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Measuring the Volatility in Ghana's Gross Domestic Product (GDP) Rate using the GARCH-type Models

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Abstract

The objective of this paper was to empirically characterize the volatility in the growth rate of real Gross Domestic Product (GDP) for Ghana in three sectors using data spanning from 2000 to 2012. The GARCH-type models(GARCH, EGARCH and GJR-GARCH) were used for the analysis of data. The results of the study present evidence that the symmetric GARCH(1, 1) structure applies reasonably well to GDP when quarterly observations are used.. As expected from financial time series, the data for the study exhibit characteristics such as leptokurtosis, clustering, asymmetric and leverage effects. It was found that there was a significant increase in volatility and leverage effect.

1. Introduction

The economy of Ghana has a diverse and rich resource base, and as such, has one of the highest GDP per capita in Africa. Ghana is one of the top–ten fastest growing economies in the world, and the fastest growing economy in Africa. Ghana remains somewhat dependent on international financial and technical assistance as well as the activities of the extensive Ghanaian diaspora. Gold, timber, cocoa, diamond, bauxite, manganese, and many other exports are major sources of foreign exchange. An oilfield which is reported to contain up to 3 billion barrels (480×10^6 m^3) of light oil was discovered in 2007. Oil exploration is ongoing and, the amount of oil continues to increase.

Gross Domestic Product (GDP) is Ghana's official measure of economic growth. There are three different approaches that can be taken to calculate GDP; the production approach, the expenditure approach, and the income approach. The approach used to calculate Ghana's GDP on a quarterly basis is the production approach. The Gross Domestic Product (GDP) in Ghana was worth 39.20 billion US dollars in 2011, according to a report published by the World Bank. The GDP value of Ghana is roughly equivalent to 0.06 percent of the world economy. Historically, from 1960 until 2011, Ghana's GDP averaged 7.15 Billion USD reaching an all time high of 39.20 Billion USD in December of 2011 and a record low of 1.20 Billion USD in December of 1960. The gross domestic product (GDP) is a measure of national income and output for a given country's economy. The gross domestic product (GDP) is equal to the total expenditures for all final goods and services produced within the country in a stipulated period of time.

The reduction in the volatility of growth rates with country size is well known. Box and Jenkins(1976) uses size measures to correct for possible heteroskedasticity in long run growth rates. Khan, et al.(2011)calculate the Spearman rank correlation coefficient of the volatility of GDP with total GDP across countries and argues that the higher output variance of smaller countries is due to their greater openness and susceptibility to foreign shocks. We argue that there is a highly structured relationship between aggregate output shocks and the size of an economy and that microeconomic models should try to explain all of these empirical regularities.

2. Materials and Method

2.1. Procedure

The time series were first analysed to identify systematic patterns (frequency components or trends) which are not salient in the time series. Autocorrelation and Crosscorrelation Functions (ACF, CCF) as well as Spectral and Cross-Spectral densities were estimated for these purposes. All estimated correlation functions (correlograms) were plotted with the 95% confidence intervals of consecutive lags in the specified range. The sample autocorrelations had been used in the earlier part of the analysis to check the stationarity of the data set and also to have a measure of the dependence considering the data as a time series.

