www.iiste.org

IISTE

Portfolio Optimization of Commercial Banks-
An Application of Genetic Algorithm

Dr. A.K.Misra
Vinod Gupta School of Management, Indian Institute of Technology Kharagpur, India
E-mail: arunmisra@vgsom.iitkgp.ernet.in

Dr. V. J.Sebastian (Corresponding author)
Institute of Management Technology Gaziabad, Delhi, India
E-mail: svj401@yahoo.com

Abstract
Portfolio optimization, in case of finance, is the trade- off between risk and return to maximize profit or return from the portfolio. Financial regulations are country specific and it depends upon the economic conditions prevailing in the country. The portfolio of a commercial bank can be constrained by regulatory prescription of exposure limits, risk weights and returns from each category of assets. Hence, optimization of return, in case of the loan portfolio, presents a challenging problem due to its large set of local extremes. In this context, Genetic Algorithm is used as a possible solution to optimize the risk-return trade-off and achieve an ideal solution for portfolio optimization.
Keywords: Portfolio Management, Risk-Return Trade Off, Commercial Banking

1. Introduction

The main goal of investors is to achieve optimal allocation of funds among various financial assets. Searching for an optimal portfolio, characterized by random future returns, seems to be a difficult task and is usually formalized as a risk-minimization problem. Commercial banks are financial intermediaries that accept deposits and channel those deposits into lending activities. Banks are a fundamental component of the financial system, and are also active players in financial markets. The essential role of a bank is to connect those who have funds (such as investors or depositors), with those who seek funds. Banking industry is highly regulated, and government restrictions on financial activities of banks have varied over time. The current set of global standards is called Basel II. Basel II is the second of the Basel Accords, which are recommendations on banking laws and regulations issued by the Basel Committee on Banking Supervision. The purpose of Basel II, which was initially published in June 2004, is to create an international standard that banking regulators can implement while creating regulations about how much capital banks need to put aside to guard against the various types of financial and operational risks banks face. Bank earn through plethora of investments made in loans and equity investments. Each category of loans and investments has its own risk weight and return and it is necessary to combine various risk categories of assets with their returns in relation to the available capital so as to maximize the risk-adjusted return and optimize the utilization of capital.   A genetic algorithm (GA) is a search technique used in computing to find exact or approximate solutions to optimization and search problems. Genetic algorithms are categorized as global search heuristics. Genetic algorithms are a particular class of evolutionary algorithms (EA) that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover.

2.   Literature Review

Modern portfolio theory provides a well-developed paradigm to form a portfolio with the highest expected return for a given level of risk tolerance. Markowitz (1952, 1959) originally formulated the fundamental theorem of mean–variance portfolio framework, which explains the trade-off between mean and variance each representing expected returns and risk of a portfolio, respectively. Although Markowitz's theory uses only mean and variance to describe the characteristics of return, his theory about the structures of a portfolio became a cornerstone of modern portfolio theory (Fama, 1970, Hakansson, 1970, Hakansson, 1974, Merton, 1990 and Mossin, 1969).

Genetic algorithm is a stochastic optimization technique invented by Holland (1975) and a search algorithm based on survival of the fittest among string structures (Goldberg, 1989). They applied the idea from biology research to guide the search to an (near-) optimal solution (Wong & Tan, 1994). The general idea was to maintain

an artificial ecosystem, consisting of a population of chromosomes. Each chromosome represents the weight of individual stock of portfolio and is optimized to reach a possible solution. Attached to each chromosome is a fitness value, which defines how good a solution the chromosome represents. By using mutation, crossover values, and natural selection, the population will converge to only one chromosomes with good fitness (Adeli & Hung, 1995). Recently, GA attracts much attention in portfolio formulations (Orito et al., 2003 and Xia et al., 2000). In the field of model solving, Arnone (Arnone et al., 1993) presented a Genetic Algorithm for an unconstrained portfolio optimization problem. However, Shoaf (Shoaf, & Foster, 1996), applied genetic algorithm, first time, to Markowitz's model. Rolland utilized 'Tabu Search' (TS) to solve Markowitz principle (Rolland, 1997). Later, to corroborate the necessity and desirability of heuristic algorithms, Mansini and Speranza proved that the portfolio selection problem with minimum transaction lots is an NP-complete problem. Subsequently, they proposed three heuristic algorithms to figure out the MAD model of Konno (Mansini, & Speranza, 1999). Afterwards, they (with Kellerer) extended their model to factor fixed transaction costs (Kellerer, Mansini, & Speranza, 1999). Since late 1990s, a number of innovative quantitative approaches to portfolio credit risk modeling have been developed (Gupton *et al*. 1997, Wilson 1997, Kealhofer 1998). Moreover, trade in financial instruments for transferring credit risk like credit default swaps, asset backed transactions, etc. have increased significantly during the last decade (Ferry, 2002). Basel Committee on Banking Supervision has declared new norms on capital regulations for Banks' exposures to credit risk. These developments have influenced the profit-related considerations; and there is an increasing demand for constrained optimization of credit portfolios of Banks.

