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The Idle/Waiting Time Operator With Applications To Multistage Flow shop Scheduling To Minimize The Rental Cost Under Specified Rental Policy Where Processing Times Are Associated With Probabilities Including Transportation Time

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Abstract

This paper is an attempt to study nX3 flow shop production scheduling problems in which the processing time is associated with their respective probabilities transportation time and job block criteria. The objective of the study is to get optimal sequence of the jobs in order to minimize the rental cost using idle/waiting time operator through iterative algorithm

Keywords: key words, orkforce sizing, job-shop production, holonic model

1. Introduction

The basic study in the field of scheduling was made by Johnson (1954) who developed a polynomial time algorithm to minimize make span in two, three stage flow shop. Convey et al (1963) formulate the integer programming model for scheduling, Ignall E and Scharge (1965) applied Branch and Bound Technique in flow shop problem. Gupta and Dudek (1971) conducted an experimental study of a comprehensive performance measure in the flow shop schedule. Maggu and Das (1977) introduced the equivalent job-block concept in the theory of scheduling. Singh T.P. & Gupta Deepak (2004) made an attempt to study the optimal two stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria

Further, we have made an attempt to find a heuristic algorithm for a multistage scheduling to minimize the rental cost under a specified policy with the application of idle/waiting time operator, the processing time is associated with probabilities and the concept of transportation time is also included. The operator theorem is very useful in economical and computational point of view and gives the optimal schedule in order to minimize the total processing times of machines. The equivalent job block has many applications in the production concern, hospital management etc. where priority of one job over other becomes significant it may arise the additional cost for providing this facility.

2. Theorem

Let n jobs 1, 2, 3, …………n are processed through two machines A & B in order AB with processing time a_i & b_i (I = 1, 2, 3, ……….n) on machine A and B respectively.

If

$(a_p, b_p)\ O_{i,w}\ (a_q, b_q) = (\alpha_\beta, b_\beta)$

then

$a_\beta = a_p + \max(a_q - b_p, 0)$

and

$b_\beta = b_q + \max(b_q – a_q, 0)$

where β is the equivalent job for job block (p, q) and p, q ϵ {1, 2, 3, …………..n}.

Proof: Starting by the equivalent job block criteria theorem for β = (p, q) given by Maggu & Das (5), we have:

$a_\beta = a_p + a_q - \min(b_{p,} a_q)$ (1)

$b_\beta = b_p + b_q \min(b_{p,} a_q)$ (2)

Now, we prove the above said theorem by a simple logic:

Case I: When $a_q > b_p$

$a_q > b_p > 0$

$\max\{ a_q > b_p, 0\} = a_q > b_p$ (3)

and

$b_p > a_q < 0$

$\max\{ b_p > a_q, 0\} = 0$ (4)

(1) $a_\beta = a_p + a_q – \min(b_p, a_q)$

$= a_p + a_q – b_p$ as $a_q > b_p$

$= a_p + \max\{a_q – b_{p,} 0\}$ using (3)

(2) $b_\beta = b_p + b_q – \min(b_p, a_q)$

$= b_p + b_q – b_p$ as $a_q > b_p$

$= b_q + (b_p – b_p)$

$= b_q + 0$

$= b_q + \max(b_p – a_q, 0)$ using (4)

CASE II: When $a_q < b_p$

$a_q – b_p < 0$

$\max(a_q – a_q, 0) = b_p - a_q$

and $b_p – a_q > 0$

$\max(b_p – a_q, 0) = b_p - a_q$

(1) $a_\beta = a_p + a_q – \min(b_p, a_q)$

$= a_p + a_q – a_q$ as $a_q > b_p$

$= a_p + a_q – a_q$ as $a_q > b_p$

$= a_p + 0$

$= a_p + \max(a_q – b_p, 0)$ using (7) (9)

(2) $b_\beta = b_p + b_q – \min(b_p, a_q)$

$= b_p + b_q – a_q$ as $a_q < b_p$

$= b_p + (b_p – a_q)$

$= b_p + \max(b_p – a_q, 0)$ using (8) (10)

CASE III: When $a_q = b_p$

$a_q – b_p = 0$

$\max(a_q - b_p, 0) = 0$ (11)

also

$b_p - a_q = 0$

$\max(b_p - a_q, 0) = 0$

(1) $a_\beta = a_p + a_q - \min(b_p, a_q)$ (12)

$= b_p + a_q - a_p$ as $b_q = a_p$

$= a_p + 0$

$= a_p + \max(a_q - b_p, 0)$ (13)

