European Journal of Business and Management
ISSN 2222-1905 (Paper) ISSN 2222-2839 (Online)
Vol.5, No.32, 2013

Modelling the Volatility of GHC_USD Exchange Rate Using Garch Model

Ebenezer Okyere[1], Alice Constance Mensah[2*], Osei Antwi[2], Prince Kumi[3]
1. Research Department, Bank of Ghana, Accra
2. Mathematics and Statistics Department, Accra Polytechnic, Accra
3. Accountancy Department, Accra Polytechnic, Accra
*E-mail of the corresponding author: alicemen_sah@yahoo.co.uk

Abstract
Modelling and forecasting the exchange rate volatility is a crucial area, as it has implications for many issues in the arena of finance and economics. Generalised Autoregressive Conditional Heteroskedasticity (GARCH) models with their modifications, is used in capturing the volatility of the exchange rates. Simple rate of returns is employed to model the currency exchange rate volatility of Ghana Cedi-United States Dollar. The daily closing exchange rates were used as the daily observations. The parameters of these models are estimated using the maximum likelihood method. The results indicate that the volatility of the GHC_USD exchange rate is persistent. The asymmetry terms for TARCH are not statistically significant. Also in TARCH case, the coefficient estimate is negative, suggesting that positive shocks imply a higher next period conditional variance than negative shocks of the same sign. This is the opposite to what would have been expected in the case of the application of a GARCH model to a set of stock returns. But arguably, neither the leverage effect or volatility feedback explanations for asymmetries in the context of stocks apply here.
Keywords: Exchange rate, volatility, GARCH model

1. Introduction

Issues related to foreign exchange rate have always been the interest of researchers in modern financial theory. Exchange rate, which is the price of one currency in terms of another currency, has a great impact on the volume of foreign trade and investment. Its volatility has increased during the last decade and is harmful to economic welfare (Laopodis 1997). The exchange rate fluctuated according to demand and supply of currencies. The exchange rate volatility will reduce the volume of international trade and the foreign investment.

Financial economists are concerned with modelling volatility in asset returns. This is important as volatility is considered a measure of risk, and investors want a premium for investing in risky assets. Banks and other financial institutions apply so-called value-at-risk models to assess their risks. Modelling and forecasting volatility or, in other words, the covariance structure of asset returns, is therefore important.

Modelling and forecasting the exchange rate volatility is a crucial area for research, as it has implications for many issues in the arena of finance and economics. In view of this, knowledge of currency volatility should assist one to formulate investment and hedging strategies.

The implication of foreign exchange rate volatility for hedging strategies is also a recent issue. These strategies are essential for any investment in a foreign asset, which is a combination of an investment in the performance of the foreign asset and an investment in the performance of the domestic currency relative to the foreign currency. Hence, investing in foreign markets that are exposed to this foreign currency exchange rate risk should hedge for any source of risk that is not compensated in terms of expected returns (Santis et al. 1998).

Foreign exchange rate volatility may also impact on global trade patterns that will affect a country's balance of payments position and thus influence the government's national policymaking decisions.

Ghana public debt in American dollar represents 50 percent of the total debt portfolio, that in euro is about 20 percent and that expressed in Britain pound is around 20 percent. As a result of that, forecasting the future movement and volatility of the Ghana cedi to United States of America dollar exchange rate is crucially important and of interest to many diverse groups including market participants and Ghana government.

The main objective of the study was to model the currency exchange rate volatility of Ghana Cedi-United States Dollar using Generalised Autoregressive Conditional Heteroskedasticity (GARCH) models.

In international capital budgeting of multinational companies, the knowledge of foreign exchange volatility will help them in estimating the future cash flows of projects and thus the viability of the projects. Consequently, forecasting the future movement and volatility of the foreign exchange rate is crucially important and of interest to many diverse groups including market participants and decision makers.

