European Journal of Business and Management
ISSN 2222-1905 (Paper) ISSN 2222-2839 (Online)
Vol.15, No.19, 2023

A Generalized Lambda Distributed (GLD) Quantile Method for Estimating Determinants of Economic Growth in Nigeria

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Abstract
This study examined the determinants of economic growth (GDP) in Nigeria using domestic debt (DDT), foreign debt (FDT), interest rate (INR), the exchange rate (EXR) and trade openness (TOP). A employed the generalized lambda distribution (GLD) quantile model was employed to explore the data collected from Central Bank of Nigeria (1986-2021). The fitted GLD quantile model fitted were GLD25Q, GLD 50Q, GLD75Q and FGLD with the location parameter of 1.8464e-08, -1.7596e-06, 1.8834e-06 and -0.0253 respectively. The scale parameter for the fitted GLD quantile model were 60332.0, 50575.0, 46236.0 and 34.1488 for the GLD25Q, GLD50Q, GLD75Q and FGLD model respectively. The space parameters denoted by $\lambda_3, and\ \lambda_4$ of the distributions and from the result GLD25Q, GLD50Q, GLD75Q showed that the economy is positively skewed with the estimated coefficient of 0.1726, 0.3605, and 0.2817 respectively while and FGLD showed that the economy in Nigeria was negatively skewed with estimated coefficient of -0.3303 as determined by the identified economic growth drivers under consideration. Therefore, it can be concluded based on the estimated location, scale and space parameters of FGLD as well as the GLD Quantile-Quantile plot that economy was growing in retrogressive direction and the need to curtail the situation. To achieve this, openness must be adopted to grow the economy in Nigeria.
Keywords: Domestic and Foreign Debt, Interest Rate, Exchange Rate, Trade Openness, Linear Regression.
DOI: 10.7176/EJBM/15-19-01
Publication date: December 31st 2023

1. INTRODUCTION

To structure Nigerian economy has been a great concern to scholars and researchers because of the several challenges possess. The economic characteristics, consequences and contradictions inherent therein was not helping matters. In early 1970s, there was a move from economy based on agriculture and its product as a mainstay to crude oil, now the sole driver of growth. Right from this period, all effort made on the economy to provide better, stable and productive growth through diversification yielded no desired outcome and achievement (Ebiwonjumi, Chifurira & Chinhamu, 2022). In spite of the huge amount of generated incomes from oil as a mainstay and the sole driver of the economy, the development and prosperity transformation of the nation and its citizens expected have be divided by failure from all fronts. In ranking, the country remains one of the household of poor in the world evidence from key indicators to measure the development of any nation.

According to Ebiwonjumi *et al.* (2022), it was emphasized that there is epileptic evidence of growth in all sector of the economy, the economy is serious faced with growth that is not sustainable and consistent as result of falling and unstable price of crude oil in the world market. No doubt, many devices, policy outlines and reforms effort but to no avail. On macroeconomic variables that includes but not limited to openness of trade, interest and exchange rate, management of debt from foreign and domestic sources through various developments, plans, programmes, visions and reforms offered at different time and level creates much hope but in actual facts, produces no or little effect. In recent times, Nigeria experienced a retarding growth in 2015 which ultimately deteriorate into harsh economic recession in 2016 and in the middle 2020 subsequently to 2018 partial relief. Agbanobi (2017) noted that changes in international monetary policy cycles serve as a foremost motivation for economic woe of the country thus severely affect financial market in Nigeria; mainly, the monetary policy instrument for Nigeria that stood at 14% in the first quarter of 2017, as against the 12% mark in the first quarter of 2016 and 13% in 2015.

Asinya and Takon (2014) and Akonji (2013) stated that economic power and attractiveness to other countries are determined by vital economic indicator called exchange rate. In this regard, Oguntegbe and Alexander (2019) viewed interest on borrowed funds and exchange rate as a contributor to economic attractiveness thus, it can be emphasized the need to formulate a workable policy to drive economy for productivity. Akinmulegun and Falana (2018); Lyndon and Ikechukwu (2019); Lawali, Machief and Aliyu (2020) emphasized the potential stability of exchange rate through reforms on monetary policies in other to enhance the output from industries. Yusuf and Mohd (2021) observed negative effect of debt from external sources and as such, long term growth is hampered

in short-term even if it is initially met for economic growth-driven. Also, there is inverse effect of debt incurred domestically in growing the economy in the long and the short term in comparison with the debt incurred from external sources. Moreso, servicing debt incurred internally creates avenue for growth retardation due to debt overhang effect.

Lawali, Machief and Aliyu (2020) opined in the course of their study that degree of openness was possible to enhance growth if the effort to diversify the economy is taking seriously and sustained. Sunday and Ahmed (2019) posited the difficult emanated from openness of the economy on economic growth to be serious whether in the short or long term. Thus, affirmed Nigeria's economy operates based on higher imports over exports and in this situation, diversification driven by export promotion must be engaged to facilitate a sustainable growing economy. However, the purpose of current study is to establish the existing relationship causal or otherwise for Nigeria's economic growth (GDP) and the identified macroeconomic variables that includes domestic debt (DDT), foreign debt (FDT), interest rate (INR), an exchange rate (EXR) and trade openness (TOP) using Generalized Lamda Distributed (GLD) quantile regression technique that was missing in most of the work and study previously done in examining the relationship among the aforementioned variables existing in literatures.

