Industrial Engineering Letters
ISSN 2224-6096 (Paper) ISSN 2225-0581 (online)
Vol.5, No.2, 2015

An Inventory Model with Periodic Demand, Constant Deterioration and Shortages

Manish Pande
Patel College of science & Technology, Ralamandal, Indore (MP)
Email:manishpandeind@gmail.com

Dr. S. S. Gautam
MGCGV University, Chitrakoot (MP)
Email:ssgcgv@rediffmail.com

Dr. N. P. Katyar
Patel College of Science and Technology, Indore (MP)
Email:np_katyar@rediffmail.com

Abstract
In this paper, an inventory model with periodic demand, constant deterioration and shortages has been proposed. The periodic demand rate increases by a constant percentage during each time interval. The proposed model will be constrained binomial geometric programming model with constant deterioration and shortage. This implies that for any exponentially growing quantity, the larger the increase in quantity, the faster it grows. A numerical analysis of the proposed model has been presented. Production rate is considered as finite and approximation procedure is used to solve the model.
Keywords: Deterioration, shortage, Inventory Control, Demand Rate, periodic, Exponential Growth.

INTRODUCTION

In real life the harvest of food grains like paddy, wheat etc. is periodic. As there are a large number of landless people in the rural area of India, there will be a constant demand of these food grains throughout the year. In the last few years inventory problem involving time variable demand pattern received attention of several researchers. Stanfel and Sivazlian (1975) discussed a finite horizon inventory problem with variable demand. Donaldson (1977) first solved the classical no shortage inventory problem analytically with linear trend in demand over a finite time horizon. However, the solution procedure requires a lot of computation time and can not be employed to determine the values of the optimal decision variable. To remove this difficulty many other researchers have proposed various other techniques to solve the same inventory problem. In the recant years Goyal et. al(1992), Dave (1989) and Datta and Pal (1992) developed models with shortages taking time proportion demand. Among these the solution procedure of Goyal et. al(1992), Dave (1989) were computationally complicated, but Datta and Pal (1992) procedure was very simple and easy to calculated the values of the decision variables. They assumed that the successive replenishment cycles were diminishing by a constant amount. In these papers the possibilities of deterioration in inventory were not taken into consideration. Dave and patel (1981) were the first to develop a no shortage inventory model for deteriorating items with time dependent demand. This model was extended by Sachan (1984) considering the backlogged shortage option. Next Haiping and Wang (1990) developed the same type of model for the prescribed finite time horizon without considering shortage. They solved the problem using dynamic programming method. Goswanmi and Chaudhuri (1991) developed an inventory model with shortage for an increasing time depended demand by taking equal cycle length of successive replenishment. However in their analysis the expressions for the holding cost and lot size per replenishment have been derived wrongly by them due to which the total cost for the system and solution are not correct.

ASSUMPTION AND NOTATIONS

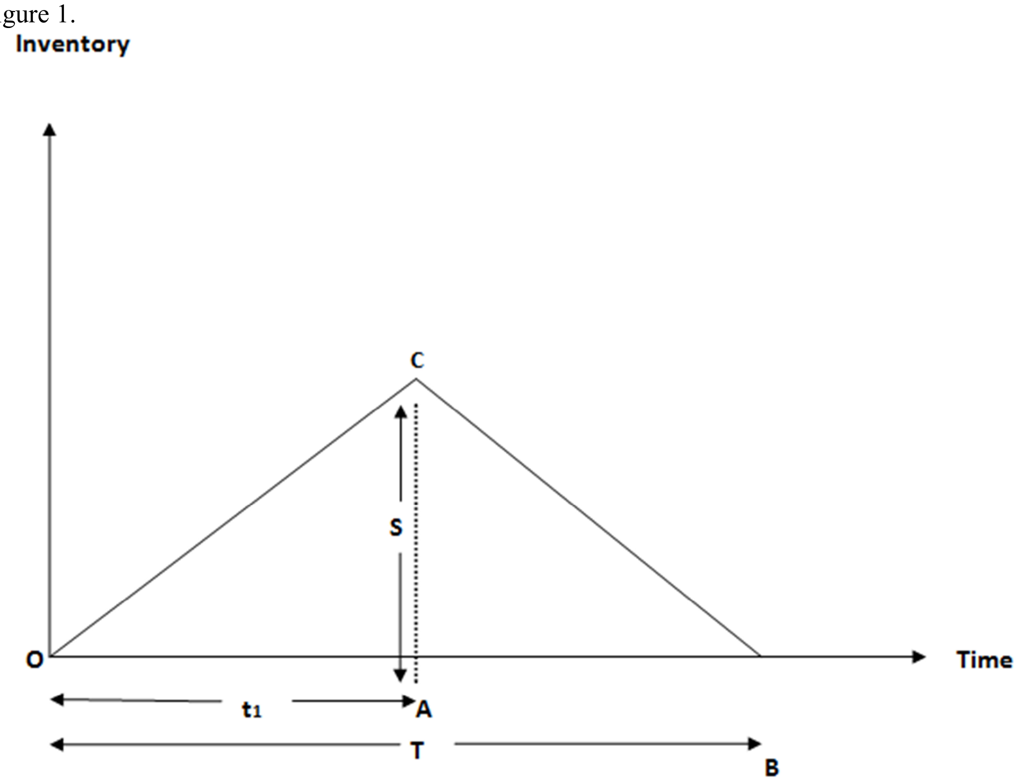
The proposed deterministic inventory model is developed under the following assumption and notations.