Seasonal dependency was proved by comparing the results of the Partial Autocorrelation Function (PACF), which considerably reduces the dependence on the intermediate elements, within the lag, and the results of the ordinary ACF (Box and Jenkins, 1976; Box and Pierce, 1970). The software routines applied validate the significance of the correlation coefficients r_k by comparing their values to the standard error of r_k, under the assumption that the series is a white noise process and that all autocorrelations are equal to zero.
The development of ARIMA models is based on the methodology described in the classical work of Box and Jenkins (1976). The procedure is applied separately to the landings and SST time series, as a univariate time series approach, taking into account only the mathematical properties of the data, without involving the biological or the physical background of the system. This kind of analysis supposes that other 'external factors' do not participate in the process development or that their contribution is stochastic.
For each developed ARIMA model the standard three-steps procedure has been followed, namely model identification, parameter estimation and finally the diagnosis of the simulation and its verification (Brockwell and Davis, 1996). As mentioned above, the input series for ARIMA needs to be differenced to achieve stationarity. The order of differencing is reflected in the *d* parameter. The general model introduced by Box and Jenkins (1976) can be summarized by the use of the following three types of parameters: the autoregressive parameters (p), the number of differencing (d), and moving average parameters (q).
In the notation introduced by Box and Jenkins, a model described as (0, 1, 2) means that it contains 0 (zero) autoregressive (p) parameters and 2 moving average (q) parameters which were computed for the series after it was differenced once. Similarly the required parameters *sp*, *sd* and *sq* of the seasonal ARIMA process are determined according to the results of the corresponding ACF and PACF. The approach used consequently was to estimate the seasonal model first, then study the residuals of this model to get a clearer view of the non-seasonal model involved. If the identification of the seasonal model was correct, these residuals showed the non-seasonal portion of the model.
After the identification of the tentative model, its parameters were estimated applying maximum-likelihood methods. The final results include: the parameter estimates, standard errors, estimate of residual variance, standard error of the estimate, log likelihood, Akaike's information criterion (AIC), Schwartz's Bayesian criterion (SBC). The minimizing of SBC and AIC were used, taking into account both how well the model fitted the observed series, and the number of parameters used in the fit.

2.2. The GARCH Models

GARCH models are used as a successful treatment to the financial data which often demonstrate time-persistence, volatility clustering and deviation from the normal distribution. Among the earliest models is Engel (1982) linear ARCH model, which captures the time varying
features of the conditional variance. Bollerslev (1986) develops Generalized ARCH (GARCH)
model, allowing for persistency of the conditional variance and more efficient testing. Engle and
Bollerslev (1986) invent the Integrated GARCH (IGARCH) model that provides consistent estimation under the unit root condition. Engle et. al. (1987) design the ARCH-in- Mean (ARCH-M) model to allow for time varying conditional mean. Nelson's (1990b) Exponential GARCH (EGARCH) model allows asymmetric effects and negative coefficients in the conditional variance function. The leveraged GARCH (LGARCH) model documented in Glosten et. al. (1993) take into account the asymmetric effects of shocks from different directions.
Since their introduction by Engle (1982), Autoregressive Conditional Heteroskedastic (ARCH) models and their extension by Bollerslev (1986) to generalised ARCH (GARCH) processes, GARCH models have been used widely by practitioners. At a first glance, their structure may seem simple, but their mathematical treatment has turned out to be quite complex. The aim of this article is to collect some probabilistic properties of GARCH processes. Although the ARCH is simple, it often requires many parameters to adequately describe the volatility process of an asset return some alternative models must be sought. Shrivastava, et al. (2010) and Hull(2006) proposed a useful extension known as the generalized ARCH (GARCH) model. An important feature of GARCH-type models is that the unconditional volatility σ depends on the entire sample, while the conditional volatilities σ_t are determined by model parameters and recent return observations.

Let $(\varepsilon t)t \in \mathbb{Z}$ be a sequence of independent and identically distributed (i.i.d.) random variables, and let $p \in \mathbb{N} = \{1, 2, 3, ...,)$ and $p \in \mathbb{N}_o = \mathbb{N} \cup \{0\}$. Further, let $\alpha_0 > 0$, $\alpha_1, ..., \alpha_{p-1} \geq 0$, $\alpha_p > 0$,

$\beta_1,...,\beta_{q-1} \geq 0$ and $\beta_q > 0$ be non-negative parameters. A GARCH(p, q) process $(X_t) t \in \mathbb{Z}$ with volatility process is $(\sigma_t) t \in \mathbb{Z}$ is then a solution to the equations:

$$X_t = \sigma_t \varepsilon_t , \quad t \in \mathbb{Z} \tag{1}$$

$$\sigma_t^2 = \alpha_t + \sum_{i=1}^{p} \alpha_i X_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-1}^2 , \quad t \in \mathbb{Z} \tag{2}$$

where the process $(\sigma_t) t \in \mathbb{Z}$ is non-negative. The sequence $(\varepsilon t) t \in \mathbb{Z}$ is referred to as the driving noise sequence. GARCH (p, 0) processes are called ARCH (p) processes. The case of a GARCH (0, q) process is excluded since in that case, the volatility equation (2) decouples from the observed process and the driving noise sequence.