Majority of studies on portfolio selection focused on equity portfolio optimization (Elton and Gruber, 1995) as per the methods developed by Markowitz (1952) Dueck and Winker (1992), Chang et al. (2000), Gilli and Këllezi (2002) for different heuristic approaches which is significantly different from credit portfolio optimization. Andersson *et al*. (2001) proposed the use of simplex algorithms in a portfolio credit risk simulation model framework while Lehrbass (1999) proposed the use of Kuhn-Tucker optimality constraints in an analytical portfolio credit risk model. The article has used Evolutionary Algorithms for solving credit portfolio optimization problems.

3. Portfolio Optimization – A Theoretical Perspective

Capital **A**sset Pricing Model (CAPM) is used to determine a theoretically appropriate required rate of return of an asset, if that asset is to be added to an already well-diversified portfolio, given the non-diversifiable risk of the asset. The model takes into account the asset sensitivity to non-diversifiable risk (also known as systematic risk or market risk), often represented by the market beta (β) as well as the expected return of the market and the expected return of a theoretical risk-free asset.

$$R_i = \alpha_i + \beta_i R_m + e_i \tag{1}$$

This model makes following assumptions:

a) $E(e_i) = 0$

b) $Cov(R_m, e_i) = 0$

c) $E(e_i, e_j) = 0$

This lead to:

$$E(R_i) = \alpha_i + \beta_i E(R_m) \tag{2}$$

$$Var(R_i) = \beta_i^2 \sigma_m^2 + \sigma_{e_i}^2 \tag{3}$$

$$Cov(R_i, R_j) = \beta_i \beta_j \sigma_m^2 \tag{4}$$

The above method drastically reduces the number of estimates to be made hence reduces both computation time and complexity of the problem. The Sharpe ratio or Sharpe index is a measure of the excess return (or Risk Premium) per unit of risk in an investment asset or a trading strategy, named after William Forsyth Sharpe. Since its revision by the original author in 1994, it is defined as:

$$S = (\frac{E(R) - RFR}{\sigma})$$

(5)

where R is the return from the asset, R_f is the return on a benchmark asset, such as the risk free rate of return, $E[R - R_f]$ is the expected value of the excess of the asset return over the benchmark return, and σ is the standard deviation of the asset. With the help of above results we can form efficient frontier as well as find the optimal portfolio through Sharpe ratio. Process is as below:

$$E(R_p) = \sum_i w_i E(R_i)$$

(6)

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij}$$

(7)

Maximization of the Sharpe Ratio from the above two inputs provides the efficient frontier.

3.1 *Genetic Algorithm Specifications*

Genetic algorithms are implemented in a computer simulation environment in which a population of abstract representations (called chromosomes or the genotype of the genome) of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem evolves toward better solutions. Traditionally, solutions are represented in binary as strings of 0s and 1s, but other encodings are also possible. The evolution usually starts from a population of randomly generated individuals and happens in generations. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), and modified (recombined and possibly randomly mutated) to form a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. If the algorithm has terminated due to a maximum number of generations, a satisfactory solution may or may not have been reached.

3.2. *Fitness Function*

A fitness function is a particular type of objective function that prescribes the optimality of a solution (that is, a chromosome) in a genetic algorithm so that that particular chromosome may be ranked against all the other chromosomes. Optimal chromosomes, or at least chromosomes which are *more* optimal, are allowed to breed and mix their datasets by any of several techniques, producing a new generation that will (hopefully) be even better.

