

Development of a Fourier Series Forecasting Model for Predicting the Sales Volume of Selected Manufacturing Company

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Abstract
The need for proper planning in a production company is pivotal; hence, proper prediction of forecasting is required to facilitate effective planning. In solving this forecasting problem, model is required to be developed to meet with specialized trend of occurrence for the output value. In this paper, a Fourier series forecasting model was developed for predicting the future sales volume of a plant /production company. A Fourier series of cosine odd function was used coupled with a straight line equation for the model development. The data were collected from Honda motorcycle sales outlet in Apapa, Lagos State, Local bottled water factory in Ado – Ekiti and a local bottling company culled from the Journal of National Institute of Engineering Management. The model was developed based on the trend of occurrence which is sinusoidal in nature. From testing and validation of the model, the result shows a high degree of dependability and correlation of 0.98 between the data of model developed and data collected from companies. Finally, the developed forecasting model can be used for budget preparation, production planning and sales budgeting in a Plant / Production Company.
Keywords: Forecasting, Sales Volume, Fourier series, sinusoidal, model

INTRODUCTION

Sales Forecasting is the basic planning document of an enterprise which establishes the level of output of the company and determines other company operating parameters such as financial and manpower requirements, raw materials and product inventory levels, operating costs and ultimately operation profits. The core of establishing a standard performance in an enterprise depends on the ability of the firm to successfully plan and to effectively carry it out. The evaluation of sales performance seeks to measure the actual sales volume of the sales department against the predetermined (forecasting) sales volume. A key factor in the success of every organization is the ability to measure its sales volume and predict future sales volume for effective coordination of the company (Aquilano *et al*, 1995). Bird (1998) observed that one of the most important aims of a standard assessment is to improve performance on sales volume and therefore productivity. While stressing the importance of evaluation, Sartain and Baker (1962) mentioned that unless we evaluate our efforts, compare actual with scheduled events, and generate necessary change to reverse undesirable trends, our hope for managerial success remains just a hope. The manager, through the controlling function, ensures that enterprises objectives are being accomplished as planned. In carrying out the planning systematically, the issue of evaluation with respect to future demand (forecasting) has to be carefully dealt with due to the importance attached to it such as minimizing cost of inventory, storages, production and others.

Oladebeye and Ejiko (2007) defined forecasting as a technique (basic planning document) for translating past experience to production of things to come which requires making estimate of the magnitude and significance involving relative and absolute forces that will influence the future of an enterprise such as the financial and man-power requirements, raw materials and product inventory levels, operating cost and ultimately operation profits (company operating parameter). Forecasting may be short, medium or long-term forecast depending on the horizon for the decision making process. This derivative prediction is based on either executive opinion or theoretical analysis, in some cases both may be utilized.

Forecasting sales volume is inherently more difficult than the construction of the subsequent sales budget. Although management exerts some degree of control over expenditures, it has little ability to direct the buying habits of individuals. The level of sales depends on the degree of the vagaries of the marketplace. Nonetheless, a sales volume forecast must attain a reasonable degree of reliability to be useful. There are a variety of forecasting techniques and methods from which the small business owner may choose. Not all of them are applicable in every situation. To allow for adequate forecasting, a business must choose those methods which best serve their purposes, utilize accurate and relevant data, and formulate honest assumptions appropriate to the market and product. Sales volume forecasts may be general if they calculate aggregate sales attainable in an industry. Conversely, forecasts may be very specific, detailing data by individual products, sales territories, types of customers, and so forth in recent decades, market analysis have increased their use of focus groups, individual surveys, interviews, and sophisticated analytical techniques aimed at identifying specific markets. A company does, however, maintain control over its production lines, prices, advertising and marketing, and the size of its sales force. After studying the underlying causes and variables in depth, the analyst uses a variety of mathematical techniques to project future trends. On the basis of these projections, management derives its sales volume forecast.