2. LITERATURE REVIEW

Researchers have carried out many studies to examine economic growth driven by various macroeconomic indicators. In this study, reviews of relevant researches conducted were presented as follows: Yusuf and Mohd (2021) evaluated debt incurred by the government and its effect on economic growth in Nigeria putting 1980-2018 into consideration. Empirical analysis of the data gathered were done using Autoregressive Distributed Lag technique and findings showed that debt incurred externally negatively affected the growing of the economy thus, hampered the long-term growth despite the fact that in the short-term growth-enhancing was observed. Also, debt incurred internally inversely affected the growth of the economy in the short and long-term in comparison with externally acquired debt. More so, the servicing of acquired debt hampered the economy growing as a resultant consequence of debt overhang. Obayori, Krokeyi, Kakain (2019) examined the impact of externally acquired debt on the growth of the economy in Nigeria spanning the period (1980-2016). The data were sourced from Central Bank Nigeria Bulletin and analyzed with the aid of Generalized Method of Moments technique. Findings showed that externally acquired debt and economic growth was positive and significant in relation.

Odubuasi, Uzoka and Anichebe (2018) investigated the effect of debt externally acquired on the economic growth in Nigeria putting the period (1981-2017) into consideration. Data sourced from CBN Statistical Bulletin and NBS were used for the study were analyzed Granger Causality and ECM techniques revealed that debt incurred externally and spending by the government on investment schemes positively and significantly affected growth of the economy. Also, result further showed the insignificant of effect on cost of servicing externally sourced debt on economic growth in Nigeria. Moh'd AL-Tamimi and Jaradat (2019) analyzed the effect of debt incurred externally on the growth of the economy in Jordan from 2010 to 2017. The result obtained using descriptive analytics method revealed that debt incurred externally was negatively and significantly affected the growing of the economy. Ademola, Tajudeen and Adewumi (2018) investigation was done on Nigeria's economic growth and external debt (1999-2015) with the aim of determining the impact of the later on the former. Cointegration and Vector Error Correction model were adopted as an econometric technique to analyze data gathered for the study. Results indicated an inverse relationship between external debt and economic growth in Nigeria.

Al Kharusi and Ada (2018) in their study determined relation externally borrowed funds by the government have with economic growth between (1990-2015). The need for the study was spurred by an uninterrupted increase in Oman's due to externally borrowed funds basically to finance yearly budget. The statistical technique adopted for the analysis done was Autoregressive Distributed Lag cointegration method. Findings revealed a negative and significant effect of externally borrowed funds by the government in growing the economy. Furthermore, gross fixed capital impact in growing the economy was positive and significant as indicated during the period under investigation. Ndubuisi (2017) focused on how externally acquired debt impacted the growth of the economy in Nigeria spanning 1985-2015. The data gathered were explored by least square regression, ADF stationary test, cointegration and error correction. Findings revealed the impact of servicing an externally incurred debt in growing the economy was negative and insignificant in Nigeria. Also, externally secured debt was found to be positive and significant in impacting the index of growing economy. Control variables have positive and significant effect in growing economy. The causality relationship between externally acquired debt and economic growth was unidirectional in the long-run.

Sunday and Ahmed (2019) investigated vigorous impact of degree of openness in relation to the growing economy in Nigeria considering the period 1980-2016. Secondary data sourced from the CBN Statistical Bulletin were explored. analytic and diagnostic technique adopted were: unit root, co-integration and error correction model. As a result, it was discovered trade openness had negative impact in growing the economy. Thus, established the overriding effect of imports on exports in Nigeria and to address this, a concerted effort must be engaged by government to make diversification through export a priority in growing the economy. Ude and Agodi

(2015) examined whether degree openness makes sense, based on economy policy on trade in Nigeria. Statistical techniques that includes autoregressive conditional heteroscedasticity (ARCH), generalized autoregressive conditional heteroscedasticity (GARCH) and granger causality were used in the study. The results showed a significant relationship between degree of openness and economy policy on trade. Also, interest paid on lending and dollar to naira rate were significant in determining economic growth. In view of this, policymakers should make friendly policy for implementation when formulating trade policies.

Ugbor (2014) determined relationship that exists between degree of openness and growth index of the economy a case in Nigeria. The study done to assess the pre (1970Q1-1985Q4) and post (1986-2011) Structural Adjustment Programme (SAP). The variables used were degree of Openness, spending by the government, and investment thus, data collected on the aforementioned were subjected to Augmented Dickey Fuller and the Pillips Perron for the unit root, granger causality, and Co-integration. In the results, degree of openness influenced economic growth in the post SAP period better than pre-SAP period. Lawali, Machief and Aliyu (2020) sought to determine impact of the exchange rate on economic growth in Nigeria (1980-2019). Unit root, co-integration, and error correction model were statistical technique applied to explore the data extracted from Statistical Bulletin of Central Bank of Nigeria. Findings, established the positive and significant impact of exchange rate on economic growth. Furthermore, the impact of degree of economy openness on economic growth was revealed to be negative. In view of this, exchange rate stability must be ensured through the monetary policies implementation, export led diversification must be pursued and sustained to enhance economic growth.