It is a desirable property that σ_t should depend only on the past innovations $(\varepsilon t - h) h \in \mathbb{N}$, that is, it is measurable with respect to σ algebra generated by $(\varepsilon t - h) h \in \mathbb{N}$. If this condition holds, we shall call the GARCH (p, q) process causal. Then (X_t) is measurable with respect to σ algebra $\sigma(\varepsilon t - h : h \in \mathbb{N}_0)$, generated by $(\varepsilon t - h) h \in \mathbb{N}_0$. Also, σ_t is independent of $(\varepsilon t + h) h \in \mathbb{N}_0$, and X_t is independent of $\sigma(\varepsilon t + h : h \in \mathbb{N})$, for fixed *t*. The requirement that all the coefficients $\alpha_1,...,\alpha_p$ and $\beta_1,...,\beta_q$ are non-negative ensures that σ^2 is non-negative, so that σ_t can indeed be defined as the square root of σ^2.

Equation(1) is the mean equation and is specified as an AR(p) process. Equation(2) is the conditional variance equation and it is specified as the GARCH(1, 1) process. Conditional variance models (Shrivastava, 2009), unlike the traditional or extreme value estimators, incorporate time varying characteristics of second moment/volatility explicitly. By successively substituting for the lagged conditional variance into equation(2), the following expression is obtained:

$$h_t = \frac{\alpha_0}{1-\beta} + \alpha_1 \sum_{i=1}^{\infty} \beta_{i-1} \varepsilon_{t-i}^2 \tag{3}$$

An ordinary sample variance would give each of the past squares an equal weight rather than declining weights. Thus the GARCH variance is like a sample variance but it emphasizes the most recent observations. Since h_t is the one period ahead forecast variance based on past data, it is called the conditional variance. The squared residual is given by:

$$v_t = \varepsilon_t^2 - h_t \tag{4}$$

Equation(4) is by definition unpredictable based on the past. Substituting equation(4) into equation(2) yields an alternative expression as follows:

$$\varepsilon_t^2 = \omega + (\alpha_1 + \beta)\varepsilon_{t-1}^2 + v_t - \beta v_{t-1} \tag{5}$$

It can immediately be seen that the squared errors followed an ARMA(1, 1) process. The autoregressive root is the sum of α_1 and β, and this is the rule which governs the persistence of volatility shocks. The Autoregressive Moving Average (ARMA) Models have been used by many researcher for forecasting(Shrivastava, et, al. , 2010; Abu and Behrooz, 2011) . Given a time series of data Z_t, the ARMA model is a tool for understanding and, perhaps, predicting future values in this series. The model consists of two parts, an autoregressive (AR) part and "a" moving average (MA) part. The model is usually then referred to as the ARMA (a, b) model where a is the order of the autoregressive part and b is the order of the moving average part. The notation ARMA (a, b) refers to the model with "a" autoregressive terms and "b" moving-average terms. This model contains the AR(a) and MA(b) models. A time series Z_t follows an ARMA (1, 1) model if it satisfies

$$Z_1 = k + \omega_t + \sum_{i-1}^{a} \beta_i Z_{t-i} + \sum_{i=1}^{b} \alpha_i \omega_{t-i} \tag{6}$$

where { ω_t } is a white noise series. The above equation implies that the forecasted value is depended on the past value and previous shocks. The notation MA(b) which refers to the moving average model of order b is written as

$$Z_t = k + \omega_t + \sum_{i=1}^{b} \alpha_i \omega_{t-i} \tag{7}$$

and the notation AR(a) which refers to the autoregressive model of order a, is as

$$Z_1 = k + \omega_t + \sum_{i-1}^{a} \beta_i Z_{t-i} \tag{8}$$

where the $\alpha_1,\ldots,\alpha_b$ are the parameters of the model, μ is the expectation of Z_t (often assumed to equal to 0), and the $\omega_t,\ldots,\omega_{t-b}$ are again, white noise error terms.