Analysts employ the indirect method by first projecting industry sales volume. From this data they project the company's share of the industry total. The direct approach, however, skips the industry projection with a straight forward estimate of sales volume for the company. Either of these methods is applicable to the causal and non- causal approaches (Waddell *et al*, 1994).

Fourier series original motivation was to solve the heat equation; it later became obvious that the same techniques could be applied to a wide array of mathematical and physical problems. The Fourier series has many applications in electrical engineering, vibration analysis, acoustics, optics, signal processing, and image processing, quantum mechanics, econometrics [Marc *et al*, 1995]. This work is aimed at developing a mathematical forecasting model for predicting the future sales volume of a plant/production company. The model is generated through the integration of a Fourier series of cosine function with a straight linear equation as this will be useful in the prediction of the future sales volume of any product or goods with a sinusoidal fluctuation in sales volume.

METHODOLOGY

The data collected from the sales department of Honda Motor Cycle sales outlet in Apapa township area of Lagos serve as the basis for the model formulation. The sales data so collected were graphically represented and observed. The graphical representation served as the function for selection of the forecasting model to be utilized. A least square method in conjunction with time series analysis was chosen for generating the trend of the sales forecast. The seasonal fluctuation observed in the data collected was addressed with time series regression model while the difference between actual sales volume and sales forecast was compared. Correlation coefficient(r) was computed to determine the degree of agreement between forecast sales volume (targets or quotas) and actual sales. Finally, a straight line trend equation from least square method was integrated with a sinusoidal equation (Fourier series function for cosine x) for our model which take care of both the upward and seasonal fluctuation in actual sales.

DEVELOPMENT OF A FOURIER SERIES FORECASTING MODEL.

From the graph in fig 2 and data represented in Table 1, the first period gives a

Upper limit = 28000 represent 'd' and Lower limit = 4000 represent 'b'

From the observed graph the occurrence of sales tends to replicate itself in an upward trend. The maximum point for the first periods is 'd' and minimum point is 'b'.

The graph shows a sinusoidal function; therefore, for continuous modeling of the system. The cosine x was integrated using Fourier series method. The equation as given by Enrique, 1992 is $\int \cos x \, dx$

2.0

where, x = (-3,-2,-1,0,1,2,3) , y = (0,-1,0,1,0,-1,0)

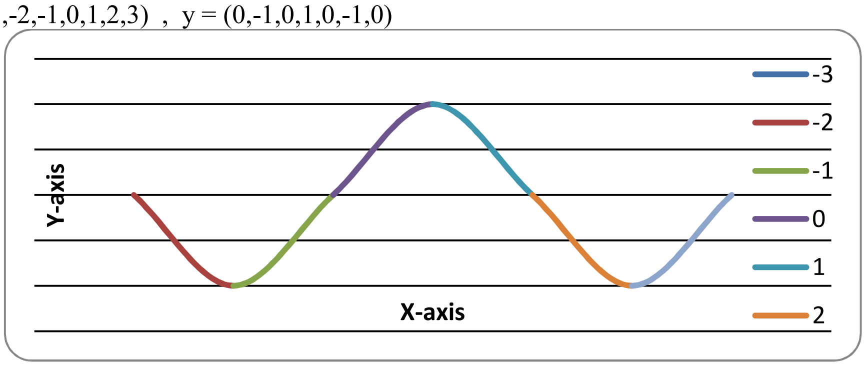

Fig.1: GRAPH OF COSINE x FUNCTION OF FOURIER SERIES

The upper boundary = 1 and lower boundary = -1 on y-axis

In this section, $f(x)$ denotes a function of the real variable x. This function is usually taken to be periodic, of period 2π, which is to say that $f(x + 2\pi) = f(x)$, for all real numbers x. We will attempt to write such a function as an infinite sum, or series of simpler 2π–periodic functions. We will start by using an infinite sum of sine and cosine functions on the interval [−π, π], as Fourier did.