(2) $b_\beta = b_p + b_q - \min(b_p, a_q)$ (12)

$= b_p + b_q - b_p$

$= b_q + (b_p - b_p)$

$= b_q + 0$

$= b_q + \max(b_p - a_q, 0)$ using (12) (14)

by (5), (6), (9), (10), (13) and (14) we conclude:

$a_\beta = a_p + a_q - \max(a_q, b_p, 0)$

$b_\beta = b_p + \max(b_p, a_q, 0)$ for all possible three cases

The theorem can be generalized for more number of job blocks as stated:

Let n jobs 1, 2, 3, ……….n are processed through two machines A & B in order AB with processing time a_i & b_i (i = 1, 2, 3, ……..n) on machine A & B respectively.

If $(a_{io}, b_{io})\ O_{i,w}\ (ai_1, bi_1)\ O_{i,w}\ (ai_2, bi_2)\ O_{i,w}$……………… $O_{i,w}\ (a_{ip}, b_{ip}) = (a_\beta, b_\beta)$

Then

$$(a_\beta = a_{io} + \sum_{J=1}^{p} \max\{a_{ij} - b_{i(j-1)}\ 0\}$$

and

$$(b_\beta = b_{ip} + \sum_{J=1}^{p} \max\{b_{i(j-1)} - a_{ij}, 0\}$$

where i_0, i_1, i_2, i_3, ……………ip є {1, 2, 3 ………..n} and β is the equivalent job for job block (i_0, i_1, i_2, i_3, ……………ip). The proof can be made using Mathematical induction technique on the lines of Maggu & Das (7).

In the light of above theorem operator $O_{i,w}$ (Idle/Waiting time Operator) is defined as follow

2.1 Defintion

Let R_+ be the set of non negative numbers. Let G = $R_+ \times R_+$. Then $O_{i,w}$ is defined as a mapping from G x G → G given by:

$O_{i,w}[(x_1, y_1), (x_2, y_2)] = (x_1, y_1)\ O_{i,w}\ (x_2, y_2)$

$= \{ x_1 + \max(x_2 - y_1, 0), y_2 + \max(y_1 - x_2, 0)]$

Where x_1, x_2, y_1, y_2 є R

2.2 Practical Situations

Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. The examination branch or a board/institute needs machine as data entry machine, computer, printer etc., on rent for computerizing & compiling examination result for secrecy point of view. Moreover in hospitals, industries concern, sometimes the priority of one job over the other Is preferred. It may be because of urgency or demand of its relative importance. Hence the job block criteria becomes significant.

3. Assumptions

1. Machine break down is not considered. This simplifies the problem by ignoring the stochastic component of the problem.
2. Jobs are independent to each other.
3. We assume rental policy that all the machines are taken on rent as and when they are required and are returned as when they are no longer required for processing. Under this policy second machine is taken on rent at time when first job completes its processing on first machine. Therefore idle time of second machine for first job is zero.
4. Pre- emption is not allowed i.e. jobs are not being split, clearly, once a job started on a machine, the process on that machine can't be stopped unless the job is completed.

3.1 Notations

S : Sequence of jobs 1,2,3,….,n

M_j : machine j, j= 1,2

A_i : Processing time of ith job on machine A.

B_i : Processing time of ith job on machine B.

$A`_i$: Expected processing time of ith job on machine A.

$B`_i$: Expected processing time of ith job on machine B.

pi : Probability associated to the processing time Ai of ith job on machine A.

q_i : Probability associated to the processing time Bi of ith job on machine B.

β: Equivalent job for job – block.

Si: Sequence obtained from Johnson's procedure to minimize rental cost.

Cj:Rental cost per unit time of machine j.

U_i:Utilization time of B (2 nd machine) for each sequence Si

t1 (Si):Completion time of last job of sequence Si on machine A.

t2(Si): Completion time of last job of sequence Si on machine B.

R(Si): Total rental cost for sequence Si of all machines.

CT(Si):Completion time of 1 st job of each sequence Si on machine A.

3.2 Problem Formulation

Let n jobs (1, 2, 3, ………….n) be processed through m machines A_j (j= 1, 2, 3, ………..m) in the order A_1, A_2, A_3……..A_m with no passing allowed. Let a_{ij} denotes the processing time job i on machine A_j with their respective probabilities p_{ij} s.t. $\sum p_{ij} = 1$ and let $t_{is\to s+1}$ denotes the transportation time of i^{th} job to transpose from A_s machine to A_{s+1} machine. Also we consider either or both of following structure relationship hold good.