Oguntegbe and Alexander (2019) analyzed the contribution of lending rate and dollar to naira rate on economic attractiveness. Data collected on lending rate, dollar to naira rate and gross domestic product the proxy for economic attractiveness spanning the period 1981-2016. The OLS technique was adopted for the analysis and found that both lending rate and dollar to naira rate had impact on competitiveness of the economy and as such, the stability of dollar to naira rate need to be rigorously pursued by the government. Lyndon and Ikechukwu (2019) studied the volatility of exchange rate, balance of trade and the growth of the economy in Nigeria. The growth of the economy was proxy by gross domestic product (GDP). Secondary data gathered from CBN Statistical Bulletins for the period of 2000-2017 were analyzed using descriptive and regression analytics technique. Results of the empirical investigation indicated the positive and significant impact of volatility of exchange rate on growth of the economy. It was also indicated that the effect of balance of trade on growth of the economy was positive but insignificant. Hence, the stability of the exchange rate must be given a necessary attention require through appropriate macroeconomic policy formulation.

Akinmulegun and Falana (2018) assessed variability in exchange rate and output from the industries in Nigeria (1986-2015). gross domestic product was used to capture growth of industrial output, while variability of exchange rate, inflation, interest rate, and net exports served as explanatory variables. Data extracted from the NBS and CBN Statistical Bulletin were analyzed using ADF and PP unit root, co-integration, granger causality and VECM. Results showed that variability of exchange rate granger caused growth of industrial output. Stated further that the positive and significant effect of variability if exchange rate on the growth of the industrial output was more visible when compare with other variables. Thus, establishing the necessity for exchange rate capabilities of improving the growth of industrial output. Ufoeze, Okuma, Nwakoby and Alajekwu (2018) investigated exchange rate variability on growth of the economy in Nigeria using data spanning (1970-2012). In the study, dollar to naira rate, inflationary rate, money supply and oil revenue were used as the explanatory variables, while the response variable, gross domestic product was used to capture the growth of the economy. CBN Statistical Bulletin was the source of data and linear regression technique was used for the analysis. A diverse finding obtained showed that to determine the growth of the economy uncontrolled exchange rate was better than fixed exchange rate.

Ebiwonjumi, Chifurira and Chinhamu (2022) investigated an efficient estimation technique for economic growth and its determinants for Nigeria in the presence of multicollinearity. The macroeconomic variables considered in the study were economic growth (RGDP), internal debt, external debt, interest rate, exchange rate and trade openness. An exploratory data analysis and the variance inflation factor carried out revealed the presence of multicollinearity. Thus, a ridge regression method was adopted and it was found that a ridge regression technique with appropriate ridge constant was a robust method that was efficient to estimate economic growth in Nigeria. However, in all the various literatures reviewed, generalized lamda distributed (GLD) technique have not been used in various analyses been carried out. Thus, in this study, generalized lamda distributed (GLD) quantile regression technique an essential technique that can be used to explore dataset tainted and contained extreme values to provide stable parameter estimate. Specifically, study that explore the causal relationship that exist among economic growth (GDP), domestic debt (DDT), foreign debt (FDT), interest rate (INR), exchange rate (EXR) and trade openness (TOP) hence, this will contribute to the existing work in existing literatures that was lacking.

3. RESEARCH METHOD

In this study, the macroeconomic data gathered from the CBN statistical bulletin that were used to examine the

existing relationship among economic growth (GDP) as endogeneous variable, domestic debt (DDT), foreign debt (FDT), interest rate (INR), exchange rate (EXR), and trade openness (TOP) between 1986-2021. The variables assumed a linear model stated in both functional and econometrical form as (1) and (2)

GDP = F(DDT, FDT, INR, EXR, TOP) (1)

$\text{GDP} = a_0 + a_1\text{DDT} + a_2\text{FDT} + a_3\text{INR} + a_4\text{EXR} + a_5\text{TOP} + \in_i$ (2)

where, a_i and $\in_i$ are the parameters to be estimated and error term respectively.

However, the multiple linear regression model stated in (1) and (2) were transformed and expressed in general form as given in (3) where Y represent the dependent variable GDP and $\boldsymbol{X}$ represent the explanatory variables DDT, FDT, INR, EXR and TOP.

$$\mathbf{Y} = \mathbf{X}'\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (3)$$

The ordinary least square estimator of $\boldsymbol{\beta}$ is given in (4)

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\mathbf{Y} \quad (4)$$

The covariance matrix of $\boldsymbol{\beta}$ can be obtained as given in (5)

$$\text{Cov}(\widehat{\boldsymbol{\beta}}) = \sigma^2(\boldsymbol{X}'\boldsymbol{X})^{-1} \quad (5)$$

where $\boldsymbol{Y}$ is an observational vector of dimension $n \times 1$, $\boldsymbol{X}$ is an $n \times p$ data matrix of regressors, $\boldsymbol{\beta}$ is a $p \times 1$ vector of regression coefficient and ε is an $n \times 1$ disturbance vector.

In this study, the ordinary least square estimator may not possess the optimum statistical property hence, the need for alternative estimator.

Generalized Lambda Distribution (GLD)

The four-parameter (quantile) GLD family is known for its high flexibility of producing distributions with a range of different shapes. Ramberg, Dudewicz, Tadikamalla and Mykytka (1979), Chalabi *et al.* (2012), and Chalabi, Scott and Würtz (2010) introduced the notation for generalized lambda distribution. Thus, Su (2007b) emphasized that the ultimate drive for the development of GLD is that the distribution is defined over all λ_3 and λ_4. The quantile probability density function of the GLD which is given as the inverse distribution function of Tukey's lambda distribution (TLD) can be expressed as (6)

$$F^{-1}(P|\lambda) = F^{-1}(P|\lambda_i) \;\forall\, i = 1,2,..4$$

$$F^{-1}(P|\lambda_i) = \lambda_1 + \frac{P^{\lambda_3} - (1-P)^{\lambda_4}}{\lambda_2} \quad (6)$$

where P are the probabilities, $P \in [0,1]$, λ_1 *and* λ_2 are the location and scale parameters while λ_3 *and* λ_4 are the shape parameters which defined the strengths of the lower and upper tail respectively. According to Chalabi et al. (2010), it can be stated that the original one-parameter TLD results in the limiting case $\lambda_1 = 0$ and $\lambda_2 = \lambda_3 = \lambda_4 = \lambda$.