Fourier's formula for 2π-periodic functions using sines and cosines

For a periodic function $f(x)$ that is integral on [−π, π], the numbers

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\partial x$, n ≥ 0 and $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\partial x$, n ≥ 1 are called the Fourier coefficients of f. One introduces the partial sums of the Fourier series for f, often denoted by (SnF) = $\frac{a_0}{2} + \sum_{n=1}^{N} [a_n \cos(nx) + b_n \sin(nx)]$, $N \geq 0$.

The partial sums for f are trigonometric polynomial. One expects that the function (SnF) approximate the function f, and that the approximation improves as N tends to infinity. The infinite sum

$$\frac{a_0}{2} + \sum_{n=1}^{N} [a_n \cos(nx) + b_n \sin(nx)]$$

is called the Fourier series of f(Stroud, 1996).

The Fourier series does not always converge, and even when it does converge for a specific value x_0 of x, the sum of the series at x_0 may differ from the value $f(x_0)$ of the function. It is one of the main questions in harmonic analysis to decide when Fourier series converge, and when the sum is equal to the original function. If a function is square-integral on the interval [−π, π], then the Fourier series converges to the function at almost every point. In engineering applications, the Fourier series is generally presumed to converge everywhere except at discontinuities, since the functions encountered in engineering are better behaved than the ones that mathematicians can provide as counter-examples to this presumption. In particular, the Fourier series converges absolutely and uniformly to f(x) whenever the derivative of f(x) (which may not exist everywhere) is square integral (Walter, 1976).

Note: - For odd number. Using Fourier function.

$a_0 = 0$, $a_n = 0$ $b_n = 1$ The function therefore is

$$f(x) = a_0 + \sum_{n=i=1}^{\alpha \leftrightarrow 0} (a_n \cos nx + b_n \sin x)$$

$$f(x) = 0 + \sum_{n=i=1}^{\alpha \leftrightarrow 0} (0 \cos nx + \sin x)$$

$$f(x) = 0 + \sum_{n=i=1}^{\alpha \leftrightarrow 0} (\sin nx) \qquad (2.1)$$

This function gives an upper and lower boundary of +1 and -1 which follows sales format but deficient of the magnitude and the linear trend.

For straight line equation,

$$Y = mx + c \qquad (2.2)$$

where, C = constant

m = sales intercept or slope of the graph

To introduce the magnitude, the difference between the upper and lower limit should be estimated.

This occurs between the 2nd and fourth terms which are 'd' and 'b'

Therefore, the mid- point value will be

$$= \frac{(d - b)}{2} \qquad (2.3)$$

Therefore the range at which the lower and upper boundary will occur will be about c, determined from the equation below

$$c = \left(\frac{d - b}{2}\right) \sum_{n=i=\theta}^{\infty \leftrightarrow 0} \sin nx \qquad (2.4)$$

Where i = θ = 0, π/2, π, $3\pi/2$, ...∞

This constant falls within the positive and negative range. To bring the constant magnitude to the positive portion of the graph, a magnitude greater than the midpoint value will be added to balance the negative effect.

Let the value be "U"

Therefore $u + y = d$ (2.5)

$u - y = b$ (2.6)

Combining the simultaneous equations 2.5 and 2.6 'U' becomes

$$U = \frac{d + b}{2} \quad (2.7)$$

$$So, \; c = u + (\frac{d - b}{2}) \sum_{n=i=0}^{\infty \leftrightarrow 0} \sin nx \quad (2.8)$$

$$c = (\frac{d + b}{2}) + (\frac{d - b}{2}) \sum_{n=i=0}^{\infty \leftrightarrow 0} \sin nx \quad (2.9)$$

Therefore, the straight line equation is

$$Y = mx + c$$

$$Y = mx + (\frac{d + b}{2}) - (\frac{d - b}{2}) \sum_{n=i=0}^{\infty \leftrightarrow 0} \sin nx \quad (2.10)$$