Beginning with the seminal works of (Mandelbrot 1963) and (Fama 1965), many researchers have found that the stylized characteristics of the foreign currency exchange returns are non-linear temporal dependence and the distribution of exchange rate returns are leptokurtic, such as (Friedman &Vandersteel 1982; Bollerslev 1987; Diebold 1988; Hsieh 1988, 1989a, 1989b; Diebold & Erlove 1989; Baillie & Bollerslev1989). Their studies have found that large and small changes in returns are ' clustered' together over time, and that their distribution is bell-shaped, symmetric and fat-tailed. These features of data are normally thought to be captured by using the Autoregressive Conditional Heteroskedasticity (ARCH) model introduced by (Engle 1982) and the Generalised ARCH (GARCH) model developed by (Bollerslev 1986), which is an extension of the ARCH model to allow for a more flexible lag structure. The use of ARCH/GARCH models and its extensions and modifications in modelling and forecasting stock market volatility is now very common in finance and economics, such as (French et al. 1987; Akgiray 1989; Lau et al.1990; Pagan & Schwert 1990; Day & Lewis 1992; Kim & Kon 1994; Franses & Van Dijk 1996 and Choo et al. 1999).

On the other hand, the ARCH model was first applied in modeling the currency exchange rate by Hsieh only in 1988. In a study done by (Hsieh 1989a) to investigate whether daily changes in five major foreign exchange rates contain any nonlinearities, he found that although the data contain no linear correlation, evidence indicates the presence of substantial nonlinearity in a multiplicative rather than additive form. He further concludes that a generalized ARCH (GARCH) model can explain a large part of the nonlinearities for all five exchange rates.

Since then, applications of these models to currency exchange rates have increased tremendously, such as by (Hsieh 1989b; Bollerslev 1990; Pesaran & Robinson 1993; Copeland et al. 1994; Takezawa 1995; Episcopos & Davies 1995; Brooks 1997; Hopper 1997; Cheung et al. 1997; Laopodis 1997; Lobo et al. 1998; Duan et al. 1999).

In many of the applications, it was found that a very high-order ARCH model is required to model the changing variance. The alternative and more flexible lag structure is the Generalised ARCH (GARCH) introduced by (Bollerslev 1986). Bollerslev et al. (1992) indicated that the squared returns of not only exchange rate data, but all speculative price series, typically exhibit autocorrelation in that large and small errors tend to cluster together in contiguous time periods in what has come to be known as volatility clustering. It is also proven that small lag such as GARCH(1,l) is sufficient to model the variance changing over long sample periods (French et al. 1987; Franses & Van Dijk 1996; Choo et al. 1999).

Even though the GARCH model can effectively remove the excess kurtosis in returns, it cannot cope with the skewness of the distribution of returns, especially the financial time series which are commonly skewed. Hence, the forecasts and forecast error variances from a GARCH model can be expected to be biased for skewed time series. Recently, a few modifications to the GARCH model have been proposed, which explicitly take into account skewed distributions. One of the alternatives of non-linear models that can cope with skewness is the Exponential GARCH or EGARCH model introduced by (Nelson 1991). For stock indices, Nelson's exponential GARCH is proven to be the best model of the conditional heteroskedasticity.

In 1987, Engle et al developed the GARCH-M to formulate the conditional mean as function of the conditional variance as well as an autoregressive function of the past values of the underlying variable. This GARCH in the mean (GARCH-M) model is the natural extension due to the suggestion of the financial theory that an increase in variance (risk proxy) will result in a higher expected return.

Choo et al. (1999) studies the performance of GARCH models in forecasting the stock market volatility and they found that the hypotheses of constant variance models could be rejected since almost all the parameter estimates of the non-constant variance (GARCH) models are significant at the 5% level;

the EGARCH model has no restrictions and constraints on the parameters;

the long-memory GARCH model is more suitable than the short-memory and high-order ARCH model in modelling the heteroscedasticity of the financial time series;

the GARCH-M is best in fitting the historical data whereas the EGARCH model is best in out-of-sample (one-step-ahead) forecasting;

the IGARCH is the poorest model in both aspects.

2.1 The GARCH (p,q) model:

The GARCH models, which are generalized ARCH models, allow for both autoregressive and moving average components in the heteroscedastic variance developed by (Bollerslev 1986). The general GARCH (p,q) model has the following form:

$$Y_t = a + \beta' X_t + u_t \qquad (1)$$

$$u_t | \Omega_t \sim \text{iid } N(0, h_t)$$

$$h_t = \gamma_0 + \sum_{i=1}^{p} \delta_i h_{t-i} + \sum_{j=1}^{q} \gamma_j u_{t-j}^2 \qquad (2)$$

$$p \geq 0, q > 0, \gamma_0 > 0, \delta_i \geq 0 \ \forall i \geq 1, i = 1, \dots, p, \qquad \gamma_j \geq 0 \ \forall j \geq 1$$