GLD Quantile Regression Model

This model is divided into two parts and is adopted to explore and estimate the economic growth (GDP) in Nigeria in relation to the macroeconomic variables such as domestic debt (DDT), foreign debt (FDT), interest rate (INR), exchange rate (EXR), and trade openness (TOP). The first part deals with the GLD regression reference line generation and the second part concentrates on finding the quantile regression coefficients based on the reference line that will be obtained from the GLD as discuss below. Thus, the quantile function for the distribution of GLD is defined as stated in (7):

$$Q(P) = \lambda_1 + \frac{\frac{P^{\lambda_3-1}}{\lambda_3} - \frac{(1-P)^{\lambda_4-1}}{\lambda_4}}{\lambda_2} \quad (7)$$

λ_1 and λ_2 are the location and scale parameters whereas λ_3 and λ_4 are the shape parameters. This distribution is most favourable because it is valid for all values of λ_3 and λ_4 where $\lambda_2 > 2$. If $\lambda_3 = \lambda_4 = 0$ the GLD parameter has the following quantile function given in (8):

$$F^{-1}(P) = \lambda_1 + \frac{\ln(p) - \ln(1-p)}{\lambda_2} \quad (8)$$

The GLD takes different quantile forms if either λ_3, λ_4 or both are equal to zero. These forms can be obtained as follows in (9, 10, 11 and 12):

if $\lambda_3 = 0,\ \lambda_4 \neq 0$

$$Q(P) = \lambda_1 + \frac{1}{\lambda_2}\left(\ln(p) - \frac{(1-P)^{\lambda_4-1}}{\lambda_4}\right), \quad 0 \le p \le 1 \quad (9)$$

If $\lambda_3 \neq 0, \lambda_4 = 0$

$$Q(P) = \lambda_1 + \frac{1}{\lambda_2}\left(\frac{(P)^{\lambda_3-1}}{\lambda_3} - \ln(1-p)\right), \quad 0 \le p \le 1 \quad (10)$$

If $\lambda_3 = 0, \lambda_4 = 0$

$$Q(P) = \lambda_1 + \frac{1}{\lambda_2}\left(\ln(p) - \ln(1-p)\right), \quad 0 \le p \le 1 \qquad (11)$$

If $\lambda_3 \neq 0, \lambda_4 \neq 0$

$$Q(P) = \frac{\lambda_2}{P^{\lambda_3-1} + (1-P)^{\lambda_4-1}} \qquad 0 \le p \le 1 \qquad (12)$$

The probability density function of the GLD can be obtained by using the relationship in (5), where $F(y) = p$ and $y = Q(P)$.

$$y = F^{-1}(y) = Q(P) \qquad (13)$$

By differentiating $Q(P)$ with respect to y the density function of GLD can be obtained as given in (14):

$$\frac{dp}{dy} = f(y) \; and \; dy = d\big(Q(p)\big) \qquad (14)$$

These two relationships give us $(y) = \frac{dp}{d(Q(p))}$. From (2), the expression in (15)

$$\frac{d(Q(p))}{dp} = \frac{\lambda_3 P^{\lambda_3-1} - \lambda_4(1-P)^{\lambda_4-1}}{\lambda_2} \qquad (15)$$

substituting the result in (7) into $f(y) = \frac{dp}{d(Q(p))}$ the density function given in (16)

$$f(y) = \frac{\lambda_2}{\lambda_3 P^{\lambda_3-1} - \lambda_4(1-P)^{\lambda_4-1}} \qquad (16)$$

where $0 \le p \le 1$.

The Maximum Likelihood Estimation

In this technique, the quantiles $Q(p_i)$ for every observation y_i, for i = 1, 2, 3,. . . n observations under a set of initial values. This involves solving expression in (7) numerically. This can be done by using the Newton-Raphson method through GLD. Having obtained the $Q(p_i)$, the substitution in the log-likelihood equation in (17) can be stated and transformed as given in (18)

$$ML = \prod_{i=1}^{n} \ln\left(\frac{\lambda_2}{\lambda_3 {p_i}^{\lambda_3-1} - \lambda_4(1-p_i)^{\lambda_4-1}}\right) \qquad (17)$$

$$ML = n\ln(\lambda_2) - \prod_{i=1}^{n} \ln\left(\lambda_3 {p_i}^{\lambda_3-1} - \lambda_4(1-p_i)^{\lambda_4-1}\right) \qquad (18)$$

This idea is valid as its maximize the likelihood in (10) and (11) using the Nelder-Mead also known as simplex search algorithm.

