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A Novel Method to Solve Assignment Problem in Fuzzy Environment

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Abstract
In the literature, there are various methods to solve assignment problems (APs) in which parameters are represented by triangular or trapezoidal fuzzy numbers. In this paper, we compare the assignment cost calculated by existing method with the assignment cost which has been found out in this paper without converting fuzzy assignment problem (AP) into crisp AP by using Fuzzy Hungarian Method (FHM), Robust's Ranking Technique and operations for subtraction and division on triangular fuzzy number (TFN) proposed by Gani and Assarudeen (2012).
Keywords: Fuzzy arithmetic, Triangular Fuzzy Number, Robust's Ranking Function, Assignment Problem.

1. Introduction

The term AP first appeared in Votaw & Orden (1952). APs are widely applied in manufacturing and service systems. An assignment problem is a special type of linear programming problem where the objective is to assign *n* number of jobs to *n* number of persons at a minimum cost (time).

Zadeh (1965) introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Since then, tremendous efforts have been spent; significant advances have been made on the development of numerous methodologies and their applications to various decision problems. Fuzzy assignment problems have received great attention in recent years. Hungarian method proposed by Kuhn (1955) is widely used for the solution to APs. Chen (1985) proposed a fuzzy assignment model that did not consider the differences of individuals and also proved some theorems. Wang (1987) solved a similar model by graph theory.

Lin and Wen (2004) investigated a fuzzy AP in which the cost depends on the quality of the job. Dubois and Fortemps (1999) proposed a flexible AP, which combines with fuzzy theory, multiple criteria decision-making and constraint-directed methodology. Huang and Xu (2005) proposed a solution procedure for the APs with restriction of qualification. Mukherjee and Basu (2010) proposed a new method for solving fuzzy APs. Kumar et al (2009) proposed a method to solve the fuzzy APs, occurring in real life situations.

Kumar and Gupta (2012) proposed two new methods for solving fuzzy APs and fuzzy travelling salesman problems. Kumar and Gupta (2011) proposed methods for solving fuzzy APs with different membership functions. K. Kalaiarasi et al (2014) proposed a fuzzy assignment model with TFN using Robust Ranking technique. In this paper we solve the fully fuzzy AP by using FHM and Operations for Subtraction and Division on TFN proposed by Gani and Assarudeen (2012).

2. Preliminaries

2.1 Fuzzy set: A fuzzy set $\tilde{A}$ in X (set of real number) is a set of ordered pairs:
$\tilde{A}$= {(x, $\mu_{\tilde{A}}$ (x)) / x ϵ X} $\mu_{\tilde{A}}$ (x) is called membership function of x in $\tilde{A}$ which maps X to [0,1].

2.2 Fuzzy Number: A fuzzy set Ã defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{A}}$: R→[0,1] has the following characteristics

(i) Ã is normal. It means that there exists an x ϵ R such that $\mu_{\tilde{A}}$ (x) = 1
(ii) Ã is convex. It means that for every x_1, x_2 ϵ R, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, λ ϵ [0,1]
(iii) $\mu_{\tilde{A}}$ is upper semi-continuous.
(iv) supp (Ã) is bounded in R

2.3 Triangular Fuzzy Number: It is a fuzzy number represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$
This representation is interpreted as membership functions and holds the following conditions
(i) a_1 to a_2 is increasing function
(ii) a_2 to a_3 is decreasing function
(iii) $a_1 \leq a_2 \leq a_3$.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{for } x > a_3 \end{cases}$$

2.4 Positive TFN: A positive TFN $\tilde{A}$ is denoted as $\tilde{A}$= (a_1, a_2, a_3) where all a_i's > 0 for all i=1, 2, 3.

2.5 Negative TFN: A negative TFN $\tilde{A}$ is denoted as $\tilde{A}$= (a_1, a_2, a_3) where all a_i's < 0 for all i=1, 2, 3.
Note: A negative TFN can be written as the negative multiplication of a positive TFN. Example: $\tilde{A}$=(−3, −2, −1) is a negative TFN this can be written as $\tilde{A}$ = − (1,2,3,).

2.6 Operation of TFN Using Function Principle:
The following are the four operations that can be performed on triangular fuzzy numbers: Let $\tilde{A}$ = (a_1, a_2, a_3) and $\tilde{B}$= (b_1, b_2, b_3) then, (i) Addition: $\tilde{A}$ + $\tilde{B}$ = (a_1+ b_1, a_2+ b_2, a_3+ b_3).
(ii) Subtraction: $\tilde{A}$-$\tilde{B}$ = (a_1- b_3, a_2- b_2, a_3- b_1).
(iii) Multiplication: $\tilde{A}$x$\tilde{B}$ = (min (a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3), a_2 b_2, max (a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3)).
(iv) Division: $\tilde{A}$/$\tilde{B}$ = (min (a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), a_2/b_2, max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3)).