2.3. Model Estimation and Evaluation

The forecast error is the difference between the realization and the forecast. Thus

$$e_\varsigma = x_{(T+\varsigma)..} - \hat{x}_{T+\varsigma}. \tag{9}$$

Assuming the model is correct, then we have

$$e_\varsigma = E[X_{T+\varsigma}] + \varepsilon_\varsigma - \hat{x}_\varsigma \tag{10}$$

We investigate the probability distribution of the error by computing its mean and variance. One desirable characteristics of the forecast $\widehat{X}_{T+\varsigma}$ is that it is unbiased. For an unbiased estimate, the expected value of the forecast is the same as the expected value of the time series. Because ε_t is assumed to have a mean of zero, an unbiased forecast implies $E[\varepsilon_\varsigma]$. The fact that the noise is independent from one period to the next period means that the variance of the error is:

$$Var[\varepsilon_t] = Var\{E[X_{T+\varsigma}] - \hat{x}_{T+\varsigma}\} + Var[\varepsilon_{T+\varsigma}] \text{ and } \sigma_{\varepsilon^2}(\varsigma) = \sigma_{E^2}(\varsigma) + \sigma^2. \tag{11}$$

The conditional-sum-of-squares is used to find starting values of parameters, then the maximum likelihood estimate for the proposed models. The procedure for choosing these models relies on choosing the model with the minimum AIC, AICc and BIC. The models are presented in Table 1 with their corresponding values of AIC, AICc and BIC. Among those possible models, comparing their AIC, AICc and BIC as shown in Table 1, ARIMA $(1,1,1)(0,0,1)_{12}$ and ARIMA $(1,1,2)(0,0,1)_{12}$ were chosen as the appropriate model that fit the data well. The basic volatility measure follows recent work by Comin and Mulani (2005, 2006) and Comin and Philippon (2005), among others. The measure of volatility is given by:

$$\bar{\sigma}_{it} = \left[\sum_{t=i}^{n} \left(\frac{\bar{Z}_{i,j+\tau}}{P_{it}} \right) \left(\gamma_{i,j+\tau} - \bar{\gamma}_{it} \right)^2 \right]^{\frac{1}{2}} \tag{12}$$

2.4. Maximum Likelihood Method

As pointed out by Bera and Higgins (1993), the GARCH models are most often estimated by maximum likelihood method. It is thus adopted in this study as well. The log likelihood function of the GARCH model based on previous period's information $f_t \mathrm{I} \psi_{t-1} \sim N(\alpha_0 + \alpha_1 f_{t-1}, l_t)$ is given by

$$l(\theta) = \frac{1}{T} \sum_{t-1}^{T} l_i(\theta)$$

where $\theta = (\xi', \gamma')$ with ξ' and γ' the conditional mean and conditional variance parameters respectively, and

$l_t(\theta) = const. - \frac{1}{2}\log(h_t) - \frac{\varepsilon_t^2}{2h_t^2}$ The likelihood function provided above is maximized using Berndt, Hall, Hall and Hausman (1974) numerical algorithm.

2.5. Model Identification

This involves the determination of the order of the AR and MA for both seasonal and non-seasonal components. This can be suggested by the sample ACF and PACF plots based on the Box-Jenkins approach. From Figures 1 and 2, the ACF plot tails of at lag 2 and the PACF plot spike at lag 1, suggesting that q= 2 and p=*1* would be needed to describe these data as coming from a non-seasonal moving average and autoregressive process respectively.
Also looking at the seasonal lags, both ACF and PACF spikes at seasonal lag 12 and drop to zero for other seasonal lags suggesting that Q= 1 and P= 1 would be needed to describe these data as coming from a seasonal moving average and autoregressive process. Hence ARIMA (1,1,2)(1,0,1) could be a possible model for the series.

3. Results of the Study

3.1. Data for the study

This study used data based on the quarterly real GDP in Ghana from three sectors(agriculture, industry and services). The source is the Ghana Statistical Service's Main Economic Indicators. The sample period is the first quarter of 2000 through the fourth quarter of 2012. Each variable is seasonally adjusted. The quarterly growth is calculated as $(Y_t - Y_{t-1}) \times 100 / Y_{t-1}$, where Y_t is the original data series (real GDP for agriculture, industry and services) at time *t*. This statistical release contains independently compiled quarterly estimates of the gross domestic product (GDP) for the period of first quarter of 1992 to second quarter of 2012. The estimates are based on the 1993 System of National Accounts (SNA), International Standard Industrial Classification Revision 4 published by the United Nations and other international organizations and Quarterly National Accounts Manual: Concepts, data sources, and compilation by International Monetary Fund (IMF). This means that the methodology, concepts and classifications, are in accordance with the guidelines and recommendations of an internationally agreed system of national accounts. The estimates of real GDP are expressed in terms of a 2006 base year.