The Empirical Likelihood Goodness of Fit Test

The hypothesis to test the goodness of fit for the GLD distribution is given as (19) and (20)

$$H_0: f = f_0 \sim GLD(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \qquad (19)$$

$$H_1: f = f_1 \sim GLD(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \qquad (20)$$

The definition of the likelihood ratio test statistic for this hypothesis is given in (21) and (22)

$$LR = \frac{\prod_{i=1}^{n} fH_1(y_i)}{\prod_{i=1}^{n} fH_0(y_i)} \qquad (21)$$

$$LR = \frac{\prod_{i=1}^{n} fH_1(y_i)}{\prod_{i=1}^{n} f(y_i|\lambda)} \qquad (22)$$

where $y_1, y_2, y_3, \dots, y_n$ follows a GLD distribution with the parameter $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ under the null hypothesis. Since f_0 and f_1 are unknown, the maximum likelihood method estimates $\boldsymbol{\lambda}$ of a GLD under the null hypothesis. Ning (2014) used the maximum empirical likelihood method to estimate the numerator as writing in (23)

$$L_f = \prod_{i=1}^{n} fH_1(y_i) = \prod_{i=1}^{n} fH_1(y_{(i)}) = \prod_{i=1}^{n} f_i \qquad (23)$$

where $y_{(1)} \le y_{(2)} \le y_{(3)} \le \dots \le y_{(n)}$ are the order statistics of the observations $y_1, y_2, y_3, \dots, y_n$. The values of f_i to maximise L_f were obtained by using the constraint $f(s)ds = 1$ corresponding to the alternative hypothesis. The value obtained is given by (24)

$$f_j = \frac{2m}{n(Y_{j+m} - Y_{j-m})} \qquad (24)$$

Where $Y_j = Y_1$, if $j \le 1$ and $Y_j = Y_n$ if $j \ge n$

The likelihood ratio statistic is given in (25) based on the maximum likelihood empirical method

$$GLD_{mn} = \frac{\prod_{j=1}^{n} \frac{2m}{n(Y_{j+m} - Y_{j-m})}}{max \prod_{j=1}^{n} f(y_j|\lambda)} \quad (25)$$

Where $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ and $0 < \lambda < 1$

Thus, in the next session, data exploration, descriptive analysis, test for outliers and multicollinearity as well as fitting GLD quantile regression models and its associated diagnostics to determine the most efficient GLD quantile regression models for estimating and predicting economic growth in Nigeria based on the identified economic growth drivers in this study.

4. Result and Discussion

Descriptive Analysis

In Table 1 the descriptive statistic such as mean, minimum, maximum, standard deviation, variance, skewness, kurtosis and others were presented for the macroeconomic variables under consideration in this study.

Table 1: Descriptive Analysis Result

	GDP	DDT	FDT	INR	EXR	TOP
Mean	10.3046	6.6288	6.6573	3.1051	4.3175	0.1665
Median	10.2054	6.7393	6.4629	3.1139	4.3803	0.1200
Maximum	11.1422	9.0867	8.4950	3.5860	6.3197	0.4600
Minimum	9.6316	3.3478	3.7246	2.4849	2.7763	0.0100
Std. Dev.	0.4503	1.4988	1.0655	0.1929	0.6951	0.1302
Skewness	0.3710	-0.4095	-0.1634	-0.5734	-0.1748	0.6710
Kurtosis	1.8572	2.5576	2.7076	4.1714	3.6271	2.1246
Jarque-Bera	10.9083	5.0906	1.1299	15.7879	3.0287	15.0811
Probability	0.0043	0.0785	0.5684	0.0004	0.2120	0.0005
Sum	1452.950	934.6548	938.6716	437.8212	608.7708	23.4700
Sum Sq. Dev.	28.3891	314.4906	158.9331	5.2110	67.6489	2.3744

Source: Researcher's Computation, 2022

Table 1 showed the descriptive analysis of results of the economic variables such as GDP, DDT, FDT, INR, EXR, and TOP under investigation in this study. The average values of RGDP during the period under study stood at 10.3046 and it ranged from 9.6315 to 11.1422. The mean value of DDT and FDT were 6.6288 and 6.6572 which were ranged between 3.3478 to 9.0867 and 3.7245 to 8.4950 respectively. While, the average values of INR, EXR and TOP were 3.1051, 4.3175 and 0.16645 respectively. It was observed that RINR, EXR and TOP were ranged from 2.4849 to 3.5860, 2.7763 to 6.3197 and 0.01 to 0.46 in that order in the study period. The values 0.4503, 1.4988, 1.0655, 0.1929, 0.6951 and 0.1302 revealed the rate at which GDP, DDT, FDT, INR, EXR and TOP deviated from their respective mean values.

The skewness and kurtosis as shown in the result gave explanation about the distribution and shape of the economic variables under investigation. The skewness result showed that the GDP (0.3710) and TOP (0.6709) were positively skewed that is, skewed to the right of the mean and it was also discovered that DDT (-0.4095), FDT (-0.1634), INR (-0.5734) and EXR (-0.1748) were negatively skewed that is skewed to the left of the mean. The kurtosis results revealed that all the macroeconomic variables under consideration were platykurtic with the kurtosis coefficient index less than 3 except INR and EXR which were mesokurtic thus, emphasized the flattering beyond the level of normal distribution.