Where, m = slope of observed graph

$$m = \frac{e - a}{x_5 - x_1} \quad (2.11)$$

Hence, the developed model is

$$Y_m = [\frac{e - a}{x_5 - x_1}]x + (\frac{d + b}{2}) - (\frac{d - b}{2}) \sum_{n=i=0}^{\infty \leftrightarrow 0} \sin nx \quad (2.12)$$

From the observed graph in fig.2,

a = 15200	e = 19815	$x_1 = 1$
d = 28000	b = 4000	$x_5 = 5$

The slope $m = \frac{e - a}{x_5 - x_1}$

$$m = 1153.75 \approx 1153.80$$

So,

$$Y = 1153.8x + (\frac{2800 + 4000}{2}) - (\frac{2800 - 4000}{2}) \sum_{n=i=0}^{\infty \leftrightarrow 0} \sin nx$$

$$Y_m = 1153.8x + 1600 - 12000 \sum_{n=i=0}^{\infty \leftrightarrow 0} \sin nx \quad (2.13)$$

Therefore, the Fourier series - straight line equation forecasting model is Y_M

Y_M =Developed Model Sales Volume Forecast. With Ya, C and Yc which are Actual Sales Volume, Multiplication Factor for Forecast and Trend Theoretical Sales Forecast Volume respectively are given in Table 2 to estimate the Actual Theoretical Sales Volume Forecast (Y_t) and the coefficient of correlation, r thus determined from equation 2.14 as given by Oladebeye and Ejiko, 2007 is 0.9823 (Very high correlation).

$$r = \sqrt{1 - \frac{\sum (y_a - y_c)^2}{\sum \left(y_a - \bar{y}_c \right)^2}} \qquad (2.14)$$

RESULTS AND DISCUSSION

The primary objective of this research project is to develop a forecasting model that will be able to forecast the future sales of a product having seasonal fluctuation in their sales volume which is as a result of fluctuation in the demand from the consumer of the product. Based on the data collected from the HONDA Motorcycle sales outlet in Apapa and the interaction with the sales manager it was observed that accurate forecasting has not been practised. Fig. 3 shows the representations of actual sales volume (Y_a) against the model sales volume (Y_m) and the theoretical sales volume (Y_t). The model sales volume was found to be closely related to the actual sales volume than that of the theoretical sales volume. To further confirm the veracity of the developed model the correlation coefficient for the developed model and the theoretical sales volume was found to be 0.98 and 0.79 respectively from the data represented in Table 2. The forecasting trends for year 2002 to 2007 of HONDA Motorcycle sales outlet show a continual increment in a sinusoidal format. The quarterly percentage increment decreases as the year progresses with an average of 42%, 30%, 23%, 19%, 16% and 14% respectively for 2002 to 2007. Across the year the sale volume ratio of 1.7, 1, 1.85 and 2.7 are usually observed indicating that the minimum sales are experience in the second quarter while the maximum sales is at the forth quarter. The developed model as a cumulative percentage error of 1.93 as against 4 of the theoretical model forecast to the actual sales volume for year 2006 and 2007 as shown in Table 2. This signifies that the developed model is highly dependable than the previous theoretical model. The data in Tables 3 and 4were collected from a local bottled water plant and Table 5 presents individually calculated data culled from Lawal, 2002 a local bottling company. The accuracy of the developed model Y_m was validated and tested which shows high correlation between the manually calculated result of 0.98 for the data of the Honda motorcycle sales outlet. Table 5 shows a comparison between the result of the developed model for the actual sale volume forecast (Y_m), the time series analysis (F_t) and the actual sales volume (Y_a) culled from Lawal, 2002. The result for the first quarter of year 1996, 1997, 1998 and 1999 show developed model forecasts of 45166, 53321, 61476 and 69632 while the time series analysis result given were 49324, 62439, 73313, 80222 for the actual sales volume for the quarter is 56399, 64912,66046, and 75483 respectively. The percentage cumulative error for the last five years of the time series forecast and the developed model values are 5.63% as against 2.56%. This shows that the model developed is more suitable and dependable, because the results are closer with relatively little error when compared with the actual sales volume. Table 6 shows the actual sales volume of the bottle water in year 2009 and the first quarter of 2010 of 5,161 as against 5,787 of time series model and 5,459 of the developed model which implies an average of 42 bottles that were not sold as against an average of 20 bottles per month when the develop model is applied.