3.3. *Encoding of a Chromosome*

The chromosome should in some way contain information about solution which it represents. The most used way of encoding is a binary string. The chromosome then could look like following pattern:

| Chromosome 1 | 1101100100110110 |
| Chromosome 2 | 1101111000011110 |

Each chromosome has one binary string. Each bit in this string can represent some characteristic of the solution, or the whole string can represent a number. The article has used following Mapping Rule:

$$x_i = x_i^l + (x_i^u - x_i^l) / (2^{l_i} - 1) \qquad (8)$$

Where: x_i = i-th chromosome or solution

x_i^l = lower bound for x_i

x_i^u = Upper Bound for x_i

l_i = length or resolution for i-th chromosome

1.

2.

3.4. *Crossover*

Crossover selects genes from parent chromosomes and creates a new offspring. The simplest way to do this is to choose randomly some crossover point and everything before this point copy from a first parent and then everything after a crossover point copy from the second parent. The Crossover would be as follows:

Chromosome 1	11011 | 00100110110
Chromosome 2	11011 | 11000011110
Offspring 1	11011 | 11000011110
Offspring 2	11011 | 00100110110

3. The crossover chosen here is scatter which means that mutation point will be randomly chosen inside a chromosome.

3.5. *Mutation*

After a crossover is performed, mutation takes place to prevent falling all solutions in population into a local optimum of solved problem. Mutation changes randomly the new offspring. For binary encoding one can switch a few randomly chosen bits from 1 to 0 or from 0 to 1. The mutation depends on the encoding as well as the crossover. Mutation can take the following shape:

Original offspring 1	1101111000011110
Original offspring 2	1101100100110110
Mutated offspring 1	1100111000011110
Mutated offspring 2	1101101100110110

3.6. *Roulette Wheel Selection*

4. Parents are selected according to their fitness. The better the chromosomes are, the more chances to be

selected they have. The algorithm for roulette wheel selection is

a) [Sum] Calculate sum of all chromosome fit nesses in population - sum S.
b) [Select] Generate random number from interval (0,S) - r.
c) **[**Loop] Go through the population and sum fit nesses from *0* **-** sum *s***.** When the sum *s* is greater then *r***,** stop and return the chromosome.

4. Empirical Design

A typical Indian bank holds a portfolio of loans and equity investments. In India banks have an obligation to provide loans to regulated sectors such as agriculture, housing, small and medium enterprises, commercial real estate etc.(Table:1). As per the concentration risk, the Banking sector regulator (Reserve Bank of India) has given different ceiling limit for each category of loans. These asset classes have different risk weights and returns. Each credit class is generally associated with a return.

Table: 1

Investment Types	Risk Weight AAA (%)	Risk-Weight AA (%)	Return (%) AAA Rating	Book-Value (%)	Regulatory Loan Requirement
SME	20	20	12.50	W1	Minimum 12%
Commercial Real Estate	20	50	14.50	W2	No limit
Large Corporation	20	20	12.00	W3	No Limit
Residential Property	20	50	14.00	W4	Minimum 10%
Consumer Credit	20	50	14.75	W5	No Limit
Regulatory Retail	20	50	12.50	W6	Minimum 18%
Equity Investment	20	50	18.00	W7	Maximum 5%
Sovereign	20	0	9.00	W8	Minimum 25%
Banks	20	50	10.00	W9	No Limit
PSE	20	50	12.50	W10	No Limit

Assets are divided into different credit classes as defined above. The returns in the table are for AAA credit rating class which is the best credit class for each segment. The portfolio allocation is to be restrained for the first two credit class in each segment i.e. AAA and AA bonds-loans. The mutation depends on the encoding as well as the crossover.

For adjusting risk of each asset class the formulation used is:

$$AR_i = R_i - CC * RW_i \qquad (9)$$

Where: AR_i = Adjusted Return

R_i = Return on *i*-th asset class

CC = Cost of capital

RW_i =Risk Weight

The paper has used AR_i in place of expected return to account for the special case of Banks. Risk of

the combined portfolio is calculated as per the following

$$E(R_i) = \alpha_i + \beta_i E(R_m) \tag{10}$$

$$Var(R_i) = \beta_i^2 \sigma_m^2 + \sigma_{e_i}^2 \tag{11}$$

$$Cov(R_i, R_j) = \beta_i \beta_j \sigma_m^2 \tag{12}$$

Betas required in the above equation have been calculated through regression from historical data. The return on market has been replaced by Prime Lending Rate taking into account the special case of bank portfolio. Outputs from equations (10), (11) and (12) are used to calculate Portfolio risk according to the equation:

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{13}$$

From the values of expected portfolio return (Adjusted return) as calculated from equation (10) and Portfolio risk calculated from equation (13) one can create fitness function and constraints needed in Genetic optimization model. Fitness function:

$$F(x) = \frac{AR_i - \text{int} - opt}{\sigma} \tag{14}$$

Where: AR_i = Adjusted Return

int = Interest cost to bank on deposits

opt =Operating cost of bank

Constraints are formulated as:

$$\sum_i w_i = 1$$

$$w_1 + w_3 \geq 0.12$$

$$w_6 \geq 0.18$$

$$0 \leq w_i \leq 1$$

Each credit class is generally associated with a given rate of return and risk level. For different book value i.e. W1,W2 W3 etc., one can get different risk-return portfolio. Each weight can be between 0 to 100%. Each credit class, the weight will be decided as per regulatory guidelines (if it is prescribed) or decided by the optimization technique. These asset classes is again divided into different credit classes as defined above. The returns for each asset class as given in the table are for AAA credit rating which is the best credit class for each segment. The portfolio allocation is to be restrained for the first two credit class in each segment i.e. AAA and AA bonds-loans.

4.1. Optimization Model: Genetic Algorithm Specifications
Population size of 30 chromosomes was taken. Each chromosome was binary encoded with string length equaling 10 to cover the range of weights from 0-100%. Elitism was set at top 3 fittest chromosomes. Elitism is a method, where the best chromosomes (or a few best chromosomes) are copied to new population. Elitism can very rapidly increase performance of GA, because it prevents losing the best found solution. Crossover probability is set to 0.4 as crossover is the main criterion for the genetic algorithm to evolve. Mutation probability is kept low with so as not to destroy better chromosomes already found. Mutation method used here is adaptive, as it randomly generates directions that are adaptive with respect to the last successful or unsuccessful generation. The feasible region is bounded by the constraints and inequality constraints. A step length is chosen along each

direction so that linear constraints and bounds are satisfied. Stopping criterion is either 100 generation reached or the best chromosome fitness –worst chromosome fitness is less than 10^{-6}, whichever criterion is reached first. Outline of basic Genetic Algorithm is:

1. **[Start]** Generate random population of *n* chromosomes (suitable solutions for the problem)
2. **[Fitness]** Evaluate the fitness *f(x)* of each chromosome *x* in the population
3. **[New population]** Create a new population by repeating following steps until the new population is complete
   a) **[Selection]** Select two parent chromosomes from a population according to their fitness (the better fitness, the bigger chance to be selected)
   b) **[Crossover]** With a crossover probability cross over the parents to form a new offspring (children). If no crossover was performed, offspring is an exact copy of parents.
   c) **[Mutation]** With a mutation probability mutate new offspring at each locus (position in chromosome).
   d) **[Accepting]** Place new offspring in a new population

4. **[Replace]** Use new generated population for a further run of algorithm
5. **[Test]** If the end condition is satisfied, **stop**, and return the best solution in current population
6. **[Loop]** Go to step **2**

5. Results & Discussions

The article used the data of a leading public sector bank of India to calculate the weights they have currently invested in Asset classes. The calculate weights and Risk-Return for their current portfolio is provided in the Table: 2.

On the same bank's data the article used the GA technique as discussed in the article. Asset classes are divided into AAA and AA and on both case scenarios, as given in Table 1, the GA technique was used. Efficient frontier was created in both cases and genetic algorithm was applied on both the efficient frontier to find the optimal portfolio weights.

Table:2

Asset Class	Weight	Asset Class	Weight
SME	8.80%	Equity Investment	6.60%
Commercial Real estate	10.00%	Regulatory Retail	11.30%
Large corporation	20.00%	Sovereign	5.00%
Residential Property	10.30%	Banks	7.10%
Consumer Credit	9.80%	PSE	11.10%
Optimal Portfolio Return	9.31%	**Optimal Portfolio Risk**	27.06%