For $p = 0$ the model reduces to ARCH (q). In the basic ARCH (1) model we assume that: $h_t = \gamma_0 + \gamma_1 u_{t-1}^2$

The simplest form of the GARCH (p,q) model is the GARCH (1,1) model for which the variance equation has the form:

$$h_t = \gamma_0 + \delta_1 h_{t-1} + \gamma_1 u_{t-1}^2 \qquad (3)$$

The GARCH in mean or GARCH-M model:

This model allows the conditional mean to depend on its own conditional variance. The GARCH-M (p,q) model has the following form:

$$Y_t = a + \beta' X_t + \theta h_t + u_t \quad (4)$$
$$u_t | \Omega_t \sim \text{iid } N(0, h_t)$$
$$h_t = \gamma_0 + \sum_{i=1}^{p} \delta_i h_{t-i} + \sum_{j=1}^{q} \gamma_j u_{t-j}^2 \quad (5)$$

The threshold GARCH (TGARCH) model:
Threshold GARCH model was introduced independently by (Zakoïan 1994; Glosten et al, 1993). The main target of this model is to capture asymmetries in terms of negative and positive shocks. To do that it simply adds into the variance equation a multiplicative dummy variable to check whether there is statistically significant difference when shocks are negative.

$$h_t = \gamma_0 + \gamma u_{t-1}^2 + \vartheta u_{t-1}^2 d_{t-1} + \delta h_{t-1} \quad (6)$$

where d_t takes the value of 1 for $u_t < 0,$ and 0 otherwise.
EGARCH model
The EGARCH or Exponential GARCH model was proposed by (Nelson 1991), and the variance equation for this model is given by:

$$\log(h_t) = \gamma + \sum_{j=1}^{q} \varsigma_j \left| \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right| + \sum_{j=1}^{q} \varepsilon_j \frac{u_{t-j}}{\sqrt{h_{t-j}}} + \sum_{i=1}^{p} \delta_i \log(h_{t-i}) \quad (7)$$

Where γ, the $\zeta' s$, $\varepsilon' s$ and $\delta' s$ are parameters to be estimated. The EGARCH model allows for the testing of asymmetries.

ESTIMATING GARCH MODELS
GARCH models are usually estimated using numerical procedures to maximize the likelihood function, which produces the most likely values of the parameters given the data. It is important to be aware that the likelihood function can have multiple local maxima, and different algorithms can lead to different parameter estimates and standard errors. Good initial estimates of the parameters are useful to ensure the global maximum is reached. It is also important to be aware that the log-likelihood function can be relatively flat in the region of its maximum value, and in this case different parameter values can lead to similar values of the likelihood function, making it difficult to select an appropriate value.

Most GARCH models are estimated using the Berndt-Hall-Hall- Hausman (BHHH) (1974) algorithm. This algorithm obtains the first derivatives of the likelihood function with respect to the numerically calculated parameters, and approximations to the second derivative are subsequently calculated. The Broyden-Fletcher-Goldfarb-Shanno (BFGS) method solves unconstrained nonlinear optimization problems by calculating the likelihood function gradient in the same way as the BHHH, but it differs in its construction of the Hessian matrix of second derivatives. The BFGS and BHHH are asymptotically equivalent, but can lead to different estimates of the standard errors in small samples.

3.0 Methodology

In this study, simple rate of returns is employed to model the currency exchange rate volatility of Ghana Cedi-United States Dollar. Consider a foreign exchange rate E_t its rate of return r_t is constructed as

$$r_t = \ln\left(\frac{E_t}{E_{t-1}}\right) \quad (8)$$

The exchange rate t denotes daily exchange rate observations.
The foreign exchange rate used in this study is focused on the Ghana Cedi to the United States dollar. This exchange rate is chosen because the Ghana public debt in American dollar represents 50 percent of the total debt portfolio . The data was collected from 18 January 2010 to 25 February 2011. The daily closing exchange rates were used as the daily observations. Data is obtained from Pacific Exchange Rate Database. The analysis is done using the econometric package EViews 5.

