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New Mixed Integer Programming for Facility Layout Design without Loss of Department Area

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Abstract
This paper proposes a New Mixed Integer Programming (NMIP) for solving facility layout problem (FLP). The formulation is extensively tested on problems from literature to minimize material handling cost when addresses on the shop floor. Every department and shop floor in tested problem were fixed dimension of length and width. Classic Mixed Integer Programming (MIP) solves FLP with fixed layout area and preset each department's lower-upper limit of length and width. As a result, there might be some departments with 5-10% area less than initial layout design requirements which leads to problem in design and construction process. It is infeasible to address the department on the actual shop floor thus adjustment of result from MIP is required. The main purpose of NMIP is to eliminate such infeasibility and show the efficiency of this model.
Keywords: mixed integer programing, facility layout design, facility layout problem, heuristics

1. Introduction

FLP is one of classical facility problems. One of the reasons is due to its contribution to saving 20–50% of the total operating cost and 15–70% of the total manufacturing cost when shop floor is well designed (Tompkins 2010; Sims 1991). In the real world environment, common framework for facility layout design starts from formulating each department then addressing all departments on the shop floor by taking smooth flow network into account to optimize material handling cost (Ioannou 2007).
To solve FLP, quadratic assignment problem (QAP) was introduced (Koopmans & Beckman 1957) to model the placement of departments to minimize material handling cost. The QAP is shown in equation (1).

$$\text{Min} \sum_{i} \sum_{j} (f_{ij}\, c_{ij})\, d_{ij} \tag{1}$$

Where f_{ij} is flow frequency from department i to department j. c_{ij} is the cost to move one unit per one distance unit from i to j and d_{ij} is the distance from i to j.
Additionally, researchers proposed various heuristics model approaches to solve FLP's objective functions (Ramkumar *et al.* 2009). In nature FLP is NP-complete problem so a variety of methodologies were introduced to deal with these complexities to find practical solution (See & Wong 2008).
MIP is one of the key methods to solve FLP. MIP solves FLP with fixed layout area and preset each department's lower-upper limit of length and width. However, this formulation does not comply with practice. Naturally, manufacturing facilities' dimensions are not equal. In this paper, NMIP formulation is proposed to solve FLP with unequal departmental dimension of length and width. The remainder of this paper is organized as follows. In Section2, review of related works and current MIP mathematical model is presented. The NMIP mathematical model is presented in Section 3. In Section 4, the result of FLP solved by NMIP is shown. Section 5 demonstrates conclusion and suggestion of future work.

2. Literature Review

2.1 Related Works

Many FLP solvers have focused on a single objective (Kazi *et al*. 2013), either quantitative (distance-based) or qualitative (adjacency-based) for efficient design of the facility layout. FLP approaches can be classified into exact approaches, heuristics approaches, and meta-heuristics approaches. The common limitation of FLP approaches is that they cannot solve FLPs with more than 20 facilities in reasonable time (Singh 2009). Many different FLP research directions have been published (Kundu & Dan 2012) such as

- Computerised technique focus; CRAFT (Armour & Buffa 1963), CORELAP (Sepponen 1969), COFAD (Tompkins & Reed 1976), ALDEP (Seehof & Evans 1967) and PLANET (Konz 1985)
- Specific characteristics study; Dynamic Layout Design (Balakrishnan & Cheng 1998), Loop Layout Design (Asef-Vaziri & Laporte 2005) and evolutionary algorithms in FLP (Pierreval *et al*. 2003)
- Meta-heuristic methods/techniques; Simulation Anneal (Coello Coello *et al*. 2007), Tabu Search (Glover & Laguna 1997), Genetic Algorithm (Goldberg 1989; Gen & Cheng 2000), ACO and PSO-based method (Ramkumar *et al*. 2008)

In the recent years, unequal area-multi-objective FLP solving is gaining interest in research world. Researchers have been trying to develop algorithms/models to find the optimal layout that is possible for practical use.