The result for the bottled water plant shows a high correlation value of 0.62; while the correlation obtained from time series analysis gives 0.62. This indicates a moderate value of correlation being estimated from Table 6.

CONCLUSION

The forecasting model developed is for product having seasonal fluctuation to be accurately predicted. This model will help its users to make a reliable decision as to actualize the volume of goods to be produced, Volume of raw materials to be purchased and, Planning scheduling of operation or processes. Inaccurate forecasting can lead to waste of resources, shortage in supply and excessive production rate, thereby leading to reduction in profitability.

The model developed has been tested and validated with some data collected and the result compared with other forecasting model using the same data. The testing and validation shows that the developed model has high correlation with the actual sales; this high level of correlation displayed by the developed model shows its level of reliability, dependability and effectiveness. For utmost performance of this model, it is important for the user to ensure that the actual quartered sales volume is sinusoidal in nature when plotted on the graph before the data are used. The model is developed for products that are seasonal in demand; from observation, the non-seasonal goods will give forecast values which are not dependable when used with the model. The use of this model will enhance effective forecasting, economical utilization of resources and it is time saving, hence, increases profitability. The model is recommended as a means for preventing deficit in forecasting sales of a company with goods having seasonal fluctuation because it is reliable, time saving and efficient. The model developed is highly suitable for small /medium and large scale enterprises whose sales outputs are sinusoidal in nature.

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Table1: Parameters for Estimating Forecasting Model

Generalized forecasting model (Y_n	Localized forecasting model (Motorcy estimate Y_{Lm})	Localized forecasting model (Bo water estimate Y_{LB})
First term "a"	15200	682
Second term "b"	4000	615
Fourth term "d"	28000	848
Fifth term "e"	19815	826
Quarter variable " x_5 "	5	5
Quarter variable " x_1 "	1	1

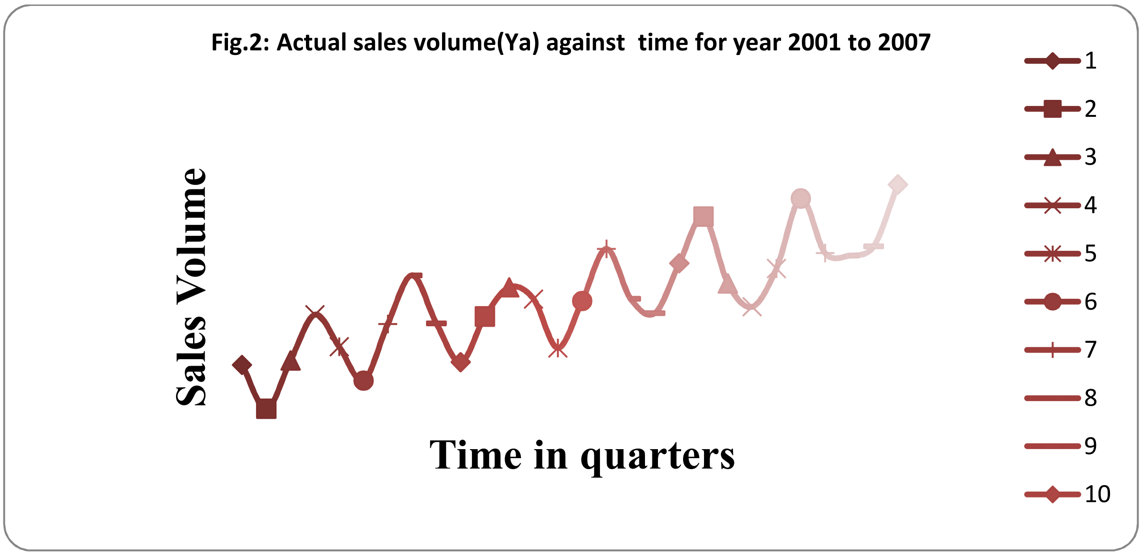
Fig.2: Actual sales volume(Ya) against time for year 2001 to 2007
Sales Volume
Time in quarters
1
2
3
4
5
6
7
8
9
10