Table 2: Correlation Matrix

	DDT	FDT	INR	EXR	TOP
DDT	1.0000	0.6073	0.1276	-0.0923	0.8200
FDT	0.6073	1.0000	0.3876	-0.2898	0.2534
INR	0.1276	0.3876	1.0000	-0.4594	-0.0089
EXR	-0.0924	-0.2898	-0.4594	1.0000	0.2416
TOP	0.8201	0.2534	-0.0089	0.2416	1.0000

Source: Researchers' Computation, 2022

The correlation coefficients presented in Table 2 showed the extent of relationship that exist among the explanatory variables under consideration such as DDT, FDT, INR, EXR and TOP. From the Table 2, it was discovered that DDT was positively correlated with FDT, INR and TOP with correlation coefficient of 0.61, 0.13 and 0.82 respectively. The study also revealed a positive correlation between the FDT and INR, FDT and TOP, EXR and TOP with correlation coefficient of 0.39, 0.25 and 0.24 respectively. Thus, the high or strong correlation between the DDT and TOP revealed the need to test for the presence of multicollinearity problem. Therefore, in order to check for the outliers and multicollinearity among the macroeconomic variables DDT, FDT, INR, EXR, and TOP used as the determinants of economic growth (GDP). Thus, in Table 4.4, the result of GLD quantile

regression was presented

GLD Quantile Regression Results

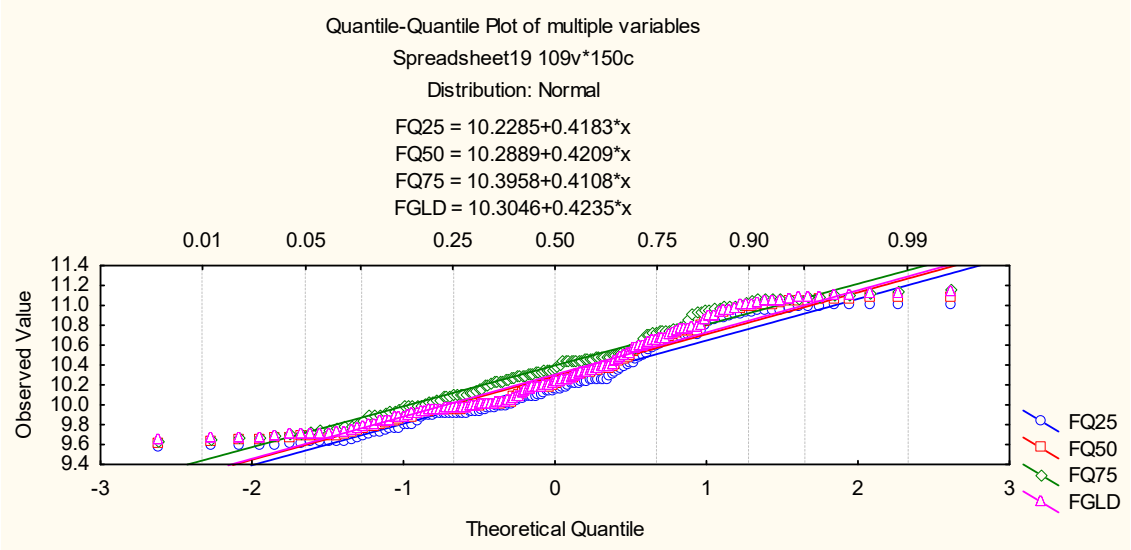
The results are displayed in the form of tables showing parameter estimates for GLD, GLD Q-Q plots for the GLD quantile models, and GLD quantile model estimated parameter and diagnostics properties such as goodness of fit, best predictors model and their quantile plots for the quarterly data on identified macroeconomic variables for investigating and predicting economic growth in Nigeria.

Table 3: GLD Quantile Models and Estimated Lambda

GLD Variables	GLD 25Q Coefficients	GLD 50Q Coefficients	GLD 75Q Coefficients	FGLD Coefficients
λ_1	1.8464e-08	-1.7596e-06	1.8834e-06	-0.0253
λ_2	6.0332e+04	5.0575e+04	4.6236e+04	34.1488
λ_3	1.7264e-01	3.6046e-01	2.8165e-01	-0.3303
λ_4	1.7417e-01	2.1353e-01	4.4265e-01	-0.5758

