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An Analytical Inventory Model for Exponentially Decaying Items under the Sales Promotional Scheme

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Abstract
In conventional EOQ model, it is implicitly assumed that, when purchaser orders and pays for Q units then the supplier supplies Q units only. But in today's modern business world, in order to survive progress and to achieve highly ambitious targets or to come out the problems of over stocking, supplier announces various sales promotional schemes. Here we have considered a scheme in which, if purchaser orders and pays for Q units then the supplier supplies (1+p)Q units, $0 \le p \le 1$. An EOQ model is developed for exponentially deteriorating items under the sales promotional schemes for constant demand rate. A hypothetical numerical example has been solved to illustrate the model.
Keywords: EOQ model, Sales promotional schemes, Exponential distribution, deteriorating rate.

1. Introduction

In the classical EOQ model, it is implicitly assumed that the quantity received exactly matches with the quantity requisitioned and there is no damage or deterioration of the units while in inventory. However, in practice, due to variety of reasons – like difference in batch size or packaging size, limited raw material, limited resources, etc., it happens that the quantity received may be different from the quantity ordered. Also it is normal tendency to adopt various sales promotional schemes by the manufacturer or supplier to the retailers for gaining the business or some times to clear the accumulated stock. One of the most common scheme is to supply some more units of commodity at free of cost with regular of promotional order by the retailer. Also units of commodity are deteriorated with respect to time while they are in inventory, usually, as the inventory time , bigger, rate of deterioration is higher so if the retailer want to take the advantage of the sales promotional scheme then the risk of deterioration will increase the cost of inventory so there is a need to balance the situation.

Silver (1976) has developed an EOQ model when the quantity received is uncertain and is a random variable with specified mean and variance. Kalro and Gohil (1982) have extended the above model by allowing shortages. Noori and Keller (1986) developed a stochastic model when quantity received is uncertain. Ghare and Schrader (1963) developed an EOQ model for exponentially decaying inventories. This model has been generalized by Covert and Philip (1973) and then again by Philip (1974) by using weibull distribution to describe time to deterioration of an inventory.

Here, an attempt is made to derived and to analyze an EOQ model when purchaser orders and pay for Q units of commodity and the supplier supplies (1+p)Q units of commodity, $0 \le p \le 1$, i.e., supply is random and the units of commodity under considerations are exponentially deteriorated for the constant demand rate. A hypothetical case is solved to illustrate the model.

2. Assumptions And Notations Followings are the notations and important assumptions for deriving the model

- demand rate of R units per time unit is known and constant during the period under consideration.
- The replenishment size Q is the decision variable. The order size is Q - units per order, however, the actual quantity received (say) Y is a normal decision variable with $E(Y) = bQ$ & $V(Y) = \sigma_0^2 + \sigma_1^2 Q^2$ (1)

where $b \ge 0$ is the bias factor, σ_0^2 and σ_1^2 are the positive known constants.
- The replenishment rate is infinite.
- Shortages are not allowed. Lead time is zero.
- At time t of a cycle, the constant fraction θ $(0 \le \theta \le 1)$ of the on hand inventory deteriorates per unit of time.
- There is no repair or replacement of the deteriorated inventory during the period under consideration.
- During sales promotional scheme at the most one order should be placed.
- The unit cost C does not depend upon the quantity ordered or received and is constant during the time under consideration.

The other notations are as follows :

- C_1 denotes the unit holding cost exclusive of interest charges.
- I_c denotes the interest charges per rupee investment per time unit.
- C_3 denoted the ordering cost.
- Q_0 the economic ordered quantity in units when the items are exponentially decaying.
- The time for which Q_0 units are sufficient to meet the demand.

- T' the time for which Q' units are sufficient to meet the demand.

3. The Mathematical Model:

In usual system, an order of $Q_0 = \sqrt{\frac{2C_3R}{(C_1+CI_c)}}$ is placed at every T time unit and the total cost of the system will be

$$K(Q_0) = CQ_0 + \frac{(C(\theta + I_C) + C_1)Q_0^2}{2R} - \frac{\theta(C_1 + CI_C)Q_0^3}{3R^2} + C_3 \quad (2)$$

The value of Q_0 is determined by using Ghare and Schrader [5] model. The objective of this mode; is to determine the special order size Q' to get maximum gain. For deriving the total cost of the system, we assume that θ^2 is very small as compare to other factors, and
we assume the series expansion and retain the terms up to θ only.
Let K(Q') denotes the total cost of the system during the time period T' when order of Q' units is placed at time T_0 and the system receives (1+p)Q' units

$$K(Q') = CQ' + \frac{((1+p)^2(C\theta + C_1) + CI_C)Q'^2}{2R} - \frac{\theta((1+p)^3C_1 + CI_C)Q'^3}{3R^2} + C_3 \quad (3)$$

Let K_1(Q') denotes the total cost of the system during the time period T' when no special order is placed during the tenure of scheme but several orders of Q_0 units are placed at every T time units, then

$$K_1(Q') = (1+p)CQ' + \frac{((1+p)C(\theta + CI_C) + C_1)Q_0Q'}{2R} - \frac{\theta(1+p)(C_1 + CI_C)Q_0^2Q'}{3R^2} + \frac{(1+p)C_3Q'}{Q_0} \quad (4)$$