Short-term indicators are used to estimate the quarterly GDP (ref Quarterly National Accounts Manual: Concepts, Data sources, and Compilation - IMF) where Annual GDP estimates are calculated independently from the quarterly estimates. Other than that, annual GDP estimates are derived as the sum of the GDP for the four quarters. The quarterly value added and GDP estimates have been seasonally adjusted. Seasonal adjustment is the process of estimating and removing seasonal effects from time series to reveal non-seasonal features. This process is to provide a clearer view of short term movements and trends and also to allow earlier identification of turning points

3.2. Empirical Results

The descriptive statistics for the GDP series are shown in Table 2. Generally, there is a large difference between the maximum and the minimum return of the index. The standard deviation is also high with regards to the number of observations including a high level of fluctuation of the yearly GDPs. The mean is close to zero and positive as is expected for a time series of returns.

There is also negative skewness, indicating an asymmetric tail which exceeds more towards negative values. The GDP series are leptokurtic, given their large kurtosis statistics as shown in Table 2. The kurtosis exceeds the normal value of three indicating that the return distribution is flat-tailed. Jarque and Bera(1980) test for normality confirms the results based on skewness and kurtosis and the series are non-normal according to Jarque and Bera which rejects normality test at 1% level of significance.

Figure 3 shows the GDP series from 2000 to 2012. Virtual inspection shows that volatility change over the time and it tends to cluster with the periods of low volatility and periods of high volatility. The volatility is relatively consistent from 2003 to the year 2009 and seems to increase in the middle of 2008 till 2010.

Table 3 shows the results of ARCH- LM test. This was done to see if there is any ARCH effect in the residuals. The ARCH- LM test for the series shows a significant presence of ARCH effect with low p-value of 0.000. The null hypothesis of no ARCH effect is rejected and a strong presence of ARCH effect is detected as is expected for most financial time series. The test is conducted at different numbers of lags. Values in parenthesis indicate the p-values. The zero p-value at all lags indicates the presence of ARCH effect in the series. Obs*R-squared is the number of observations multiplied by the R-squared value. The results in Table 3 confirm that the GARCH-type models can be applied to the GDP series. In most empirical implementations, the values, $p \leq 2$ and $q \leq 2$, are sufficient to model the volatility which provides a sufficient tradeoff between flexibility and parsimony (Knight and Satchell, 1998). The symmetric GARCH and nonlinear asymmetric EGARCH, and GJR-GARCH models were examined at different lags for $p \leq 2$ and $q \leq 2$.

Table 4 shows the results of GARCH model estimation. The AR order for the mean equation is selected by the Akaike Information Criterion (AIC) criterion and is found to be three for all the sectors. The AIC is a measure of the relative goodness of fit of a statistical model. In the general case, the AIC is equal to $2k - 2In(L)$, where k is the number of parameters in the statistical model, and L is the maximized value of the likelihood function for the estimated model.

The GARCH(1, 1), EGARCH(1, 1) and GJR-GARCH(1, 1) were found to be the most successful models according to AIC. As they have the smallest value while satisfying restriction such as non-negativity for symmetric GARCH. The models were estimated for the series using Quasi-Maximum likelihood assuming the Gaussian normal distribution.

Data in Table 5 clearly indicates that the ARCH and GARCH terms are both significant for the service sector with coefficient of 0.8811 and 0.0901 respectively. Similar parameter estimates are obtained for both coefficients in the agricultural sector, whereas the estimates are quite discrepant in the industrial sector, with relatively large estimates for ARCH (0.3682) and a small estimate for GARCH(0.4539). Moreover, the sum of α_1 and β_1, a parameter that shows the persistence of volatility is relatively high in the service and agricultural sectors, but relatively low in the industrial sector. It should be noted that AR order for the mean equation selected by the AIC criteria is found to be three for all data sets. The number in parentheses below the parameter estimates are standard errors obtained from the heteroskedasticity consistent covariance matrix of the parameters.