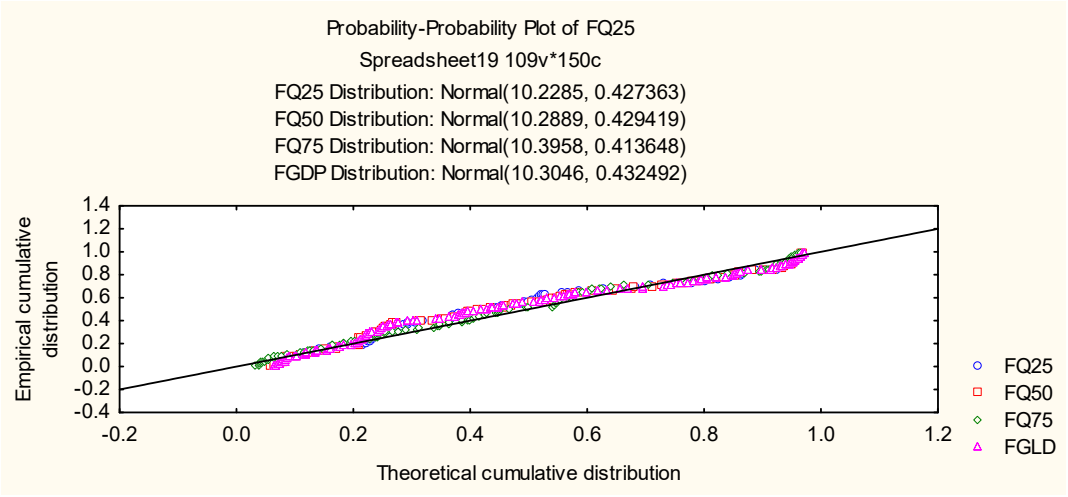
Source: Researcher's Computation, 2022

Table 3 showed the estimates for the generalized lambda distribution (GLD) $\lambda_1, \lambda_2, \lambda_3, and\ \lambda_4$ for the GLD quantile model fitted for the investigating economic growth in Nigeria with respect to the identified macroeconomic variables such as DDT, FDT, INR, EXR and TOP. From the result, $\lambda_1\ and\ \lambda_2$ represent the location and scale parameters while $\lambda_3, and\ \lambda_4$ represent the space parameters. The GLD quantile model fitted were GLD25Q, GLD50Q, GLD75Q and FGLD with the location parameter (mean) of 1.8464e-08, -1.7596e-06, 1.8834e-06 and -0.0253 respectively. The scale parameter (variance) for the fitted GLD quantile model were 60332.0, 50575.0, 46236.0 and 34.1488 for the GLD25Q, GLD50Q, GLD75Q and FGLD model respectively. The space parameters denoted by $\lambda_3, and\ \lambda_4$ were the skewness (spread) and the kurtosis (peakness) of the distributions and from the result GLD25Q, GLD50Q, GLD75Q showed that the economy is positively skewed with the estimated coefficient of 0.1726, 0.3605, and 0.2817 respectively while and FGLD showed that the economy in Nigeria was negatively skewed with estimated coefficient of -0.3303 as determined by the identified economic growth drivers under consideration. Thus, it can be deduced from the macroeconomic data set under consideration that economic growth in Nigeria assumed a unimodal distributional form.

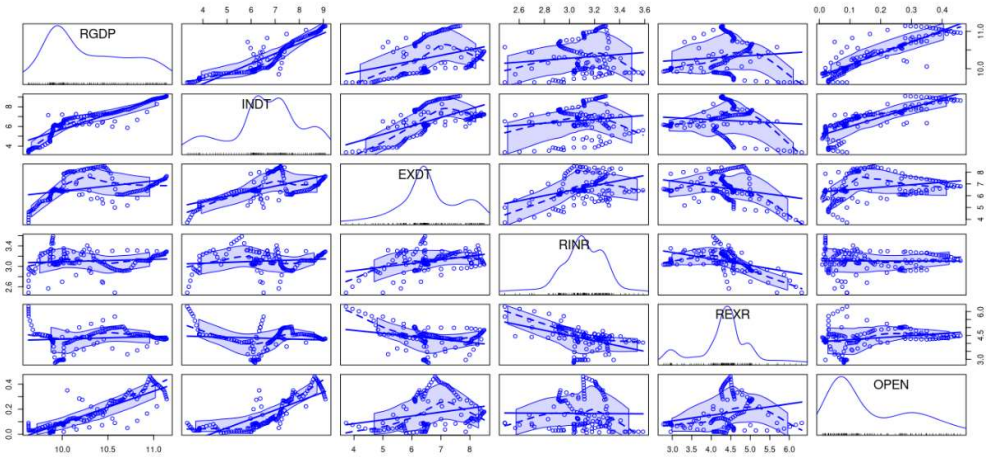


Source: Researcher's Computation, 2022

Figure 1: Q,Q Plots GLD Quantile Models



Source: Researcher's Computation, 2022
Figure 2: Probability Plots for the GLD Quantile Models



Source: Researcher's Computation, 2022
Figure 3: Scattered Plots for the variables and GLD Regression Models

Table 4: GLD Quantile Models and Estimated Coefficient

Variables	GLD25Q Coefficients	GLD50Q Coefficients	GLD75Q Coefficients	FMKL-FGLD Coefficients
C	8.1465	7.8604	6.7134	8.1492
DDT	0.2035	0.1794	0.3180	0.1514
FDT	-0.0679	-0.0392	-0.0922	-0.0247
INR	0.2294	0.2942	0.4901	0.2449
EXR	0.0599	0.0799	0.1629	0.0608
TOP	1.2848	1.4510	-0.2183	1.7597
Bias COR	-9049.803	-6321.909	-3467.66	-14.5995

Source: Researcher's Computation, 2022

Table 4 revealed the GLD quantile estimated parameters for each variable: domestic debt (DDT), foreign debt (FDT), interest rate (INR), exchange rate (EXR) and trade openness (TOP) used as the determinants of Nigeria's GDP. It must be noted that, data used were model using GLD25Q, GLD50Q, GLD75Q and FGLD quantile regression. Thus, from the result presented in Table 4.6, the coefficients of the GLD quantile regression models, it was discovered that the DDT was positive and contributed 20.35, 17.94, 31.80 and 15.14 percent to the GDP as revealed by the GLD25Q, GLD50Q, GLD75Q and FGLD regression model respectively. The FDT was negative and as such weakened the GDP in Nigeria by 6.79, 3.92, 9.22 and 2.47 percent as indicated by the fitted GLD25Q, GLD50Q, GLD75Q and FGLD quantile regression model respectively. The INR was revealed to be positively

related to the GDP to turn of 22.94, 29.42, 49.01 and 24.49 percent as observed from the result of the fitted GLD25Q, GLD50Q, GLD75Q and FGLD regression model respectively. Also, EXR was in turn positive in relation to the GDP with the contribution of 5.99, 7.99, 16.29 and 6.08 percent as showed by the GLD25Q, GLD50Q, GLD75Q and FGLD regression model respectively. The TOP was positive for all the fitted GLD quantile models with the contribution of 128.48, 145.10 and 175.97 percent to GDP except the GLD75Q which was negative with -21.83 percent contribution to the GDP in Nigeria. This shows the importance of economic trade openness to the development and the growth of economy in Nigeria. Having obtained the contributions of the economic growth determinants identified in this study to the economic growth by the various fitted GLD quantile model, it is imperative to examine the goodness of fit and best estimator to fit GLD quantile model that is the most efficient in determining the economic growth in Nigeria based on the data set under study.

