breuer, P., Haksar, V., Helbling, T., Medas, P., Sommer, M., (2015), "Global Implications of Lower Oil Prices", IMF Staff Discussion Note. Available at https://www.imf.org/external/pubs/ft/sdn/2015/sdn1515.pdf, accessed on 23/02/2017

Kilian, L., (2009), "Not all Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market", American Economic Review, 99, 1053-1069.

McDonald, R.L., and Siegel, D.R., (1985), "Investment and The Valuation of Firms When There is an Option to Shut Down", International Economic Review 26 (2), 331–349 (Jun/85).

Nigeria Budget and National Planning Office: Speech of President Muhammadu Buhari on the 2017 budget of Recovery and Growth at Joint Session of the National Assembly 14th December 2016: Available at http://nationalplanning.gov.ng/index.php/news-media/news/current-news/715-speech-of-h-e-president-muhammadu-buhari-on-the-2017 budget-of-recovery-and-growth-at-joint-session-of-the-national-assembly-14th-december-2016 accessed on 2nd January 2017

Paddock, J. L, Siegel, D. R., and Smith, J. L., (1988), "Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases", The Quarterly Journal of Economics 103, 479–508 (Aug./88).

Landon, S., and Smith, C., (2010), "Government revenue volatility: the case of Alberta, an energy dependent economy", Economic and Research Institute; ISSN: 2031-4892.

Larson, D., and Varangis, P., (1996), "Dealing with commodity price uncertainty", World Bank Policy Research Working Paper 1667, International Economics Department, Commodity Policy and Analysis Unit.

Larson, D., Varangis, P., and Yabuki, N., (1998), "Commodity risk management and development", World Bank Policy

Lewis, K. A., (2010), "The fundamental theorem of asset pricing: discrete time", Mathematics in Finance Working Paper Series, Courant Institute of Mathematical Sciences, New York University, Pp. 1 – 12

McCallion, Pauline, (2009), "Mexico hedges oil price at $57 in 2010," Energy Risk, 9 December

Merton, R. C., (1990), "Continuous time finance", Blackwell Publishers, Massachusetts, U.S.A

Schwartz, E.S., (1997), "The Stochastic Behaviour of Commodity Prices: Implications for Valuation and Hedging", Journal of Finance 52 (3), 923 – 973.

Swidler, S., Buttimer, R. J., and Shaw, R., (1999), "Government hedging: Motivation, implementation, and evaluation", Public Budgeting & Finance, Winter, 75 – 90.

Theil, H., (1961), "Henri Theil's Contributions to Economics and Econometrics: Econometric Theory and Methodology", Raj, B and Koerts, J (Eds) Kluwer Academic Publishers

Postali, F. A. S., and Picchetti, P., (2006), "Geometric Brownian Motion and Structural Breaks in Oil Prices: A Quantitative Analysis", Energy Economics 28 (2006) 506 – 522

Reichsfeld, D. A., and Roache, S. K., (2011), "Do Commodity Futures Help Forecast Spot Prices?", IMF Working Paper 11/254. International Monetary Fund.

U.S Energy Information Administration (EIA) Website http://www.eia.gov/