Table 6 gives the residual diagnostics corresponding to the estimates in Table 5. The Ljung-Box test is used to check the autocorrelation of the residuals (Ljung and Box, 1979) and the Jarque-Bara test is used to check the normality of the residuals(Jarque and Bara, 1987). The enteries in Table 5 are the p-values and $LB^2(12)$ values. The $LB^2(12)$ is the Ljung-Box test of order 12 using squared standardized residuals. As Table 6 indicates, the null hypothesis of no autocorrelation is not rejected for all three sectors at 1% significance level.

The result of non-normality in residuals shows that the GARCH effect is insufficient to capture the characteristics of the distribution.

The LB^2 (12) is the Ljung-Box test of order 12 using squared standardized residuals and the normality test is obtained from Jarque-Bera statistic. Entries represent corresponding p-values. P-values less than 0.05 imply the hypothesis of remaining no ARCH effect is rejected and the hypothesis of normal distribution is rejected at the 5% level of significance.

The variance in the low volatility state is estimated at 0.4861 for the service sector, 0.4625 for the agricultural sector, and 0.2534 for the industrial sector, and all these values are significant at the 5% significance level. The variance in the high volatility state is estimated at 2.0811 for the service sector, 2.8925 for the agricultural sector, and 1.2336 for the industrial sector, and all these values are significant at the 5% significance level. It is noted that the variance in the high volatility state($S_t = 2$) in the service, agricultural and industrial sectors are more than four, six and eight times as great as the variances in the low volatility states respectively. It is then possible to quantify the differences or the breaks in the variance of the GDP process reported in other studies(Kim, et.al. 2001; Bhar, et. al.2009)

4. Conclusion

The ARCH and the GARCH models have been applied to a wide range of time series analysis but applications in finance have been particularly successful and have been the focus of this paper. Recent studies have uncovered evidence of a structural break in the variance of GDP process in many countries.

The paper used the GARCH – type models to characterize the volatility in the growth rate of real GDP in three sectors. The main objects of interest were the unconditional volatility(σ)and conditional volatilities (σ_t). The value that σ_t is set to can in some case make a large difference. For example, global volatility started picking up with the advent of the 2007 crisis, peaking up in 2008. In such cases where there is a clear structural break in volatility, the GARCH model experience difficulties since it is based on the assumption of average volatility being constant.

Volatility models are estimated by maximum likelihood(ML)where parameter estimates are obtained by numerically maximizing the likelihood function with an algorithm called the optimizer. Previous work has documented the usefulness of a GARCH(1, 1) model without asymmetry in the innovation. In the absence of market shocks GARCH variance will eventually settle to a steady state value. This is the value $\bar{\sigma}^2$ such that $\sigma_t^2 = \bar{\sigma}^2$ for all *t*.

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Table 1: AIC and BIC for the Suggested ARIMA Model

Model	AIC	AICc	BIC
$MA(1,1,1)(0,0,0)_{12}$	723.46	723.74	746.57
$MA(1,1,1)(0,0,0)_{12}$	723.54	723.86	751.24
$MA(1,1,1)(0,0,0)_{12}$	723.41	724.45	756.19
$MA(1,1,1)(0,0,0)_{12}$	723.75	724.82	761.21
$MA(1,1,1)(0,0,0)_{12}$	723.85	735.39	753.02

Table 2: Descriptive Statistics of GDP of the three Sectors (agriculture, industry and services)

Statistic	Sectors		
	Agriculture	Industry	Services
Mean	0.006429	0.008126	0.002443
Standard Deviation	0.000013	0.000002	0.000027
Maximum	0.064365	0.057323	0.032394
minimum	-0.001265	-0.000213	-0.001219
Skewness	-0.544954	-0.646754	-0.564648
Kurtosos	10.142612	9.342675	11.242609
Jarque-Dera	2598.313	3546.325	32578.856
Probability	0.000000	0.000000	0.000000

Table 3: ARCH LM test

Number of lags	1992 to 2012		
	1	5	10
F statistic	69.987	54.342	26.654
	(0.000)	(0.000)	(0.000)
Obs*R-squared	58.643	214.873	246.832
	(0.000)	(0.000)	(0.000)