*2.7 Operations for Subtraction and Division on TFN (*Gani & Assarudeen 2012)*:*
2.7.1 Subtraction: Let $\tilde{A}$ =(a_1, a_2, a_3) and $\tilde{B}$= (b_1, b_2, b_3) then, $\tilde{A}$-$\tilde{B}$ = (a_1-b_1,a_2-b_2,a_3-b_3). The new subtraction operation exist only if the following condition is satisfied DP($\tilde{A}$)≥DP($\tilde{B}$) where DP($\tilde{A}$)=$\frac{a_3-a_1}{2}$, DP($\tilde{B}$)=$\frac{b_3-b_1}{2}$ and DP denotes Difference point of a TFN.
2.7.2 Division:
Let $\tilde{A}$=(a_1, a_2, a_3) and $\tilde{B}$= (b_1, b_2, b_3) then, $\tilde{A}$/$\tilde{B}$= (a_1/b_1, a_2/b_2, a_3/b_3)
The new Division operation exists only if the following conditions are satisfied $\left|\frac{DP(\tilde{A})}{MP(\tilde{A})}\right| \ge \left|\frac{DP(\tilde{B})}{MP(\tilde{B})}\right|$ and the negative TFN should be changed into negative multiplication of positive number as per note in definition 2.5 where MP($\tilde{A}$) =$\frac{a_3+a_1}{2}$, MP($\tilde{B}$) =$\frac{b_3+b_1}{2}$ DP($\tilde{A}$)=$\frac{a_3-a_1}{2}$, DP($\tilde{B}$)=$\frac{b_3-b_1}{2}$ and MP denotes Midpoint of a TFN.

3. Robust's Ranking Techniques (Kalaiarasi et al 2014)

Robust's ranking technique which satisfies costs, linearity, and additives properties and provides results which are consistent with human intuition. Give a convex fuzzy number ã, the Robust's Ranking Index is defined by R (ã) =$\int_0^1 0.5(a_\alpha^L, a_\alpha^U)$, where (a_α^L, a_α^U) is the α – level cut of the fuzzy number ã. In this paper we use this method for ranking the objective values. The Robust's ranking index R (ã) gives the representative value of the fuzzy number ã. It satisfies the linearity and additive property.

4. Fuzzy Hungarian method (Prabakaran & Ganesan)

The steps of FHM are as follows:
Step 1: Determine the fuzzy cost table from the given problem. If the number of sources is equal to the number of destinations go to step 3. If the number of sources is not equal to the number of destinations go to step 2.
Step 2: Add a dummy source or dummy destination, so that the fuzzy cost table becomes a fuzzy square matrix. The fuzzy cost entries of dummy source/destinations are always fuzzy zero.
Step 3: Subtract the row minimum from each row entry of that row.
Step 4: Subtract the column minimum of the resulting fuzzy AP after using step 3 from each column entry of that column. Each column and row now has at least one fuzzy zero.
Step 5: In the modified fuzzy assignment table obtained in step 4, search for fuzzy optimal assignment as follows.
(a) Examine the rows successively until a row with a single fuzzy zero is found. Assign the fuzzy zero and cross off all other fuzzy zeros in its column. Continue this for all the rows
(b) Repeat the procedure for each column of reduced fuzzy assignment table.
(c) If a row and / or column have two or more fuzzy zeros assign arbitrary any one of these fuzzy zeros and cross

off all other fuzzy zeros of that row/column. Repeat (a) through (c) above successively until the chain of assigning or cross ends.

Step 6: If the number of assignments is equal to n, the order of the fuzzy cost matrix, fuzzy optimal solution is reached. If the number of assignments is less than n, the order of the fuzzy zeros of the fuzzy cost matrix, go to the step 7.

Step 7: Draw the minimum number of horizontal and / or vertical lines to cover all the fuzzy zeros of the reduced fuzzy assignment matrix. This can be done by using the following: (i) Mark rows that do not have any assigned fuzzy zero. (ii) Mark columns that have fuzzy zeros in the marked rows. (iii) Mark rows that do have assigned fuzzy zeros in the marked columns. (iv) Repeat (ii and iii) above until the chain of marking is completed. Draw lines through all the unmarked rows and marked columns. This gives the desired minimum number of lines.

Step 8: Develop the new revised reduced fuzzy cost matrix as follows: Find the smallest entry of the reduced fuzzy cost matrix not covered by any of the lines. Subtract this entry from all the uncovered entries and add the same to all the entries lying at the intersection of any two lines.

Step 9: Repeat step 6 to step 8 until fuzzy optimal solution to the given fuzzy AP is attained.