2.2 Mixed-Integer Programming (MIP)

Last two decades, MIP formulation was presented (Montreuil 1990). This model used a distance-based algorithm. The advantages of MIP are as follows; all departments are rectangular in shape, no need to enter an initial layout, Inter-department is guaranteed non-overlapping and high reliability by computerized simulation. Konak *et al.* (2006) developed MIP formulation to solve unequal area FLPs based on the flexible bay structure (FBS), which can solve FLP with upto14 facilities only. MIP utilized mathematic formulation to find decision variables that can minimize objective function as shown below;

Parameter, let:

B_x be the shop floor length (x-axis),
B_y be the shop floor width (y-axis),
A_i be the area of department i,
Ll_i be the lower limit on the length of department i,
Lu_i be the upper limit on the length of department i,
Wl_i be the lower limit on the width of department i,
Wu_i be the upper limit on the width of department i,
f_{ij} be the flow frequency from department i to department j,
c_{ij} be the cost to move one unit per one distance unit from department i to department j,
M is a large number.

Decision variable, let:

α_i be the x-coordinate of the centroid of department i,
β_i be the y-coordinate of the centroid of department i,
x'_i be the x-coordinate of the left (west) side of department i,
x''_i be the x-coordinate of the right (east) side of department i,
y'_i be the y-coordinate of the bottom (south) side of department i,
y''_i be the y-coordinate of the top (north) side of department i.

Binary variable, let:

zx_{ij} be equal 1 if department i positions strictly to the east of department j, otherwise 0,
zy_{ij} be equal 1 if department i positions strictly to the north of department j, otherwise 0.

Non-negative variable, let: $\overset{+}{\alpha}_{ij}$, $\overset{-}{\alpha}_{ij}$, $\overset{+}{\beta}_{ij}$, $\overset{-}{\beta}_{ij}$.

Objective function:

$$\text{Min} \sum_i \sum_j f_{ij}\, c_{ij} \left(\overset{+}{\alpha}_{ij} + \overset{-}{\alpha}_{ij} + \overset{+}{\beta}_{ij} + \overset{-}{\beta}_{ij}\right) \tag{2}$$

Subject to:

$$Ll_i \le (x''_i - x'_i) \le Lu_i \quad \text{for all } i \tag{3}$$
$$Wl_i \le (y''_i - y'_i) \le Wu_i \quad \text{for all } i \tag{4}$$
$$(x''_i - x'_i) * (y''_i - y'_i) = A_i \quad \text{for all } i \tag{5}$$
$$0 \le x''_i \le x'_i \le B_x \quad \text{for all } i \tag{6}$$
$$0 \le y''_i \le y'_i \le B_y \quad \text{for all } i \tag{7}$$
$$\alpha_i = 0.5\, x'_i + 0.5 x''_i \quad \text{for all } i \tag{8}$$
$$\beta_i = 0.5\, y'_i + 0.5 y''_i \quad \text{for all } i \tag{9}$$
$$\alpha_i - \alpha_j = \overset{+}{\alpha}_{ij} - \overset{-}{\alpha}_{ij} \quad \text{for all } i \text{ and } j,\ i \ne j \tag{10}$$
$$\beta_i - \beta_j = \overset{+}{\beta}_{ij} - \overset{-}{\beta}_{ij} \quad \text{for all } i \text{ and } j,\ i \ne j \tag{11}$$
$$x''_j \le x'_i + M\,(1 - zx_{ij}) \quad \text{for all } i \text{ and } j,\ i \ne j \tag{12}$$
$$y''_j \le y'_i + M\,(1 - zy_{ij}) \quad \text{for all } i \text{ and } j,\ i \ne j \tag{13}$$
$$zx_{ij} + zx_{ji} + zy_{ij} + zy_{ji} \ge 1 \quad \text{for all } i \text{ and } j,\ i < j \tag{14}$$
$$\alpha_i, \beta_i, x'_I, x''_I, y'_I, y''_i \ge 0 \quad \text{for all } i \tag{15}$$
$$\overset{+}{\alpha}_{ij}, \overset{-}{\alpha}_{ij}, \overset{+}{\beta}_{ij}, \overset{-}{\beta}_{ij} \ge 0 \quad \text{for all } i \text{ and } j,\ i \ne j \tag{16}$$

zx_{ij} , zy_{ij} 0/1 integer for all i and j, $i \neq j$ (17)