As seen in Figure-1 with increasing risk, return of the portfolio also increases. The portfolio risk increases from 0%, when all the asset value is invested in sovereign bonds, to 60%, when whole portfolio is invested in Equity investments.
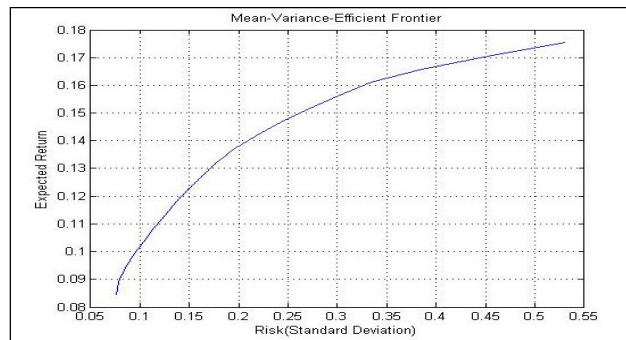
Fig 1: Efficient Frontier (Asset Class -AAA)

Optimal Portfolio according to genetic algorithm is:

Table:3

Asset Class	Weight	Asset Class	Weight
SME	14.95%	Commercial Real estate	4.80%
Large corporation	6.05%	Residential Property	5.10%
Consumer Credit	10.80%	Regulatory Retail	18.00%
Equity Investment	8.22%	Sovereign	9.05%
Banks	9.78%	PSE	13.25%
Optimal Portfolio Risk	**13.41%**	**Optimal Portfolio Return**	**11.89%**

Similarly, efficient frontier, when all asset classes are AA.
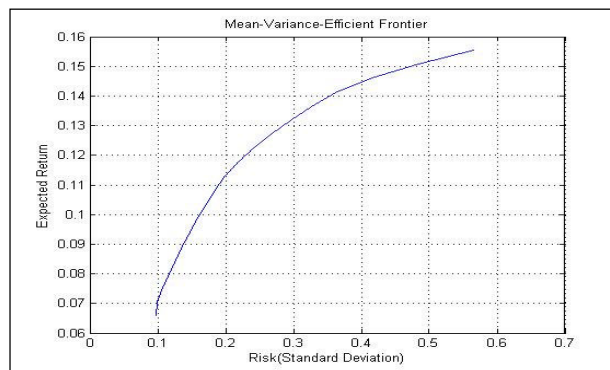

Fig 2: Efficient Frontier (Asset Class -AA)

The Figure-2 is steeper than Figure-1 indicating declining return with increasing risk. Optimal Portfolio according to genetic algorithm is:

Table: 4

Asset Class	Weight	Asset Class	Weight
SME	13.61%	Commercial Real estate	4.18%
Large corporation	6.00%	Residential Property	5.21%
Consumer Credit	9.82%	Regulatory Retail	18.00%
Equity Investment	7.71%	Sovereign	21.56%
Banks	4.51%	PSE	9.40%
Optimal Portfolio Risk	**14.61%**	**Optimal Portfolio Return**	**11.05%**

A comparison has been carried between current portfolio (practice by the bank) and the portfolios that have been created by GA through the standard method of Sharpe ratio (Table:5). For estimation of Sharpe ratio, the article has used Yield on 1-Year Government Security as risk free interest rate.

Table: 5

Scenario	Risk Free Rate	Portfolio Risk	Portfolio Return	Sharpe Ratio
Portfolio is AAA	6.65%	13.41%	11.89%	39.1%
Portfolio is AA	6.65%	14.61%	11.05%	30.1%
Actual Portfolio	6.65%	27.06%	9.31%	9.8%

The Sharpe ratio of current portfolio is least than the portfolios created by GA. If the bank maintained AAA credit rating for its portfolio, the Sharpe ratio would be 39.10% and if the bank maintained AA credit rating for its portfolio, the Sharpe ratio would be 30.10%. With the down gradation of credit rating, portfolio risk is increasing along with declining return on the portfolio. With the increasing portfolio risk, the bank needs to keep more capital to maintain regulatory capital adequacy and the cost of more capital reduce the portfolio return.

5. Conclusion
Banks are highly regulated industry with plethora of regulatory prescriptions which governed their day-to-day functioning. Regulatory guidelines on asset concentration, credit allocation, credit rating and capital adequacy influence banks' portfolio risk and return. With multiple constrains optimization of banks' risk-return is a challenging task. In this context, Genetic Algorithm provides ideal solution. The article has built portfolio with mean-variance dominating for both AAA rating and AA rating. The GA technique applied to a leading bank of India. Portfolio designed as per Indian Banking Regulations has been outperformed the current portfolio of the bank. This model can be further improved if optimization is also done inside each asset class taking into account all the credit class of each asset.