5. Numerical Example (Kalaiarasi et al 2014)

Example 5.1: A company has four sources S1, S2, S3, S4 and destinations D1, D2, D3, D4. The fuzzy transportation cost for unit quantity of product from ith sources jth destinations is Cij.

where Cij,

$$\begin{bmatrix} (1,5,9) & (3,7,11) & (7,11,15) \\ (4,8,12) & (1,5,9) & (4,9,13) \\ (0,4,8) & (3,7,11) & (6,10,14) \\ (6,10,14) & (0,4,8) & (4,8,12) \end{bmatrix} \begin{matrix} (2,6,10) \\ (2,6,10) \\ (3,7,11) \\ (-1,3,7) \end{matrix}$$

Solution : Step 1. Row reduction. Subtract the minimum element of each row for all elements of that row

$$\begin{bmatrix} (0,0,0) & (2,2,2) & (6,6,6) \\ (3,3,3) & (0,0,0) & (3,4,4) \\ (0,0,0) & (3,3,3) & (6,6,6) \\ (7,7,7) & (1,1,1) & (5,5,5) \end{bmatrix} \begin{matrix} (1,1,1) \\ (1,1,1) \\ (3,3,3) \\ (0,0,0) \end{matrix}$$

Step 2. Column Reduction. Subtract the minimum element of each column for all elements of that column

$$\begin{bmatrix} (0,0,0) & (2,2,2) & (2,2,3) \\ (3,3,3) & (0,0,0) & (0,0,0) \\ (0,0,0) & (3,3,3) & (2,2,3) \\ (7,7,7) & (1,1,1) & (1,1,2) \end{bmatrix} \begin{matrix} (1,1,1) \\ (1,1,1) \\ (3,3,3) \\ (0,0,0) \end{matrix}$$

Step 3. Making assignments and drawing minimum possible horizontal and vertical lines covering all fuzzy zeros.

$$\begin{bmatrix} (0,0,0) & (2,2,2) & (2,2,3) \\ (3,3,3) & (0,0,0) & (0,0,0) \\ (0,0,0) & (3,3,3) & (2,2,3) \\ (7,7,7) & (1,1,1) & (1,1,2) \end{bmatrix} \begin{matrix} (1,1,1) \\ (1,1,1) \\ (3,3,3) \\ (0,0,0) \end{matrix}$$

Step 5. Modify the above table by subtracting the smallest uncovered fuzzy number from all the elements not covered by lines and adding same at the intersection of the two lines.

$$\begin{bmatrix} (0,0,0) & (1,1,1) & (1,1,2) \\ (4,4,4) & (0,0,0) & (0,0,0) \\ (0,0,0) & (2,2,2) & (1,1,2) \\ (7,7,7) & (0,0,0) & (0,0,1) \end{bmatrix} \begin{matrix} (1,1,1) \\ (2,2,2) \\ (3,3,3) \\ (0,0,0) \end{matrix}$$

Step 6. Making assignments and drawing minimum possible horizontal and vertical lines covering all fuzzy zeros.

$$\begin{bmatrix} (0,0,0) & (1,1,1) & (1,1,2) & (1,1,1) \\ (4,4,4) & (0,0,0) & (0,0,0) & (2,2,2) \\ (0,0,0) & (2,2,2) & (1,1,2) & (3,3,3) \\ (7,7,7) & (0,0,0) & (0,0,1) & (0,0,0) \end{bmatrix}$$

Step 7. Modify the above table by subtracting the smallest uncovered fuzzy number from all the elements not covered by lines and adding same at the intersection of the two lines.

$$\begin{bmatrix} (0,0,0) & (0,0,0) & (0,0,1) & (0,0,0) \\ (5,5,5) & (0,0,0) & (0,0,0) & (2,2,2) \\ (0,0,0) & (1,1,1) & (0,0,1) & (2,2,2) \\ (8,8,8) & (0,0,0) & (0,0,1) & (0,0,0) \end{bmatrix}$$

Step 8. Making assignments

$$\begin{bmatrix} (0,0,0) & (0,0,0) & (0,0,1) & (0,0,0) \\ (5,5,5) & (0,0,0) & (0,0,0) & (2,2,2) \\ (0,0,0) & (1,1,1) & (0,0,1) & (2,2,2) \\ (8,8,8) & (0,0,0) & (0,0,1) & (0,0,0) \end{bmatrix}$$

The fuzzy optimal total cost $\tilde{a}_{12}+\tilde{a}_{23}+\tilde{a}_{31}+\tilde{a}_{44}$ = (3,7,11) + (4,9,13) + (0,4,8) + (-1,3,7)
= (6,23,39)

Now, by using Robust ranking technique then R(6,23,39) = 22.75

6. Result

In this section we will compare the assignment cost which has been found out in example 5.1 with the assignment cost calculated by existing method (Kalaiarasi et al 2014)

	Existing Method (Kalaiarasi et al 2014)	Proposed Method
Example 5.1	Assignment Cost = 23	Assignment Cost = 22.75

7. Conclusion

In this paper, we solve the fully fuzzy AP by using FHM proposed by Prabakaran & Ganesan and Operations for Subtraction and Division on TFN proposed by Gani and Assarudeen (2012). By comparing the results of the proposed method and existing method, it is shown that it is better to use the proposed methods instead of existing method. Also proposed method is easy to apply and understand.

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