(1) Replenishment rate is finite.
(2) The lead time is zero.
(3) T is the fixed duration of each production cycle.
(4) θ is the constant rate of deterioration.
(5) $q(t)$ is the inventory level at any time t.
(6) S is the initial inventory after fulfilling back orders.
(7) D is the total amount of deteriorated units.
(8) K is the average total cost.

(9) C is the cost of each unit.
(10) C_1 is the holding cost per unit per unit time.
(11) C_2 is the shortage cost per unit per unit time.
(13) Q is the total inventory produced at the beginning of each period.

MATHEMATICAL FORMULATION AND ANALYSIS FOR THE SYSTEM

Let $q(t)$ is the inventory level at any time$(0 \le t \le \pi)$. We consider Q as total and Sis the initial inventory after fulfilling backorders. Inventory level gradually decreases during time $(0, \frac{\pi}{2})$ due to market demand and deterioration and ultimately falls to zero at$\frac{\pi}{2}$. Shortage occurs during period$(\frac{\pi}{2},\pi)$which are fully backlogged. The cycle then repeats itself. We have to determine optimal value of S. this model has been shown graphically in figure 1.



The differential equation governing the system in time interval $(0,\pi)$ are given by

$$\frac{dq(t)}{dt} + \theta q(t) = -asint \qquad 0 \le t \le \frac{\pi}{2} \quad \text{.........} (1)$$

$$\frac{dq(t)}{dt} = -asint \qquad \frac{\pi}{2} \le t \le \pi \quad \text{......} (2)$$

Equation (1) is a linear differential equation.
Its solution is given by

$$q(t)e^{\theta t} = -\int asinte^{\theta t}\ dt + c$$

$$q(t)e^{\theta t} = \frac{a}{1+\theta^2}[cost - \theta sint] + c$$

$$q(t) = \frac{ae^{-\theta t}}{1+\theta^2}[cost - \theta sint] + c.e^{-\theta t} \quad \text{..........} (3)$$

where c is the constant of integration. Solution of equation (2)is

$$q(t) = -a\int sint\ dt + c$$

$$q(t) = acost + c' \quad \text{.........} (4)$$

where c′ is the constant of integration

Applying the boundary condition q (t) =S at t=0 in equation (3) we get

$$S = \frac{a}{1+\theta^2}[1] + c$$
$$c = S - \frac{a}{1+\theta^2}$$

Equation (3)is reduces to

$$q(t) = \frac{ae^{-\theta t}}{1+\theta^2}[cost - \theta sint] + \left(S - \frac{a}{1+\theta^2}\right).e^{-\theta t}$$

$$q(t) = Se^{-\theta t} + \frac{ae^{-\theta t}}{1+\theta^2}[cost - \theta sint] - \frac{a}{1+\theta^2}e^{-\theta t} \quad \ldots\ldots\ldots\ldots (5)$$

