Vibrations of Systems with One Degree- of- Freedom in the Presence of Coulomb Friction

Dr. falah Mustafa al-saraireh
Mechanical Engineering Department, faculty of Engineering, Mutah university.jordan

Abstract
An analytical solution for the motion equations of a single-mass mechanical system damped by Coulomb friction force is derived.

Theory
Coulomb friction forces are used to obtain many technical solutions.
To control the vibrations of anvil hammers caused by friction forces, the free vibrations of the anvil hammers are damped after impact during the double stroke of the falling part.
The technical literature, does not give an analytical solution for the equation of motion of a mechanical system with one degree of freedom in the presence of Coulomb friction forces. A monograph [2] gives the following equation of motion for mass **m**, located on a horizontal surface, attached to a vertical wall with a spring stiffness of **K** :-

$$mx \qquad \kappa x \qquad F \tag{1}$$

Where ***F*** is the force of Coulomb friction.
The authors of [2] proposed that (1) be solved numerically by converting it into the form

$$x' = -\int\left(\frac{k}{m}x \mp \frac{F}{m}\right)dt \tag{2}$$

In .Yablonski, and Noreika [3], §10 is devoted to vibrations in the presence of Coulomb friction, The differential equation of free vibrations has the form :-

$$mx'' + cx = F_{R(x')} \tag{3}$$

Where $F_R(x')$ is the resistance force (friction), which is a nonlinear function of the generalized velocities x .
Further, the authors of [3] "state that" An exact solution of the differential equation (10.1) by means of elementary functions, in most cases is impossible
Approximate methods of solving this equation by graphical or numerical
Integration, although it is possible with sufficient accuracy for practical applications, but sometimes cumbersome and require lengthy calculations. "

Analysis:
This paper presents a solution for the free vibrations of a system (see Fig.1a) using operational calculus. The equation of motion of the system has the form

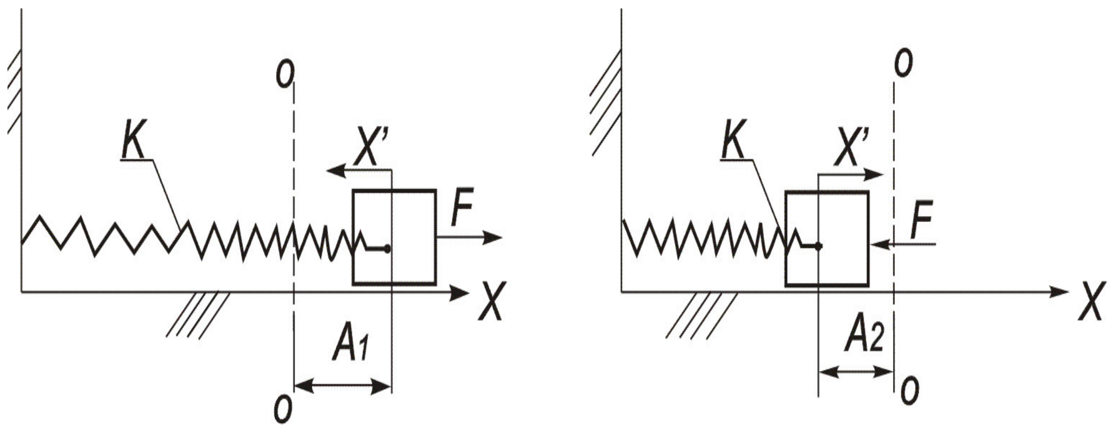
$$x'' + \omega^2 x = \frac{F}{m}\mu(t) \tag{4}$$

Where $\omega = \sqrt{\frac{k}{m}}$ - is the angular velocity, F -is the friction force, and (t) is the unit Heaviside function,

$$\mu(t) = \begin{cases} 1, t > 0 \\ 0, t < 0 \end{cases}$$

Performing the Laplace [4] transform of equation (4) with the initial conditions $x_0 = A_1, x'_{0=0}$ we obtain

$$x_{(s)} = (s^2 + \omega^2) - SA_{1=}\frac{F}{m} \cdot \frac{1}{S} \tag{5}$$



(a) (b)

Fig.1). position of the mass relative to the axis of static equilibrium
(Assuming no friction): a) at the beginning of motion, and b) late in the first half period of oscillation.