Table 5: Goodness of Fit for the fitted GLD Quantile Models

	GDPQ25	**GLDQ50**	**GLDQ75**	**FGLD**
MSE	0.0250	0.0209	0.0289	0.0225
MAE	0.1034	0.0875	0.1212	0.0921
MRSE	0.0002	0.0002	0.0003	0.0002
MRAE	0.0101	0.0085	0.0117	0.0089
COR	0.9511	0.9474	0.9478	0.9425

Source: Researcher's Computation, 2022

In the Table 5, the goodness of fit of the GLD quantile models were presented and it was determined by mean square error (MSE), mean absolute error (MAE), mean relative square error (MRSE) and mean relative absolute error (MRAE). Thus, from the result the MSE for the fitted GLD25Q, GLD50Q, GLD75Q and FGLD quantile model were 0.0250, 0.0209, 0.0289 and 0.0225 respectively. The MAE for the fitted GLD25Q, GLD50Q, GLD75Q and FGLD quantile model were 0.1034, 0.0875, 0.1212 and 0.0921 respectively thus indicating that GLD50Q and FGLD regression model were the most efficient GLD quantile model for examining and predicting the economic growth in Nigeria based on the smallest MSE and MAE of the GLD quantile models.

Table 6: Best Predictors for the GDP based on the fitted GLD Quantile Model

Model	F-value	p-value
GLDQ25	163.5235	0.0000
GLDQ50	151.3470	0.0000
GLDQ75	145.4918	0.0000
FGLD	160.1817	0.0000

Source: Researcher's Computation, 2022

The best estimator for the economic growth in Nigeria based on the fitted model was presented in the Table 6 to reveal the important plot based on F-Statistic values. In the table the F-Statistic value with p-value < 0.05 were 163.5235, 151.3470, 145.4918 and 160.1817 for the fitted GLD25Q, GLD50Q, GLD75Q and FMKL-FGLD quantile model respectively. Thus, it can be emphasized based on this result, important plot for F-Statistic value and the goodness of fit obtained by MSE and MAE that FGLD quantile model was the best and most efficient to estimate determinant of economic growth in Nigeria based on the data set under consideration for this study.

5 CONCLUSSION

In this study, an estimation of economic growth's parameters in Nigeria in relation to the identified determinants of economic growth such as DDT, FDT, INR, EXR, and TOP was investigated. Consequently, an efficient estimate of parameter for DDT, FDT, INR, EXR, and TOP on GDP were obtained using GLD quantile regression technique. Thus, a GLD25Q, GLD50Q, GLD75Q and FGLD quantile model that produce efficient parameter estimate for the data set under investigation were fitted.

The GLD quantile model fitted were GLD25Q, GLD50Q, GLD75Q and FGLD with the location parameter of 0.00000002, -0.000018, 0.000002 and -0.0253 respectively. The scale parameter for the fitted GLD quantile model were 60332.0, 50575.0, 46236.0 and 34.1488 for the GLD25Q, GLD50Q, GLD75Q and FGLD model respectively. The space parameters denoted by $\lambda_3, and\ \lambda_4$ of the distributions and from the result GLD25Q, GLD50Q, GLD75Q showed that the economy is positively skewed with the estimated coefficient of 0.1726, 0.3605, and 0.2817 respectively while and FGLD showed that the economy in Nigeria was negatively skewed with estimated coefficient of -0.3303 as determined by the identified determinants of economic growth in this study. However, it was discovered from the FGLD regression model was the most efficient and optimal technique to estimate the impact of DDT, FDT, INR, EXR, and TOP on economic growth in Nigeria as revealed in this study. This assertion was based the F-Statistic value with p-value < 0.05 were 163.5235, 151.3470, 145.4918 and 160.1817 for the fitted GLD25Q, GLD50Q, GLD75Q and FGLD regression model respectively.

Thus, it can be emphasized based on this result, important plot for F-Statistic value and the goodness of fit obtained by MSE and MAE that FGLD quantile model was the best and most efficient in estimating and

determining economic growth in Nigeria. Further examination revealed that the DDT was positive and contributed 15.14 percent to the GDP as revealed by the FGLD regression model. The FDT was negative and as such weakened the GDP in Nigeria by 2.47 percent as indicated by the fitted FGLD regression model. The INR was revealed to be positively related to the GDP to the turn of 24.49 percent as observed from the result of the fitted FGLD regression model. Also, EXR was in turn positive in relation to the GDP with the contribution of 6.08 percent as showed by the FGLD regression model. The TOP was positive for the fitted FGLD regression model with the contribution of 175.97 percent to GDP in Nigeria. Therefore, it can be emphasized based on the estimated location, scale and space parameters of FGLD as well as GLD Quantile-Quantile plots that economy was growing in retrogressive direction and the need to curtail the situation. To achieve this, the trade openness must be adopted to grow the economy in Nigeria.

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