$\min (a_{is}p_{is}+ t_{is\to s+1}) \geq \max (a_{is}p_{is+1}+ t_{is\to s+1})$

for (s = 1, 2, 3 ……….m – 2)

$\min \{t_{is\to s+1} + a_{i(r+1)}) p_{i(r+1)}\} \geq \max (a_{is}p_{is+1}+ t_{is\to s+1})\geq \max (a_{is}p_{ir}+ t_{ir\to r+1})$

for (r = 1, 2, 3 ……….m – 1)

The Mathematical model of the problem in the matrix form can be stated as

Job	Machine A_1		$t_{n\to2}$	Machine A_2		$t_{i2\to3}$	Machine A_3	
i	a_{ij}	p_{i1}		a_{i2}	p_{12}		a_{i3}	p_{i3}
1	a_{11}	p_{11}	$t_{11\to2}$	a_{12}	p_{12}	$t_{12\to3}$	a_{13}	p_{13}
2	a_{11}	p_{11}	$t_{21\to2}$	a_{22}	p_{12}	$t_{22\to3}$	a_{23}	p_{23}
3	a_{11}	p_{11}	$t_{31\to2}$	a_{32}	p_{12}	$t_{32\to3}$	a_{33}	p_{33}
	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--
n	a_{n1}	p_{n1}	$t_{n1\to2}$	a_{n2}	p_{12}	$t_{n2\to3}$	a_{n3}	p_{n3}

Let α = (p, q) be an equivalent job block in which job p is given priority on a job q. And we assume machine while processing the job, gets break down for a fixed interval of time (a, b) hours by virtue of government policy due to electric cut. Our objective is to find the optimal schedule of all the jobs which minimize the total idle time of each machine.

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3.3 Optimal Schedule Procedure

Optimal schedule procedure can be decomposed into the following steps:

Step 1

Define expected processing time A'_i & B'_i on machine A & B respectively as follows:

$A'_i = A_i \times p_i$

$B'_i .= B_i \times q_i$

Step 2

Define two fictitious machines G & H with processing time G_i & H_i for job I on G & H respectively, defined as:

$G_i = A'_{i1} + t_{i1\to2} + A'_{i2}$

$H_i = t_{i1\to2} + A'_{i2} + t_{i2\to3}$

Step 3

Determine equivalent jobs for each job block using operator theorem and concept of the idle/waiting time operator $O_{i,w}$ as per definition

Step 4 Using Johnson's two machine algorithm [5] obtain the sequence Si , while minimize the total elapsed time.

Step 5 Observe the processing time of 1 st job of S1 on the first machine A. Let it be α.

Step 6 Obtain all the jobs having processing time on A greater than α. Put these job one by one in the 1 st position of the sequence S1 the same order. Let these sequences be S2, S3, S4,…Sr

Step 7 Prepare in-out table for each sequence Si (i=1,2,…r) and evaluate total completion time of last job of each sequence t1 (Si) & t2(Si) on machine A & B respectively.

Step 8 Evaluate completion time CT(Si) of 1 st job of each sequence Si on machine A.

Step 9 Calculate utilization time Ui of 2 nd machine for each sequence Si as:

$U_i = t_2(S_i) - CT(S_i)$ for i=1,2,3,…r.

Step 10 : Find Min $\{U_i\}$, i=1,2,…r. let it be corresponding to i=m,then S_m is the optimal sequence for minimum rental cost.

Min rental cost = $t_1(S_m)\times C_1 + U_m \times C_2$

Where C_1 & C_2 are the rental cost per unit time of 1 st & 2 nd machine respectively

4. Numerical illustration

Consider 5 jobs and 3 machines problem to minimize the rental cost. The processing times with their respective associated probabilities are given as follows:

JOBS	MACHINE1		$t_{i\,1\to 2}$	MACHINE 2		$t_{i\,2\to 3}$	MACHINE3	
	A	P_i1		B	P_i2		C	P_i3
1	70	0.1	4	50	0.2	6	55	0.2
2	80	0.3	3	40	0.3	8	50	0.2
3	55	0.2	6	40	0.1	4	40	0.3
4	65	0.2	5	30	0.2	8	65	0.2
5	140	0.2	7	50	0.2	4	150	0.1

Obtain the optimal sequence of jobs and minimum rental cost of the complete set up, given rental costs per unit time for machines M_1 , M_2 and M_3 15, 25and 35 units respectively, and jobs (2,4) are to be processed as an equivalent group jobs