Applying the boundary condition at $t = \frac{\pi}{2}$ at $q(t) = 0$ in equation (4)we get

$$0 = 0 + c'$$
$$or \;\; c' = 0$$

Therefore,

$$q(t) = acost \qquad \frac{\pi}{2} \le t \le \pi \quad \ldots\ldots\ldots (6)$$

$$Now \; since \; at \; t = \frac{\pi}{2} \quad q(t) = 0$$

so equation (5)reduces to

$$0 = Se^{-\theta\frac{\pi}{2}} + \frac{ae^{-\theta\frac{\pi}{2}}}{1+\theta^2}\left[-\frac{\pi}{2}\right] - \frac{a}{1+\theta^2}e^{-\theta\frac{\pi}{2}}$$

therefore

$$S = \frac{a}{1+\theta^2}\left[1 + \frac{\pi}{2}\right] \ldots\ldots\ldots\ldots\ldots (7)$$

$$s = \frac{a(2+\pi)}{2(1+\theta^2)}$$

Hence deteriorated amount of inventory will be

$$D = S - \int_0^{\frac{\pi}{2}} a \, sint \, dt$$
$$= S + a$$
$$= a\left[\frac{a(2+\pi)}{2(1+\theta^2)} + 1\right]$$
$$= a\left[\frac{(2+\pi) + 2 + 2\theta^2}{2(1+\theta^2)}\right]$$
$$= \frac{a}{2}\left[\frac{(4+\pi)+2\theta^2}{2(1+\theta^2)}\right] \quad \ldots\ldots\ldots\ldots \quad (8)$$

Total average number of units in inventory during $(0, \frac{\pi}{2})$

$$q_1(t_1) = \frac{1}{T}\left[\int_0^{\frac{\pi}{2}} q(t)dt\right] = \frac{1}{T}\left[\int_0^{\frac{\pi}{2}}\left\{Se^{-\theta t} + \frac{ae^{-\theta t}}{1+\theta^2}[cost - \theta sint] - \frac{a}{1+\theta^2}e^{-\theta t}\right\}dt\right]$$

$$= \frac{1}{T}\left[\frac{Se^{-\theta t}}{-\theta} + \frac{a}{(1+\theta^2)}\left\{\frac{e^{-\theta t}}{(1+\theta^2)}[(sint - \theta cost)] + \frac{\theta e^{-\theta t}}{(1+\theta^2)}(cost + \theta sint)\right\} + \frac{ae^{-\theta t}}{\theta(1+\theta^2)}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{T}\left[\frac{Se^{-\theta t}}{-\theta} + \frac{ae^{-\theta t}}{(1+\theta^2)^2}[(1+\theta^2)sint] + \frac{ae^{-\theta t}}{\theta(1+\theta^2)}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{T}\left[\frac{Se^{-\theta t}}{-\theta} + \frac{ae^{-\theta t}}{(1+\theta^2)}[sint] + \frac{ae^{-\theta t}}{\theta(1+\theta^2)}\right]_0^{\frac{\pi}{2}}$$

Using equation (7) above equation become

$$q_1(t_1) = \frac{1}{T}\left[\frac{a(2+\pi)}{2(1+\theta^2)}\frac{e^{-\theta t}}{-\theta} + \frac{ae^{-\theta t}}{(1+\theta^2)}[sint] + \frac{ae^{-\theta t}}{\theta(1+\theta^2)}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{ae^{-\theta t}}{T(1+\theta^2)}\left[sint + \frac{1}{\theta} - \frac{(2+\pi)}{2\theta}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{ae^{-\theta t}}{T(1+\theta^2)}\left[\frac{2\theta sint + 2 - 2 - \pi}{2\theta}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{ae^{-\theta t}(2\theta sint - \pi)}{2T(1+\theta^2)\theta} \quad \dots\dots\dots\dots\dots\dots\dots (9)$$

Also average number of units in shortage during $\left(\frac{\pi}{2}, \pi\right)$ *are*

$$q_2(t_1) = \frac{1}{T}\int_{\frac{\pi}{2}}^{\pi} q(t)dt \;= \frac{1}{T}\int_{\frac{\pi}{2}}^{\pi} a\, cost\, dt$$

$$= \frac{1}{T}\,[a\, sint]_{\frac{\pi}{2}}^{\pi}$$

$$= \frac{a}{T}\left[sin\pi - sin\frac{\pi}{2}\right] \;= \frac{-a}{T} \dots\dots\dots\dots\dots.. (10)$$