The objective function given by equation (2) is the distance-based algorithm's objective shown earlier as equation (1). Constraint (3) and (4) ensure that each department's length-width is within specified bounds. Equation (5) expresses each department's area. Constraint (6) and (7) ensure that each department locates within shop floor's bound. Constraint (8) and (9) define coordinate (x, y) of each department's centroid. Constraint (10) and (11) help for linearization to avoid absolute value operator in objective function (2). Constraint (12), (13) and (14) ensure that each department is non-overlap by forcing a separation at least in the east-west or north-south direction. Equation (15) and (16) ensure the non-negative constraints. Lastly constraint (17) designates binary variables.

3. New Mixed-integer programming (NMIP)

This paper proposes NMIP which a few parameters in classic MIP are ignored. In order to solve unequal departmental dimension of length and width, NMIP ignores parameters A_i, Ll_i, Lu_i, Wl_i and Wu_i while adds Sh_i and Lng_i (the shorter and longer side length of department i). Moreover NMIP adds one more binary variable Hor_{ij} (equal 1 if department i strictly to the longer side of department j on x-axis, otherwise 0). (-) indicates the parameters that are ignored parameters while (+) indicates the parameters and variable that are added. The proposed model is shown as follows:

Parameter, let:

B_x be the shop floor length (x-axis),
B_y be the shop floor width (y-axis),
(-) A_i be the area of department i,
(-) Ll_i be the lower limit on the length of department i,
(-) Lu_i be the upper limit on the length of department i,
(-) Wl_i be the lower limit on the width of department i,
(-) Wu_i be the upper limit on the width of department i,
(+) Sh_i be the shorter side length of department i,
(+) Lng_i be the longer side length of department i,
f_{ij} be the flow frequency from department i to department j,
c_{ij} be the cost to move one unit per one distance unit from department i to department j,
M is a large number.

Decision variable, let:

α_i be the x-coordinate of the centroid of department i,
β_i be the y-coordinate of the centroid of department i,
x'_i be the x-coordinate of the left (west) side of department i,
x''_i be the x-coordinate of the right (east) side of department i,
y'_i be the y-coordinate of the bottom (south) side of department i,
y''_i be the y-coordinate of the top (north) side of department i.

Binary variable, let:

zx_{ij} be equal 1 if department i positions strictly to the east of department j, otherwise 0,
zy_{ij} be equal 1 if department i positions strictly to the north of department j, otherwise 0.
(+) Hor_{ij} be equal 1 if department i positions strictly to the longer side of department j on x-axis, otherwise 0.

Non-negative variable, let: $\overset{+}{\alpha}_{ij}$, $\overset{-}{\alpha}_{ij}$, $\overset{+}{\beta}_{ij}$, $\overset{-}{\beta}_{ij}$.

Objective function:

$$\text{Min} \sum_i \sum_j f_{ij}\, c_{ij}\, (\overset{+}{\alpha}_{ij} + \overset{-}{\alpha}_{ij} + \overset{+}{\beta}_{ij} + \overset{-}{\beta}_{ij}) \quad (2)$$

Subject to:

$x''_i - x'_i \geq Hor_i * Lng_i + (1 - Hor_i) * Sh_i$	for all i	(18)
$y''_i - y'_i \geq Hor_i * Sh_i + (1 - Hor_i) * Lng_i$	for all i	(19)
$(x''_i - x'_i) + (y''_i - y'_i) = Lng_i + Sh_i$	for all i	(20)
$0 \leq x''_i \leq x'_i \leq B_x$	for all i	(6)
$0 \leq y''_i \leq y'_i \leq B_y$	for all i	(7)
$\alpha_i = 0.5\, x'_i + 0.5x''_i$	for all i	(8)
$\beta_i = 0.5\, y'_i + 0.5y''_i$	for all i	(9)

$\alpha_i - \alpha_j = \alpha^+_{ij} - \alpha^-_{ij}$ for all i and j, $i \neq j$ (10)

$\beta_i - \beta_j = \beta^+_{ij} - \beta^-_{ij}$ for all i and j, $i \neq j$ (11)

$x''_j \leq x'_i + M(1 - zx_{ij})$ for all i and j, $i \neq j$ (12)

$y''_j \leq y'_i + M(1 - zy_{ij})$ for all i and j, $i \neq j$ (13)

$zx_{ij} + zx_{ji} + zy_{ij} + zy_{ji} \geq 1$ for all i and j, $i < j$ (14)

$\alpha_i, \beta_i, x'_I, x''_I, y'_I, y''_i \geq 0$ for all i (15)

$\alpha^+_{ij}, \alpha^-_{ij}, \beta^+_{ij}, \beta^-_{ij} \geq 0$ for all i and j, $i \neq j$ (16)

zx_{ij}, zy_{ij} 0/1 integer for all i and j, $i \neq j$ (17)

Hor_i 0/1 integer for all i (21)

The objective function shown in equation (2) is the distance-based algorithm's objective shown earlier as equation (1). Constraint (3), (4) and (5) were not used in this NMIP. Constraint (18) and (19) define the shape of each department that is horizontal oriented or vertical oriented shape. Equation (20) ensures each department's perimeter is not over than fixed dimension. Constraint (6) and (7) ensure that each department locates within shop floor's boundary. Constraint (8) and (9) define coordinate (x, y) of each department's centroid. Constraint (10) and (11) ensure linearization to avoid absolute value operator in objective function (2). Constraint (12), (13) and (14) ensure that each department is non-overlap by forcing a separation at least in the east-west or north-south direction. Equation (15) and (16) ensure the non-negative constraints. Lastly, constraint (17) and (21) designate binary variables.

4. FLP Solved by NMIP

Shown in Table 1, FLP data sheet from (Tompkins 2010) is to be solved by NMIP; however, two parameters – shorter and longer side of each department – are added. We run this NMIP model by A Modelling Language for Mathematical Programming (AMPL/CPLEX) version 12.2. The problem is to address the department A to H on the shop floor with objective to minimize material handling cost. The department is 240 meters in x-axis and 320 meters in y-axis. We assume the cost to move one unit per one distance unit (c_{ij}) from department i to j is equal to US$1.

Table 1. Datasheet of FLP

					Flow frequency (trips per day)							
Department	no.	short side	long side	Area	A	B	C	D	E	F	G	H
A	1	100	120	12,000	0	45	15	25	10	5	0	0
B	2	80	100	8,000	0	0	0	30	25	15	0	0
C	3	60	100	6,000	0	0	0	0	5	10	0	0
D	4	100	120	12,000	0	20	0	0	35	0	0	0
E	5	80	100	8,000	0	0	0	0	0	65	35	0
F	6	60	200	12,000	0	5	0	0	25	0	65	0
G	7	60	200	12,000	0	0	0	0	0	0	0	0
H	8	40	50	2,000	0	0	0	0	0	0	0	0

The result of AMPL/CPLEX is shown in Figure 1. The material handling cost (objective function) is $51,700.