From (5) we find the reaction

$$x(s) = \frac{F}{m} \cdot \frac{1}{s(s^2+\omega^2)} + A_1 \frac{s}{s^2+\omega^2} \tag{6}$$

After performing the inverse laplace transform of equation (6) ,we obtain the equation of motion of the mass as a function of time

$$x = \frac{F}{m\omega^2}\left((1 - cos\omega t) + A_1 cos\omega t\right) \tag{7}$$

Since $m\omega^2 = k$, we present equation (7) as

$$x = \left(A_1 - {}^{F}\!/_{k}\right) cos\omega t + {}^{F}\!/_{k} \tag{8}$$

If mass m is inclined from the static equilibrium position by the amount $A_1 = {}^{F}\!/_{k}$, then after substituting A_1 in (8) we obtain

$$x \qquad F / \kappa \tag{9}$$

And the weight will remain stationary.

This corresponds to the condition $F = A_1 \kappa$, which is the reaction force of the spring on the value of its deformation ,and equals A_1, i.e. the elastic deformation force equals the force of friction.

Analysis of (8) shows that mass ***m*** will be at rest under condition that its initial displacement is no more F / κ , i.e. when $A_1 \le {}^{F}\!/_{k}$.

Let us examine the vibration of the system when - A_1 $\quad F / \kappa$

Based on the mass motion equation (8) we find that the equation of velocity is

$$x' = -\left(A_1 - {}^{F}\!/_{k}\right)\omega \sin \omega t \tag{10}$$

To determine the maximum displacement of the mass, we equate its speed to zero A_1 $\quad F / \kappa$, the maximum displacement will be at sin $\omega t = 0$, i.e., $\omega t = \pi$ after substituting this in (8) we get the value of the anvil motion relative to the position of static equilibrium; the second amplitude is negative and equal to

$$-A_2 = \frac{2F}{k} - A_1 \tag{11}$$

From (11) we find that the decrease in amplitude occurs for the value

$$A_1 - A_2 = \frac{2F}{k} \tag{12}$$

The second mass displacement will occur from the position shown in (Fig. 1b). The friction force changes its direction, so that it is opposite to the direction of the velocity, and the equation of motion will have the form

$$x'' + \omega^2 x = -\frac{F}{m}\mu(t) \tag{13}$$

Subjecting equation (7) to the Laplace transform [5] with the initial conditions $x_0 = -A_2$, $x'_{0=0}$ we obtain

$$x_{(s)}(s^2 + \omega^2) + sA_2 = -\frac{F}{m} \cdot \frac{1}{s} \tag{14}$$

and define the reaction as

$$x_{(s)} = -\frac{F}{m} \cdot \frac{1}{s(s^2+\omega^2)} - A_2 \frac{S}{S^2+\omega^2} \tag{15}$$

After the inverse Laplace transform [5] and simple algebraic manipulations, we obtain the equation of motion of the mass as a function of time

$$x = \left(F/_k - A_2\right)cos\omega t - F/_k \tag{16}$$

And the velocity equation

$$x' = \left(A_2 - F/_k\right)sin\omega t \tag{17}$$

The mass will not move when its velocity vanishes, i.e $x'_0 = 0$, which corresponds to the condition $\omega t = \pi$, After substituting this in (16) we find the value of the amplitude

$$A_3 = A_2 - \frac{2F}{k} \tag{18}$$

The amplitude decreases at the value

$$A_2 - A_3 = \frac{2F}{k} \tag{19}$$

The subsequent displacement will correspond to the initial position of the mass shown in Fig.1,(a). The initial displacement is equal to A3, The equation of motion corresponds to equation (4),and the mass displacement corresponds to equation (8), which uses A_1 instead of A_3. Amplitude A_4 is less than A_3 by $2F\ /\ \kappa$ .Each subsequent amplitude at a time equal to half of the oscillation period, $t = \frac{\pi}{\omega}$, will be reduced by $2F / \kappa$, This pattern can be represented as follows

$$\left.\begin{aligned} A_1 - A_2 &= \frac{2F}{k} \\ A_2 - A_3 &= \frac{2F}{k} \\ &\cdots \\ A_n - A_{n+1} &= \frac{2F}{k} \end{aligned}\right\} \tag{20}$$