4.1 Results

Figure 1 shows daily observed exchange rates of the Ghana Cedi to the United States dollar, covering the period. The returns of GHC_USD are plotted as shown in figure 2. It can be seen that there are certain periods that have higher volatility (and therefore are riskier) than others. This means that the expected value of the magnitude of the disturbance terms can be greater at certain periods compared to others. The spikes in Figure 2 suggest that the daily return series are not random walk processes, and that there exists significant volatility clustering.
Figure 3 is the graph of conditional standard deviation for an ARCH(1) model of the GHC_USD returns. The graph provides a conditional variance series which is very spiky.

Some descriptive statistics of the rate of returns are given in Table 1. The mean and variance are quite small. The excess kurtosis indicates the necessity of fat-tailed distribution to describe these variables. The skewness of -0.085 indicates that the distribution of rate of returns for GHC_USD is negatively skewed.

Testing for ARCH (1), the R-squared from Table 2 is 29.27 and has a probability limit of 0.000. This clearly suggests that ARCH effects are present. This led to the indication that GARCH-type model is appropriate for the data.

Table 3 displays the result of GARCH (1,1). The coefficients on both the lagged squared residual and lagged conditional variance terms in the conditional variance equation are highly statistically significant. Unlike, as is typical of GARCH model estimates for financial asset returns data, the sum of the coefficients on the lagged squared error and lagged conditional variance is very close to unity (approximately 0.96). This implies that shocks to the conditional variance will be highly persistent.

Table 4 shows the EGARCH (1,1) and TARCH(1,1) results. In both specifications, the asymmetry terms RESID(-1)/@SQRT(GARCH(-1)) for EGARCH and RESID(-1)^2*(RESID(-1)<0) for TARCH are not statistically significant. Also in TARCH case, the coefficient estimate is negative, suggesting that positive shocks imply a higher next period conditional variance than negative shocks of the same sign.

This is the opposite to what would have been expected in the case of the application of a GARCH model to a set of stock returns. But arguably, neither the leverage effect or volatility feedback explanations for asymmetries in the context of stocks apply here. For a positive return shock, this implies more Ghana Cedi per dollar and therefore a strengthening dollar and a weakening Cedi. Thus the results suggest that a strengthening dollar (weakening Cedi) leads to higher next period volatility than when the Cedi strengthens by the same amount.

4.2 GARCH (1,1) Static forecasts

Fig.4 shows GARCH (1,1) forecast. It is evident that the variance forecasts gradually fall over the out-of sample period, although since these are a series of rolling one-step a head forecasts for the conditional variance, they show much more volatility than for the dynamic forecasts. This volatility also results in more variability in the standard error bars around the conditional mean forecasts. The conditional variance forecasts provide the basis for the standard error bands that are given by the dotted red lines around the conditional mean forecast. Because the conditional variance forecasts rise gradually as the forecast horizon increases, so the standard error bands widen slightly. The forecast evaluation statistics that are presented in the box to the right of the graphs are for the conditional mean forecasts.

5. Conclusion

The analysis revealed that there is significant evidence of volatility clustering, a result that is different from the previous literature. There is a strong presence of serial correlation and in estimating the volatility parameter a series of GARCH models have been investigated. The results indicate that the volatility of the GHC_USD exchange rate is persistent. The asymmetry terms for TARCH are not statistically significant. Also in TARCH case, the coefficient estimate is negative, suggesting that positive shocks imply a higher next period conditional variance than negative shocks of the same sign.

References

Akgiray, V. 1989. Conditional heteroskedasticity in time series of stock returns: Evidence and forecasts. Journal of Business 62: 55-80.

Baillie, R. T. and Bollerslev T. 1989. The message in daily exchange rates: A conditional variance. Journal of Business and Economic Statistics 7: 297-305.

Berndt, E. R., Hall, B. H., Hall, R. E. and Haussmann, J. A. 1974. 'Estimation and Inference in Nonlinear Structural Models', Annals of Economic and Social Measurement, Vol. 4 pp. 653–665.

Bollerslev, T. 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31: 307-327.

Bollerslev, T. 1987. A conditional heteroskedastic time series model for speculative prices and rates of return. Review of Economics and Statistics 69: 542-547.

Cheung, Y. W. and Wong C. Y. P . 1997. The performance of trading rules on four Asian currency exchange rates. Multinational Finance Journal 1(1): 1-22.

CHoo, W. C., Ahmad M. I. and Abdullah M. Y. 1999. Performance of GARCH models in forecasting stock market volatility. Journal of Forecasting 18: 333-343.

