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Effect of Nonlinear Thermal Radiation on Stagnation Flow of a Casson Fluid towards a Stretching Sheet

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Abstract
In this study, we analyzed the stagnation-point flow of a casson fluid towards a stretching sheet with nonlinear thermal radiation and induced magneticfield effects. The nonlinear partial differential equations are transformed into a set of nonlinear ordinary differential equations using self-suitable transformations, which are then solved numerically using Runge-Kutta based shooting technique. The effects of different non-dimensional parameters on velocity, induced magneticfield and temperature profiles along with the friction factor and local Nusselt number are discussed through graphs and tables. Comparisons of the present results with the existed literature were presented. Result indicate that the casson parameter have tendency to control the flow.
Keywords: Induced magneticfield, thermal radiation, casson fluid, stretching Sheet.

1. Introduction

Now a days the cooling of electronic devices is one of the important requirements in micro-electromechanical systems, nuclear reactors vehicle thermal management etc. The magneto hydrodynamics (MHD) deals with the motion of highly conducting fluids in the presence of magnetic field. The flow of the conducting fluid across the magneticfield causes the electric current, which has the capacity to change the influence of magneticfield and the flow behaviour of the fluid. It is important to study the flow characteristics of electrically conducting magneto-nanofluids. The magneto nanofluids have pivotal importance in the process of targeted drug release, asthma treatment, cancer therapy and in the construction of power generators. Flow due to stretching surface has many important applications in plastic and metal industries. Also, the glasses blowing, continuous casting of metals and spinning of fibers involve the concept of flow through stretching sheet. In all the above applications of flow through stretching sheet, the quality of the final product depends upon the rate of heat transfer at the stretching sheet.

The effect of non-linear thermal radiation on different types of fluid flows through various channels was studied by Hayat et al. (2015), Shehzad et al. (2014) and Mustafa et al. (2014). Cortell (2014) studied the nonlinear heat transfer effect on fluid flow over a stretching surface. Hayat et al. (2015a) discussed the MHD peristaltic flow of a nanofluid with slip effects and joule heating. The electro osmotic flow analysis of casson fluid flow past a micro slit channel was discussed by Chiu-On Ng (2013). Sulochana et al. (2015) discussed the influence of aligned magneticfiled on the flow through vertical surface in porous medium with heat source. The flow characteristics on stagnation-point flow of casson fluid past a stretching surface in the presence of viscous dissipation effect was considered by Mustafa et al. (2012). An unsteady heat transfer characteristics on fluid flow past a permeable stretching surface with thermal radiation and non-uniform heat source/sink was considered by Pal (2011). Stagnation point flow of MHD Nanofluid flow past a non-isothermal stretching surface in presence of induced magneticfield effect was investigated by Pal and Mandal (2015). Jayachandra Babu et al. (2015) discussed the effect of radiation and viscous dissipation on stagnation-point flow of a micropolar fluid over a nonlinearly stretching surface.

Mohan Krishna et al. (2013, 2014) discussed the flow of dusty and nano fluids through different channels in the presence of transverse magneticfiled. Aligned magneticfield, radiation and chemical reaction effects on dusty viscous flow over a flat plate were studied by Ramana Reddy et al. (2014). Sandeep et al. (2012, 2013) illustrated the radiation effects on the flows through porous media. An unsteady MHD free convection flow of a dissipative fluid past a vertical plate through porous media with constant heat flux was discussed by Sugunamma and Sandeep (2011). Radiation and inclined magnetic field effects on free convective flow past an impulsively moving vertical plate in a porous medium was studied by Sandeep and Sugunamma (2014). Raju et al. (2015) studied the influence of aligned magnetic field and radiation on the flow of ferrofluids over a flat plate with non-uniform heat source/sink. Sandeep and Sulochana (2015) presented dual solutions of radiative MHD nanofluid flow over an exponentially stretching sheet. Raju et al. (2015) illustrated the effects of radiation, inclined magnetic field and cross-diffusion effects on flow over a stretching surface. Stagnation point flow and heat transfer behavior of Cu-water nanofluid towards horizontal and exponentially stretching/shrinking cylinders was studied by Sulochana and Sandeep (2015). Sandeep and Sulochana (2015) analyzed the effects of MHD on the flow of a dusty nanofluid over a stretching surface with volume fraction of dust particles.