In equation (10) *negative sign shows the shortage.*
Finally average total cost per unit time is given by

$$K = \frac{ae^{-\theta t}(2\theta sint - \pi)C_1}{2T(1+\theta^2)\theta} - \frac{aC_2}{T} + \frac{aC}{2}\left[\frac{(4+\pi)+2\theta^2}{2(1+\theta^2)}\right] \dots\dots\dots\dots (11)$$

Here C is the total cost of each unit, C_1 *is holding cost per unit per unit time,* C_2 *is the shortage cost per unit perunit time.*
To get the optimum value of t one need to apply the criteria of optimization as

$$\frac{dK}{dt} = 0$$

$$i.e., \frac{-ae^{-\theta t}\theta(2\theta sint - \pi)C_1}{2T(1+\theta^2)\theta} + \frac{ae^{-\theta t}(2\theta cost)C_1}{2T(1+\theta^2)\theta} = 0 \dots\dots\dots\dots\dots\dots.. (12)$$

The equation (12)*is a transcedental equation. It can be solved numerically to get optimum value of t.* (*say* t^*).

Also

$$\frac{d^2K}{dt^2} = \frac{ae^{-\theta t}\theta^2(2\theta sint - \pi)C_1}{2T(1+\theta^2)\theta} - \frac{ae^{-\theta t}\theta(2\theta cost)C_1}{2T(1+\theta^2)\theta} + \frac{-2ae^{-\theta t}\theta^2(cost)C_1}{2T(1+\theta^2)\theta} - \frac{2ae^{-\theta t}sintC_1}{2T(1+\theta^2)\theta}$$

Verify the optimum value of t^*

Therefore total optimum cost, $K^* = \frac{ae^{-\theta t^*}(2\theta sint^* - \pi)C_1}{2T(1+\theta^2)\theta} - \frac{aC_2}{T} + \frac{aC}{2}\left[\frac{(1+\pi)+2\theta^2}{(1+\theta^2)}\right]$

Conclusion

In this chapter a level inventory model with periodic demand and constant deterioration ratio has been developed under certain assumption. Cost minimization technique has been adopted to obtain the optimal value of stock, true and total cost. Deterministic cases of demand $(asint)$are considered by allowing shortages. Expressions for initial inventory total number of deteriorated units and total minimum average cost are obtained.

Reference

[1] Datta T.K. and Pal A.K. (1992) A note on a replenishment policy for an inventory model with linear trend in demand and shortages.Journal of Opl. Res. Soc. 43, 993-1001.

[2] Dave U. (1989) On a heuristic inventory replenishment rule for items with a linearly increasing demand incorporating shortages.J. Opl. Res. Soc. 40, 827-830.
[3] Dave U. (1989) A deterministic lot size inventory model with shortages and a linear trend in demand.Navel Res. Logist, 36, 507-514.
[4] Donaldson W. A. (1977) Inventory replenishment policy for a linear trend in demand, an analytical solution. Operational Res. Quarterly (28), 663-670.
[5] Goyal S.K. Morin, D. and Nebebe, F. (1992)The finite horizon trened inventory replenishment problem with shortage.J. Opl. Res. Soc. 43, 1173-1178.
[6] Haping XU and HSU-(PIN) BEN Wang (1990) An economic ordering policy model for deteriorating items with time proportional demand. Euoropean Journal of Operation Research, (46), 21-27.
[7] Goswami, A. and chaudhari, K. (1991) EOQ model for an inventory with a linear trend in demand and finite rate of replenishment considering shortages. Int. J. Syst. Sci. 22, 181-187.
[8] Sachan R. S. (1984) On (T, Si) policy inventory model for deteriorating items with time proportional demand. J. Opl. Res. Soc. 1013-1019.
[9] Sharma A. K., Kumar Naresh (2001) Optimum ordering interval with known demand for items with variable rate of deteriorating and shortages. Applied Science Periodicals III (2), 95-100.