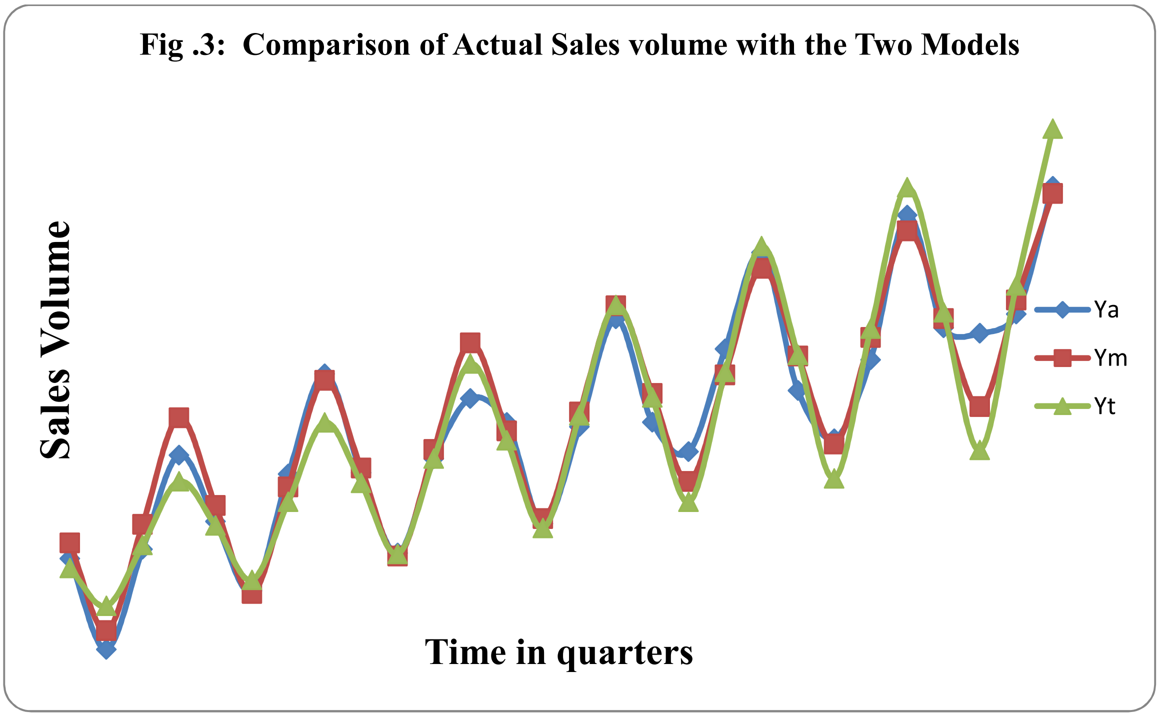
Fig .3: Comparison of Actual Sales volume with the Two Models
Sales Volume
Time in quarters
Ya
Ym
Yt

Table 2: Comparison of Quarterly Forecast Time Series Analysis and Developed Model with Actual Quarterly Sales Volume for Year 2001 To 2007.