References

Adeli, H, & S. Hung, (1995), *Machine learning: neural networks, genetic algorithms, and fuzzy systems*, Wiley, New York

Aiello, S & N Chieffe,(1999), International Index Funds and the Investment Portfolio, *Financial Services Review,* 8, 27–35.

Bogle, J.C, (1998), The implications of style analysis for mutual fund performance, *Journal of Portfolio Management,* 24 (4), 34–42

Chang, K.P, (2004), Evaluating mutual fund performance: An application of minimum convex input requirement set approach, *Computers and Operations Research,* 3, 929–940

Fama, E.F,(1970), Multi period consumption–investment decisions, *American Economic Review* 60, 163–174.

Ferry, J (2002), New players, New Rules, *Risk,* 15 (5), 14–15

Gilli,M & E. Këllezi,(2002), Portfolio Optimization with VaR and expected shortfall. In: Kontoghoirghes, E., Rustem, B. and Siokos, S., *Computational Methods in Decision-making, Economics and Finance*, Kluwer, Dordrecht, 167–183.

Goldberg, D.E,(1989), *Genetic algorithms in search, optimization and machine learning*, Addison-Wesley, New York

Hakansson, N, (1970), Optimal Investment and Consumption Strategies under Risk for a class of Utility Functions, *Econometric,* 38, 587–607.

Hakansson, N, (1974), Convergence in multi period portfolio Choice, *Journal of Financial Economics,* 1, 201–224.

Holland, J.H,(1975), Adaptation in natural and artificial systems: An introductory analysis with applications to

biology, control and artificial intelligence, University of Michigan Press

Kealhofer,S, (1998), *Portfolio Management of Default Risk*, KMV Corporation, San Francisco

Lensberg, T, Eilifsen, A and T. McKee, (2006), Bankruptcy Theory Development and Classification via Genetic Programming, *European Journal of Operational Research,* 169 (2), 677–697.

Markowitz, H, (1952), Portfolio Selection, *Journal of Finance,* 7, 77–91.

Markowitz, H, (1959), *Portfolio Selection: Efficient Diversification of Investments,* Wiley, New York

Merton, R.C,(1990), *Continuous time finance*, Basil Blackwell, Oxford

Mossin, J, (1969), optimal multi period portfolio policies, *Journal of Business,* 41, 215–229.

Orito,Y, Yamamotod, H & G Yamazaki (2003), Index fund selections with genetic algorithms and heuristic classifications, *Computers and Industrial Engineering,* 45, 97–109.

Sharpe, W,(1971), A linear programming approximation for the general portfolio analysis problem, *Journal of Financial and Quantitative Analysis,* 6, 1263–1275.

Sharpe,W, (1966), Mutual fund Performance, *Journal of Business,* 39, 119–138.

Speranza, M.G, (1993), Linear Programming Models for Portfolio Optimization, *Finance,* 14, 107–123.

Speranza,M.G, (1996), A Heuristic Algorithm for a Portfolio Optimization model Applied to the Milan Stock Market, *Computers and Operations Research,* 23, 433–441

Wilson, T, (1997), Portfolio Credit Risk, *Risk* 10 (10), 56–61.

Wilson, T, (1997), Portfolio Credit Risk, *Risk,* 10 (9),111–119.

Wong, F & C. Tan, (1994), Hybrid Neural, Genetic, and Fuzzy Systems In: G.J. Deboeck, (Editor), *Trading on the edge*, Wiley, New York, 243–261

CALL FOR PAPERS

The IISTE is currently hosting more than 30 peer-reviewed academic journals and collaborating with academic institutions around the world. There's no deadline for submission. **Prospective authors of IISTE journals can find the submission instruction on the following page:** http://www.iiste.org/Journals/

The IISTE editorial team promises to the review and publish all the qualified submissions in a **fast** manner. All the journals articles are available online to the readers all over the world without financial, legal, or technical barriers other than those inseparable from gaining access to the internet itself. Printed version of the journals is also available upon request of readers and authors.

IISTE Knowledge Sharing Partners

EBSCO, Index Copernicus, Ulrich's Periodicals Directory, JournalTOCS, PKP Open Archives Harvester, Bielefeld Academic Search Engine, Elektronische Zeitschriftenbibliothek EZB, Open J-Gate, OCLC WorldCat, Universe Digtial Library , NewJour, Google Scholar