In this study, we analyzed the stagnation-point flow of a casson fluid towards a stretching sheet with nonlinear thermal radiation and induced magneticfield effects. The nonlinear partial differential equations are transformed into a set of nonlinear ordinary differential equations using self-suitable transformations, which are then solved numerically using Runge-Kutta based shooting technique. The effects of different non-dimensional parameters on velocity, induced magneticfield and temperature profiles along with the friction factor and local Nusselt number are discussed through graphs and tables.

2. Mathematical Analysis

Consider a steady, incompressible, electrically conducting stagnation-point flow of a Casson fluid towards a stretching sheet in the presence of induced magneticfield. The stretching sheet is considered along the $x-$axis and $y-$axis is normal to it. It is assumed that the applied magneticfield is of uniform strength H_0. It is also assumed that induced magneticfield is applied in $y-$direction and the parallel component H_1 approaches the value $H_e = H_0$ in the free stream flow and normal component of the induced magneticfield H_2 vanishes near the wall. Under the above assumptions the governing boundary layer equations are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial H_1}{\partial x}+\frac{\partial H_2}{\partial y}=0, \tag{2}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}+u_e(x)\frac{\partial u_e(x)}{\partial x}+\frac{\mu}{4\pi}\left(H_1\frac{\partial H_1}{\partial x}+H_2\frac{\partial H_1}{\partial y}-H_e\frac{\partial H_e}{\partial x}\right), \tag{3}$$

$$u\frac{\partial H_1}{\partial x}+v\frac{\partial H_1}{\partial y}=H_1\frac{\partial u}{\partial x}+H_2\frac{\partial u}{\partial y}+\alpha_1\frac{\partial^2 H_1}{\partial y^2}, \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^*}{3k^*\rho c_p}\frac{\partial}{\partial y}\left(T^3\frac{\partial T}{\partial y}\right), \tag{5}$$

Subject to the boundary conditions

$$\left.\begin{array}{ll} u=u_w(x)=cx, v=0, \dfrac{\partial H_1}{\partial y}=H_2=0, T=T_w & \text{at } y=0, \\ u=u_e(x)=ax, H_1=H_e(x)=H_0 x, T=T_\infty, & \text{as } y\to\infty, \end{array}\right\} \tag{6}$$

where u, v are the velocity components of the fluid in x, y directions, H_1, H_2 are the magnetic components in x, y directions, ρ and μ are the fluid density and dynamic viscosity respectively, σ is the electrical conductivity, ρc_p is the specific heat capacitance, T is the fluid temperature, k is the effective thermal conductivity, σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively, α_1 is the magnetic diffusivity of the nanofluid, which is given by $\alpha_1 = 1/4\pi\sigma$.

To convert the governing equations into set of nonlinear ordinary differential equations, we now introduce the following similarity transformation.

$$\left.\begin{array}{l} u=cxf'(\eta), v=-\upsilon^{1/2}c^{1/2}f(\eta), \eta=\upsilon^{-1/2}c^{1/2}y, T=T_\infty(1+(\theta_w-1)\theta) \\ H_1=H_0 x g'(\eta), H_2=-H_0\upsilon^{1/2}c^{-1/2}g(\eta), \theta(\eta)=(T-T_\infty)/(T_w-T_\infty), \end{array}\right\} \tag{7}$$

Substituting equations (6) and (7) into (1)-(5), equations (1) and (2) satisfies automatically. Now, the equations (3)-(5) will be transformed into the following nonlinear coupled ordinary differential equations:

$$\left(1+\frac{1}{\beta}\right)f'''-\left(f'^2-ff''\right)+B\left(g'^2-gg''-1\right)+\left(\frac{a}{c}\right)^2=0, \tag{8}$$

$$\lambda g'''+fg''-f''g=0, \tag{9}$$

$$\theta''+R\left[\left(1+(\theta_w-1)\theta\right)^3\theta''+3(\theta_w-1)\theta'^2\left(1+(\theta_w-1)\theta\right)^2\right]+\Pr f\theta'=0, \tag{10}$$

With the transformed boundary conditions

$$\left.\begin{array}{ll} f=0, f'=1, g=0, g''=0, \theta=1, & \text{at } \eta=0, \\ f'=a/c, g'=1, \theta=0, & \text{as } \eta\to\infty, \end{array}\right\} \quad (11)$$

where β is the casson parameter, a,c are constants, B is the magnetic parameter, λ is the reciprocal magnetic Prandtl number, R is the radiation parameter, θ_w is the temperature parameter, $\Pr$ the Prandtl number, which are represented below.