Copeand, L. S. and Wang P. 1994. Estimating daily seasonality in foreign exchange rate changes. Journal of Forecasting 13(6): 519-528.

Day, T. E. and Lewis C. M. 1992. Stock market volatility and information content of stock index options. Journal of Econometrics 52: 267287

Diebold, F. X. and Nerlove M.. 1989. The dynamics of exchange rate volatility : A multivariate latent variable factor ARCH model. Journal of Applied Econometrics 4(1): 1-21.

Diebold, F. X. 1988. Empirical Modeling of Exchange Rate Dynamics. Berlin: Springer-Verlag.

Duan , J. C. and Jaso Z. W. 1999. Pricing foreign currency and cross-currency options under GARCH. The

Journal of Derivatives (Fall).
Engle, R. F. 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation. Econometrica 50: 987-1008.
Episcopos, A. and Davies J. 1995. Predicting returns on Canadian exchange rates with artificial neural networks and EGARCH-M models. Neural Computing and AppLUations (forthcoming).
Fama, E. F. 1965. 'The Behavior of Stock Market Prices', Journal of Business, Vol. 38, No. 1, pp. 34–105.
Franses, P. H. and van Dijk, D. 1996. Forecasting Stock Market Volatility Using
Non-Linear GARCH Models, Journal of Forecasting 15, 229—35
French, K. R., Schwert G. W. and Stambaugh R. F.1987. Expected stock returns and volatility. Journal of Financial Economics 19: 3-30.
Frjedman, D. And Vandersteel S. 1982. Short-run fluctuations in foreign exchange rates : Evidence from the data 1973 - 1979. Journal of International Eonomics 13: 171-186.
Glosten, L. R., Jagannathan, R. and Runkle, D. E. (1993) On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, The Journal of Finance 48(5), 1779—1801
Hopper, G. P. 1997. What determines the exchange rate: Economics factors or market sentiment. Business Review: 17-29. Federal Reserve Bank of Philadelphia.
Hsieh, D. A. 1988. The statistical properties of daily foreign exchange rates: 1974 - 1983. Journal of International Economics: 129-145.
Hsieh, D. A. 1989a. Modeling hetereskedasticity in daily foreign exchange rates. Journal of Business and Economic Statistics 7: 306-317.
Hsieh d. A. 1989b. Testing for non-linear dependence in daily foreign exchange rates. Journal of Business 62: 339-368.
Laopodls, N. T. 1997. U.S. dollar asymmetry and exchange rate volatility. Journal of Applied Business Research 13(2) Spring: 1-8.
LAu, A., H. LAu and Wingender J. 1990. The distribution of stock returns: New evidence against the stable model. Journal of Business and Economic Statistics 8 (April): 217-223.

Mandelbrot, B. 1963. "The Variation of Certain Speculative Prices." Journal of Business. 36, 394-419.
Nelson, D. B. 1991. Conditional Heteroskedasticity in Asset Returns: A New Approach, Econometrica 59(2), 347—70
Pesaran, B. and Robinson G. 1993. The European exchange rate mechanism and the volatility of the sterling-deutschmark exchange rate. Economic Journal 103: 1418-1431.
Santis, G. D. and Gerard B. 1998. How big is the premium for currency risk? Journal of Financial Economics 49: 375-412.
Takezawa, N. 1995. A note on intraday foreign exchange volatility and the informational role of quote arrivals. Economics Letters 48: 399-404.
Zakoian, J.M. 1994. 'Threshold Heteroskedastic Models', Journal (Economic Dynamics and Control, 18, pp. 931-55.