Adding, the right and left sides we obtain

$$A_1 - A_{n+1} = n\frac{F}{k} \tag{21}$$

In (21) the last amplitude is $A_{n+1} < \frac{2F}{k}$. If the first amplitude A_1 is divided by $2F / \kappa$, the integer part of the resulting number will be an integer of the half-period of the oscillations, Then

$$A_{n+1} = A_1 - n\left(\frac{2F}{k}\right) \tag{22}$$

Where n integer equals to half-periods of oscillations.
When n – is an odd number, the mass is located on the side where the vibration began, It is on the opposite side when n is an even number.
The mass will stop within the dead zone, i.e.. When x F / κ . The precise coordinates of the mass at rest are found from the conditions when the potential energy of a spring compressed at an amount A_{n+1} , is spent on the work of friction, which can be expressed by the following equation

When $\frac{1}{2}kA_{n+1} < F$,

$$\frac{1}{2}kA_{n+1}^2 - \frac{1}{2}x^2k = F(A_{n+1} - x) \tag{23}$$

When $\frac{1}{2}kA_{n+1} > F$,

$$\frac{1}{2}kA_{n+1}^2 = \frac{1}{2}x^2k + F(A_{n+1} + x) \tag{24}$$

From (23), (24) we find (x) the position at the moment (when the anvil finally stops. i.e..there is no vibration).
In technical solutions, the friction force is used for damping vibrations. In such cases, $A_{n\ 1}$ 0, and equation (21) is used to determine the necessary friction force, created by friction devices at a given time of oscillation damping.

Conclusions

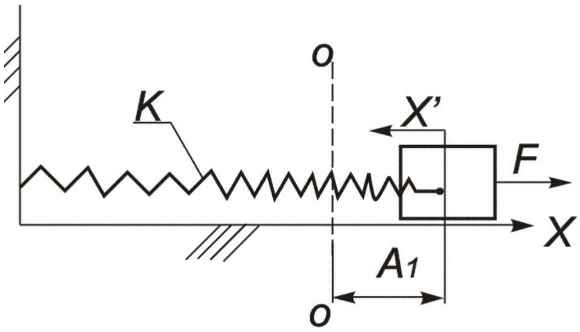
Using the operational calculations allows us to solve the differential equation of motion of a mass in the presence of Coulomb friction.

We found that during each half-cycle of oscillation, the amplitude is reduced by twice the amount of elastic deformation of the spring caused by the force, which equals the friction force numerically.

This paper shows the analytical dependence of the relationship between the half periods of vibration, friction, and the first and last oscillation amplitudes.

References

1. DP38111A (Ukraine). MPK7V21J12/00. Acoustic insulation no-anvil hammers. /R.I .Ray, Y. Gutko, O. Tkachuk (Ukraine) Publ. 04/15/01. Bul. № 4.
2. Mechanical vibrations I.E. Morse, R.T. Hinkle / Moscow: Mashinostroenie, 1966. 508P.
3. . A. Yablonski Course in the theory of oscillations / / A.A Yablonski, S.S Nareyko / Moscow: Higher School, 1975. 248P.
4. Corn Handbook of mathematics / G. Corn and T. Corn / / Moscow: Nauka, 1984. 832P.



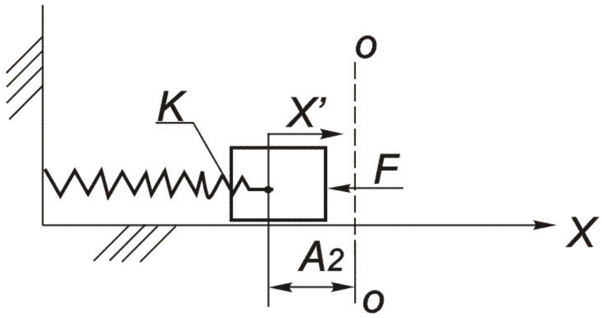


FIG 1) a) b)