$$B=\frac{\mu H_0^2}{4\pi\rho c^2}, \lambda=\frac{1}{4\pi\sigma\upsilon}, R=\frac{16\sigma^* T_\infty^3}{3kk^*}, \theta_w=\frac{T_w}{T_\infty}, \Pr=\frac{\nu}{\alpha}, \quad (12)$$

For physical quantities of engineering interest are the shear stress coefficient C_f (friction factor) and the local Nusselt number Nu_x are given by

$$\mathrm{Re}_x^{1/2} C_f = f''(0), \quad (13)$$

$$\mathrm{Re}_x^{-1/2} Nu_x = -\theta'(0), \quad (14)$$

Where Re_x is the local Reynolds number, given by

$$\mathrm{Re}_x=\frac{u_w x}{\upsilon},$$

3. Results and Discussion

The system of nonlinear ordinary differential equations (8) – (10) with the boundary conditions (11) has been solved numerically using Runge-Kutta based shooting technique. Further the effects of various physical parameters like casson parameter β, magneticfield parameter B, temperature parameter θ_w, radiation parameter R and stretching ratio a/c on velocity, induced magneticfield and temperature profiles. For numerical results we considered $\theta_w=1.1,\ a/c=0.5,\ \beta=0.5, R=1, \lambda=0.1$. These values are kept as common in entire study except the varied values as displayed in the respective figures and tables.

Figs. 1 and 2 illustrate the effect of casson parameter β on velocity and induced magneticfield profiles. It is observed that an increase in the casson parameter declines the velocity as well as induced magneticfield profiles. This agrees the general physical behaviour of the non-Newtonian parameter. The effect of radiation parameter R on temperature profiles of the flow is shown in Fig. 3. It is found that an increase in the radiation parameter leads to an increase in the temperature profiles of the flow.

Figs. 4 and 5 depict the influence of magneticfield parameter on velocity and induced magneticfield profiles. It is evident that an increase in the magneticfiled parameter causes to increase in the velocity and induced magneticfield profiles. This is due to the hike in the retardation force acts along the flow. Fig. 6 displays the effect of temperature parameter on the temperature profiles of the flow. It is clear that an increase in the temperature parameter enhances the temperature profiles of the flow. This may happen due to the enhanced thermal conductivity of the flow.

Fig. 7 and 8 reveal the influence of the stretching ratio parameter on velocity profiles respectively and induced magneticfield profiles. It is interesting to note that an increase in the value of a/c enhances the momentum boundary layer thickness and depreciates the induced magneticfield profiles. But we have noticed an opposite results to the above with an increase in the reciprocal magnetic Prandtl number, which are displayed in Figs. 9 and 10.

Finally the effects of various physical parameters on friction factor (C_f) and local Nusselt number (Nu_x) are shown in Table 1. It is evident that an increase in the casson parameter and reciprocal magnetic Prandtl number reduces the friction factor and heat transfer rate. A raise in the magneticfiled parameter increases the friction factor and local Nusselt number. Stretching ratio parameter have tendency to increase the heat transfer rate and reduce the friction factor. Table 2 shows the validation of the present results with the existed literature under some special assumptions. We found an excellent agreement of the present results with the existed results.