Table 4: GARCH-type Models

Coefficients	GARCH-type Models		
	GARCH	EGARCH	GJE-GARCH
ϕ_0	0.00915	0.00718	0.02026
ϕ_1	0.01417	0.07617	0.06121
ϕ_2	0.06071	0.00341	0.09317
ϕ_3	0.08691	0.08192	0.00231
α_0	0.00361	0.00622	0.03432
α_1	0.09017	0.08216	0.09541
β_1	0.02018	0.03421	0.06123

Table 5: GARCH(1, 1) estimation of real growth rate in GDP

Model Parameters	SECTORS		
	Service	Agriculture	Industry
ϕ_0	0.2416*	0.5411*	0.4409*
	(0.1013)	(0.1093)	(0.1342)
ϕ_1	0.1417**	0.1317	0.1114*
	(0.0819)	(0.0718)	(0.0819)
ϕ_2	0.3071*	0.2161	0.2112*
	(0.0875)	(0.0913)	(0.0801)
ϕ_3	0.2596*	0.1387*	0.2596
	(0.0786)	(0.0236)	(0.0702)
α_0	0.0238	0.0332	0.0318
	(0.0284)	(0.0185)	(0.0211)
α_1	0.0901*	0.0811**	0.0901*
	(0.0385)	(0.0283)	(0.0475)
β_1	0.8901*	0.8721*	0.7803*
	(0.0432)	(0.0337)	(0.0451)

** significant at 5% level; * significant at 10% level

Table 6: Diagnostic test using standardized residuals from GARCH(1, 1) model

	SECTOR		
	Service	Agriculture	Industry
$LB^2(12)$	0.954	0.487	0.312
Normality test	0.000	0.000	0.000

Figure 1: ACF of First Order Difference Series

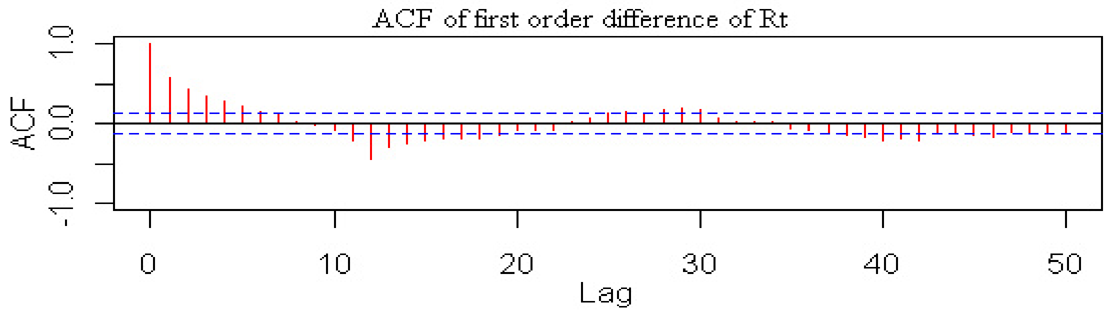

Figure 2: PACF of First Order Difference Series

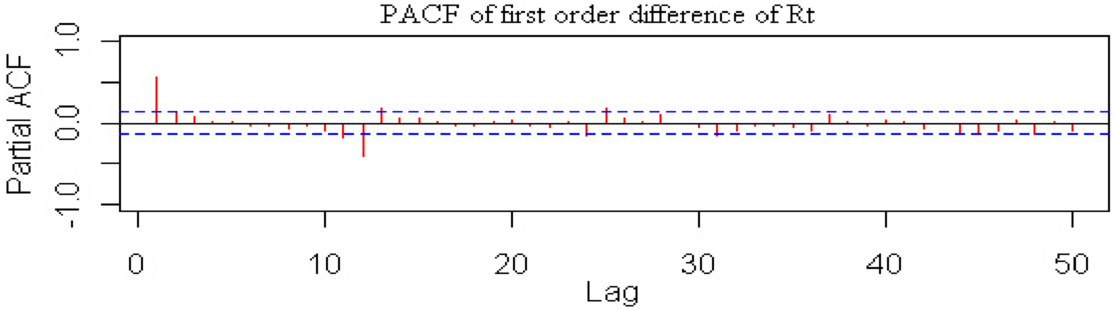

Figure 3: GDP series from 1992 to 2012