Appendix

Fig. 1: Daily Observed Exchange Rates of the GHC - USD

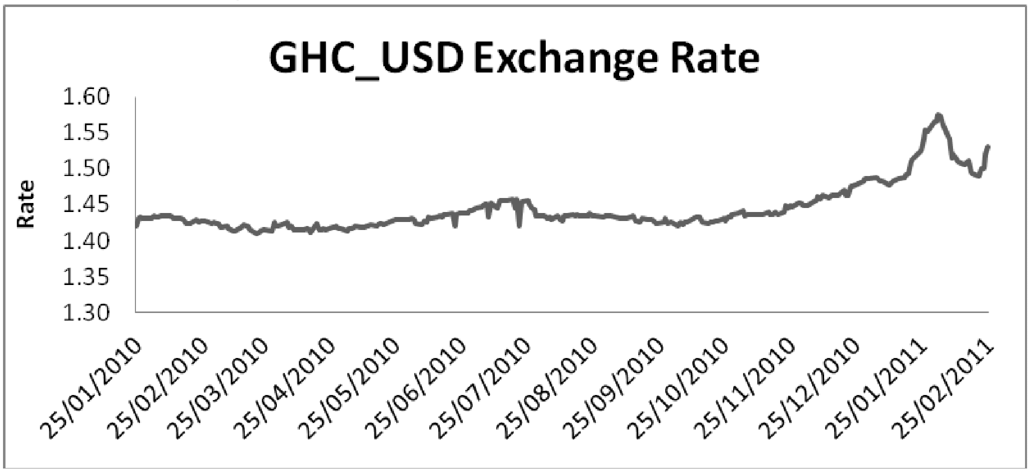

Fig. 2: Returns of GHC – USD Exchange Rates

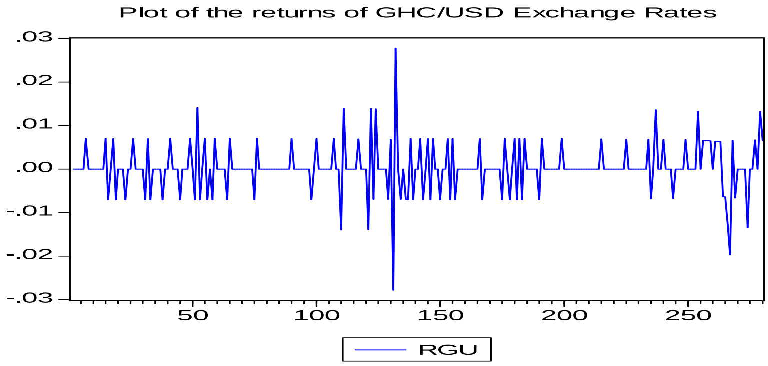

Fig. 3: Conditional Standard Deviation for ARCH of the GHC – USD Returns

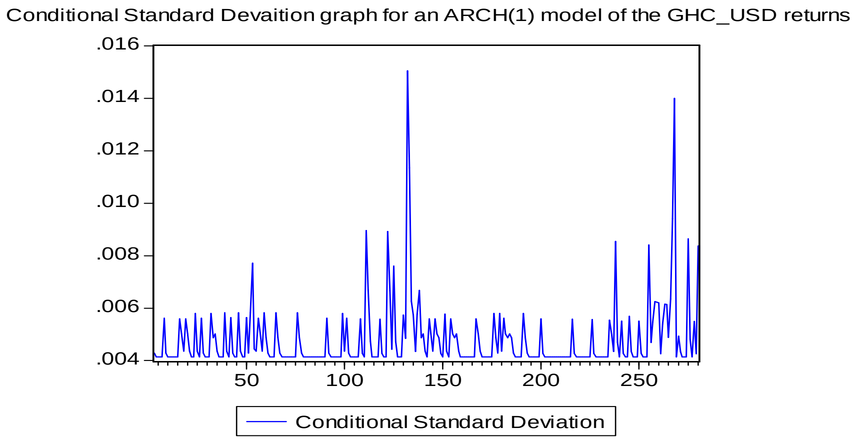

Fig 4:GARCH (1.1) Forecast

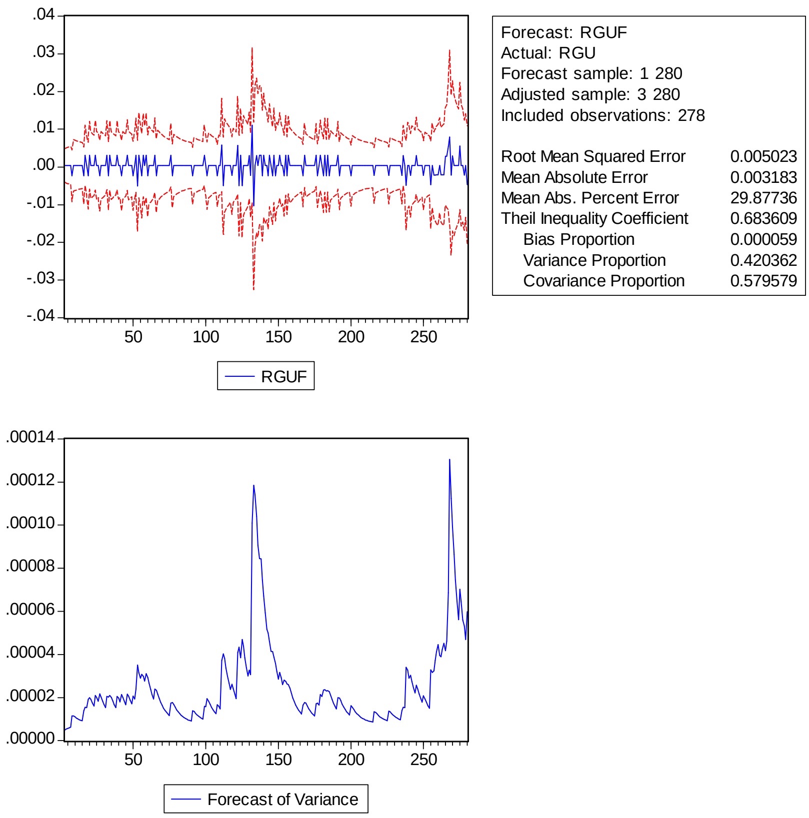

Table 1: Descriptive Statistics of GHC_USD returns (RGU)

Mean	0.000267
Median	0.000000
Maximum	0.027780
Minimum	0.027780
Std. Dev.	0.005287
Skewness	0.005287
Kurtosis	8.806400
Jarque – Bera	392.2645
No. Observations	279

Table 2: Testing for ARCH (1) effects in the GHC_USD

F – Statistic	32.49187	Probability	0.00000
Obs R-squared	29.26987	Probability	0.00000

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.71E-05	4.02E-06	4.251844	0.0000
RESID^2(-1)	0.325937	0.057180	5.700164	0.0000

Table 3: A GARCH(1,1) model for the GHC_USD returns

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000401	0.000249	1.608605	0.1077
RGU(-1)	0.385038	0.076813	-5.012639	0.0000
	Variance Equation			
C	1.16E-06	4.16E-07	2.776306	0.0055
RESID(-1)^2	0.113125	0.029840	3.790995	0.0002
GARCH(-1)	0.850657	0.036891	23.05887	0.0000

RGU = C(1) + C(2)*RGU(-1)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)
Substituted Coefficients:
RGU = 0.0004012284202 - 0.385037599*RGU(-1)