SOL. As per step 1: Expected processing time are as under:

jobs	A_i^1	$t_{i\,1\to 2}$	A_i^2	$t_{i\,2\to 3}$	A_i^3
1	7	4	10	6	11
2	24	3	12	8	10
3	11	6	4	4	12
4	13	5	6	8	13
5	28	7	10	4	15

As Per Step 2, we defined two fictitious machines Gi and H_i

Jobs	G_i	H_i
1	27	31
2	47	33
3	25	26
4	32	32

5	49	36

As Per step 3: The expected processing time for the equivalent job on fictitious machine

$(a_p, b_q)\ O_{i,w}\ (a_q, b_q) = (a_\beta, b_\beta)$

$= \{(a_p + \max(a_q - b_p, 0), (b_q + \max(b_p - a_q, 0)\}$

$\{(a_2+\max(a_4-b_2, 0), (b_4+\max(b_2-a_4, 0)\}$

$\{47+\max(32-33, 0), 32+\max(33-32, 0)$

$(47, 32+1)$

$(47, 33)$

As per step4:

Jobs	G_i	H_i
1	27	31
α	47	33
3	25	26
5	49	36

As per step 5, using Johnson technique, we make a sequence S_1

S_1=3,1,5,α

S_1=3,1,5,2,4

Other optimal sequences for minimize rental cost, are

S_2=1, 3,5,2,4

S_3= 5, 3,1,2,4

S_4=2, 4,3,1,5

As per step we prepare in-out table for Si (i=1, 2, 3…r)

In- out table for sequence S_1= S_1=3, 1,5,2,4

Jobs	MACHINE 1		$ti_{1\to2}$	MACHINE 2		$ti_{2\to3}$	MACHINE3	
	IN	OUT		IN	OUT		IN	OUT
3	0	11	6	16	20	4	24	36
1	11	18	4	22	32	6	38	49
5	18	46	7	53	63	4	67	82
2	46	70	3	73	85	8	93	103
4	70	83	5	88	108	4	112	125

Total elapsed time= 125

Total idle time for machine 1 = 125-83 = 42

Total idle time for a machine 2 =125-52=73

Total idle time for a machine 3 =61

In- out table for sequence S_2=1, 3,5,2,4

Jobs	MACHINE 1		$ti_{1\to2}$	MACHINE 2		$ti_{2\to3}$	MACHINE3	
	IN	OUT		IN	OUT		IN	OUT
1	0	7	4	11	21	6	27	38
3	7	18	6	24	28	4	38	50
5	18	46	7	53	63	6	67	82
2	46	70	3	73	85	8	93	103
4	70	83	5	88	94	8	102	115

Total elapsed time=115

Total idle time for a machine 1 =115-83=32

Total idle time for a machine 2 =115-42=73

Total idle time for machine 3 =115-54 =61

In- out table for sequence S_3=5, 3,1,2,4

Jobs	MACHINE 1		$ti_{1\to2}$	MACHINE 2		$ti_{2\to3}$	MACHINE3	
	IN	OUT		IN	OUT		IN	OUT
5	0	28	7	35	45	4	49	64
3	28	39	6	45	49	4	64	76
1	39	46	4	50	60	6	76	87
2	46	70	3	73	85	8	93	103
4	70	83	5	88	98	4	104	119

Total elapsed time=117

Total idle time for a machine 1 =117-83=34

Total idle time for a machine 2 =117-52=65

Total idle time for machine 3 =117-54=61

In- out table for sequence S_4=2, 4,3,1,5

Jobs	MACHINE 1		$ti^{1\rightarrow 2}$	MACHINE 2		$ti^{2\rightarrow 3}$	MACHINE3	
	IN	OUT		IN	OUT		IN	OUT
2	0	24	3	27	39	8	47	57
4	24	37	5	42	48	8	57	70
3	37	48	6	54	58	4	70	82
1	48	55	4	59	69	6	82	93
5	55	83	7	90	100	4	104	119

Total elapsed time=119

Total idle time for a machine 1 =119-83=36

Total idle time for a machine 2 =119-58=61

Total idle time for machine 3 =117-57=62

The total utilization of machine 1 is fixed 83 units and minimum utilization time of machine 2 and 3 are 61, 62 for sequence S_4.

Therefore the optimal sequence is **S_4**=2, 4,3,1,5

Total Rental Cost =83X15 +61 X 25 +35 X 62 =4940 Units.

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