The gain due to taking advantage of the special sales promotional scheme is

$$G(Q') = K_1(Q') - K(Q')$$

$$= pCQ' + \frac{(1+p)C(\theta + CI_C) + C_1)Q_0Q'}{2R} + \frac{\theta((1+p)^3C_1 + CI_C)Q'^3}{3R^2} - \frac{((1+p)^2(C\theta + C_1) + CI_C)Q'^2}{2R} - \frac{\theta(1+p)(C_1 + CI_C)Q_0^2Q'}{3R^2} + \frac{(1+p)C_3Q'}{Q_0} - C_3 \quad (5)$$

For maximization, $\frac{\partial G(Q')}{\partial Q'} = 0$ gives the optimum value of $Q' = Q_0'$ and is a solution of the quadratic equation,

$$XQ'^2 + YQ' + Z = 0 \quad (6)$$

Where,

$$X = \frac{\theta((1+p)^3C_1 + CI_c)}{R^2} \quad (7)$$

$$Y = \frac{((1+p)^2(C\theta + C_1) + CI_C)}{R} \quad (8)$$

$$Z = pC + \frac{(1+p)C(\theta + CI_C) + C_1)Q_0}{2R} - \frac{\theta(1+p)(C_1 + CI_C)Q_0^2}{3R^2} + \frac{(1+p)C_3}{Q_0} \quad (9)$$

the maximum gain is obtained by substituting the value of Q' in equation (6). The gain is maximum iff

$$\frac{\partial^2 G(Q')}{\partial Q'^2} \le 0$$

i.e.,
$$-\frac{((1+p)^2(C\theta+C_1)+CI_C)}{R}+\frac{2\theta((1+p)^3C_1+CI_C)Q'}{R^2}\le 0 \quad (10)$$

Case 1: If $\theta = 0$ i.e., if the items are not deteriorated then the optimum order quantity is given by the equation (11).

$$Q' = \frac{pCR+(1+p)\sqrt{2(C_1+CI_C)C_3R}}{(1+p)^2C_1+CI_C} \quad (11)$$

The respective equation of gain is given by the equation (12).

$$G_1(Q') = \left(pC + \frac{(1+p)}{R}\sqrt{2(C_1+CI_C)C_3R} \right)Q' - \frac{((1+p)^2C_1+CI_C)Q'^2}{2R} - C_3 \quad (12)$$

The gain is maximum iff $\frac{\partial^2 G_2(Q_0)}{\partial Q_0^2} \le 0$

CASE 2: When usual EOQ (Q_0 units)are ordered during the tenure of scheme then the system receives (1+p) Q_0 units then the gain is given by the equation (13).

$$G_2(Q_0) = G_1(Q') - p\left(K(Q_0) - \frac{(2+p)C_1Q_0^2}{2R} \right) \quad (13)$$

The gain is maximum iff $\frac{\partial^2 G(Q')}{\partial Q'^2} \le 0$

Note that here if p=0 then

$$Q' = Q_0 = \sqrt{\frac{2C_3R}{C_1+CI_c}}$$

which the usual economic ordered quantity. and obviously $K(Q') = K_1(Q') = K(Q_0)$ and, $G_1(Q') = G_2(Q') = 0$

When $\theta = 0$ and $p = 0$ i.e., when the items are not deteriorated and vendor does not announce any sales promotional schemes then the derived ordered quantity reduces to the classical ordered quantity and the corresponding gain reduces to zero.

4. Hypothetical Example and iterpretations

Consider an inventory system with following parameters
Unit cost C = Rs. 20 per unit.
Demand Rate R = 12000 units per year.
Replenishment cost C_3 = Following table shows the different values of optimum order quantity to be procured during sales promotional scheme and the optimum value of gain for the different values of the sales promotional factor p and the deterioration factor is calculated. Also in each case the value of economic order quantity is calculated and the value of corresponding gain is also obtained.
Rs. 300 per order.
Interest Charges I_c = 0.18 per rupee per year.
Inventory Holding Cost C_1 = Re. 0.25 per unit per year.
By varying the values of sales promotional factor and the deterioration factor, the following table is obtained

		Sales Promotional Factor p	
Deterioration factor	Quantity	0.10	0.20
0.01	Q_0	180	180
	Q’	478	734
	G	1627.42	4483.74
0.02	Q_0	178	178
	Q’	466	713
	G	1576.69	4311.23
0.03	Q_0	176	176
	Q’	454	693
	G	1529.61	4152.91
0.04	Q_0	175	175
	Q’	443	674
	G	1485.81	4007.22

Interpretations From the above table it can be observed that as the sales promotional factor p increases and the deterioration factor remains same then there is no much more difference when Q_0 are ordered but there is significant difference in the order quantity Q’ and the respective gain. Also when p remains same and as deterioration rate increases both the order quantities and gain decreases significantly.

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