GARCH = 1.155819627e-006 + 0.1131246474*RESID(-1)^2 + 0.8506571715*GARCH(-1)

Table 4: An EGARCH (1,1) model for the GHC_USD returns

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000561	0.000284	1.976267	0.0481
RGU(-1)	0.379770	0.070113	-5.416543	0.0000
	Variance Equation			
C(3)	-0.895740	0.300548	-2.980352	0.0029
C(4)	0.214272	0.047863	4.476768	0.0000
C(5)	0.022457	0.055872	0.401932	0.6877
C(6)	0.929182	0.026723	34.77124	0.0000

Estimation Equation:
RGU = C(1) + C(2)*RGU(-1)

LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Substituted Coefficients:
RGU = 0.0005612265355 - 0.3797695977*RGU(-1)

LOG(GARCH) = -0.8957401191 + 0.214271649*ABS(RESID(-1)/@SQRT(GARCH(-1))) + 0.0224566917*RESID(-1)/@SQRT(GARCH(-1)) + 0.9291823468*LOG(GARCH(-1))

TARCH (1,1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000473	0.000273	1.731090	0.0834
RGU(-1)	0.402434	0.073105	-5.504913	0.0000
	Variance Equation			
C	9.51E-07	4.16E-07	2.285572	0.0223
RESID(-1)^2	0.140964	0.055101	2.558298	0.0105
RESID(-1)^2*(RESID(-1)<0)	-0.066617	0.074149	-0.898431	0.3690
GARCH(-1)	0.865843	0.036413	23.77849	0.0000

Estimation Equation:
RGU = C(1) + C(2)*RGU(-1)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)
Substituted Coefficients:
RGU = 0.000473286153 - 0.4024342848*RGU(-1)
GARCH = 9.508618687e-007 + 0.1409638545*RESID(-1)^2 - 0.06661731731*RESID(-1)^2*(RESID(-1)<0) + 0.8658425379*GARCH(-1)