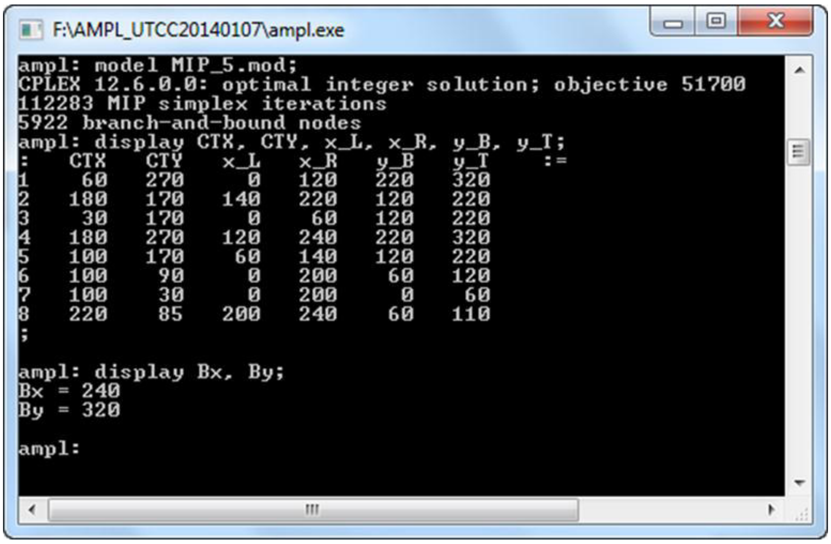

Figure 1. Result from AMPL/CPLEX

From the result of Figure 1, we can draw the facility layout of department A to H on the shop floor as shown in Figure 2.

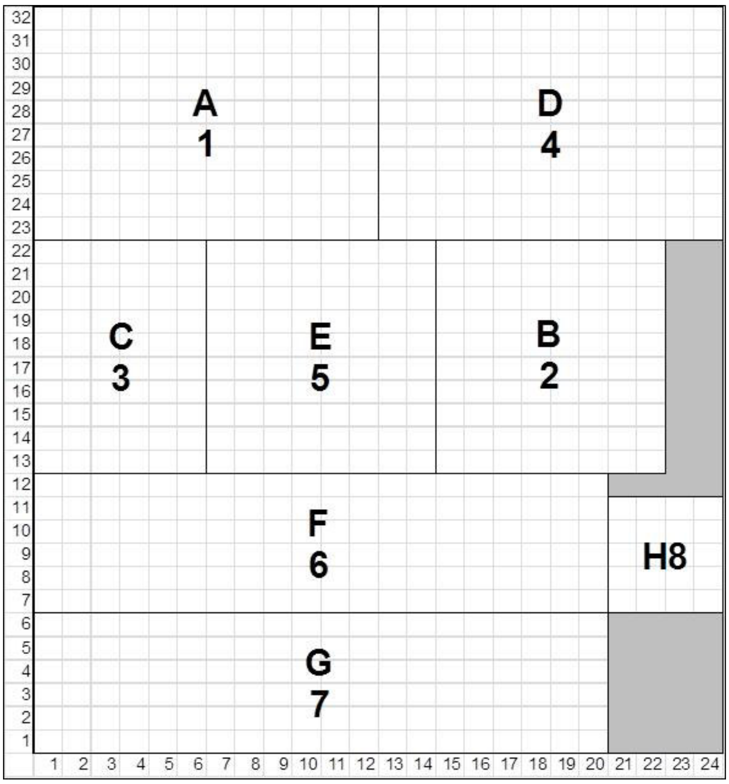

Figure 2. The Layout of Department A to H

5. Conclusion

Result from using NMIP has revealed the significance of the merit of this model; it eliminates a weak point of the classic MIP. NMIP can solve the FLP to minimize material handling cost and design the layout of department A to H without any loss of any department's required area within the boundary of the shop floor's dimension.

Suggested future works are to vary shop floor's area (B_x and B_y) by using heuristics algorithm to reduce the total area requirement, finding the relationship between shop floor's area and material handling cost, and limitation of number of departments that can be solved by this model.

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