X	Y_M	y_a	C	y_C	Y_t
1	17,153.8	15200	0.997	14053	14011
2	6307.8	4000	0.608	15374	9347
3	19461.4	16353	1.011	16695	16877
4	32615.2	28000	1.374	18016	24754
5	21769	19815	0.997	19337	19279
6	10922.8	11205	0.608	20658	12560
7	24076.6	25634	1.011	21979	22221
8	37230.4	38041	1.374	23300	32014
9	26384.2	25798	0.997	24621	24547
10	15538	15900	0.608	25942	15773
11	28691.8	27550	0.011	27263	27563
12	41845.6	34997	1.374	28584	39274
13	30999.4	32008	0.997	29905	29815
14	20153.2	19526	0.608	31226	18985
15	33307	31500	1.011	32547	32905
16	46460.8	44806	1.374	33868	46535
17	35614.6	32040	0.997	35189	35083
18	24738.4	28412	0.608	36510	22198
19	37922.2	41108	1.011	37831	38247
20	51076	53001	1.374	39152	53795
21	40229.8	35982	0.997	40473	40352
22	29383.6	30015	0.608	41794	25111
23	42537.4	39784	1.011	43115	43589
24	55691.2	57618	1.374	44436	61055
25	44845	43724	0.997	45757	45620
26	33998.8	34052	0.608	47078	28623
27	47526	45424	1.011	48399	48931
28	60306.4	61202	1.374	49720	68315

Table 3: Actual Sales Volume from a Local Bottled Water Plant

Months	Year 2007	Year 2008	Year 2009	Year 2010
January	178	275	373	478
February	118	319	381	213
March	386	232	373	361
April	139	615	351	468
May	244	351	199	326
June	232	365	297	
July	205	359	183	
August	364	418	329	
September	222	257	238	
October	213	506	380	
November	405	303	493	
December	230	643	512	

Table 4: Yearly Quarter from Bottled Water Plant for year 2007 to 2010

Quarter	Year 2007	Year 2008	Year 2009	Year 2010
Jan-Mar	682	826	1127	1052
Apr-Jun	615	1331	847	
Jul-Sept	791	1034	750	
Oct-Dec	848	1452	1385	

Table 5: Local Bottling Company Sales Volume for year 1995 - 1999

Years	Quarters	Actual sales (Y_a)	Target forecast (Y_t)	Developed model forecast (Y_m)	Comment on developed model forecast.
1995	1	32696	-	37011	Low
	2	26712	-	30790	Low
	3	26827	22530	41088	Low
	4	43232	50676	51387	Low
1996	1	56399	49325	45166	Very Good
	2	42890	43538	38945	Very Good
	3	28948	31958	49244	Low
	4	67934	65320	59542	Very Good
1997	1	64912	62439	53321	Very Good
	2	49236	53991	55360	Low
	3	38319	37636	57399	Low
	4	83039	76972	67698	Very Good
1998	1	66046	73313	61476	Very Good
	2	65831	60811	55255	Very Good
	3	38412	42229	65554	Low
	4	89873	85394	75853	Very Good
1999	1	75483	80222	69632	Very Good
	2	70107	67096	63410	Very Good
	3	44966	46671	73709	Low
	4	95584	94970	84008	Very good
		∑=1107446	∑=1045091	∑=1135848	

Table 6: Comparison between time series analysis method and the developed model Y_m of the sales volume forecast with the actual sales volume of bottled water plant.

Year	Quarters	Actual sales(Y_a)	Forecast Y_c .C	Developed model (Y_m)
2007	1	682	738	768
	2	615	781	687
	3	791	824	840
	4	848	867	992
2008	1	826	910	991
	2	1331	953	831
	3	1031	996	984
	4	1452	1039	1136
2009	1	1127	1082	1056
	2	847	1125	975
	3	750	1168	1128
	4	1385	1211	1180
2010	1	1052	1201	1120
	2		1269	1119
	3		1178	1272
	4		1622	1424
2011	1		1344	1366
	2		1438	1263
	3		1329	1416
	4		1825	1568