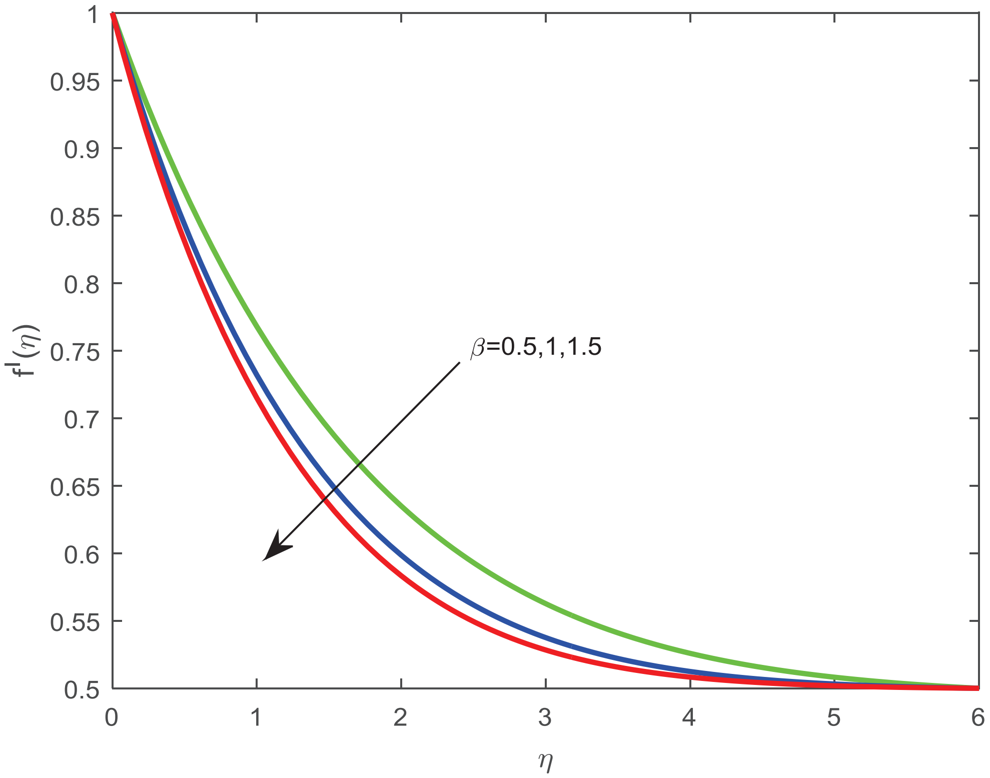

Fig.1 Velocity field for different values of casson parameter

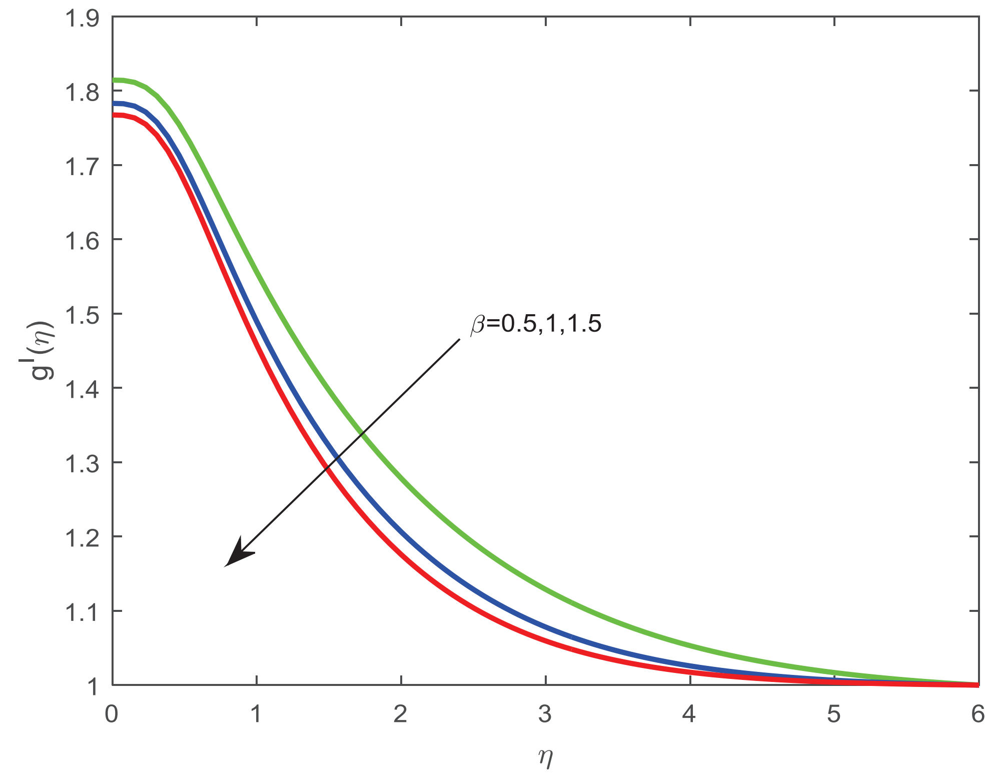

Fig.2 Induced magneticfiled for different values of casson parameter

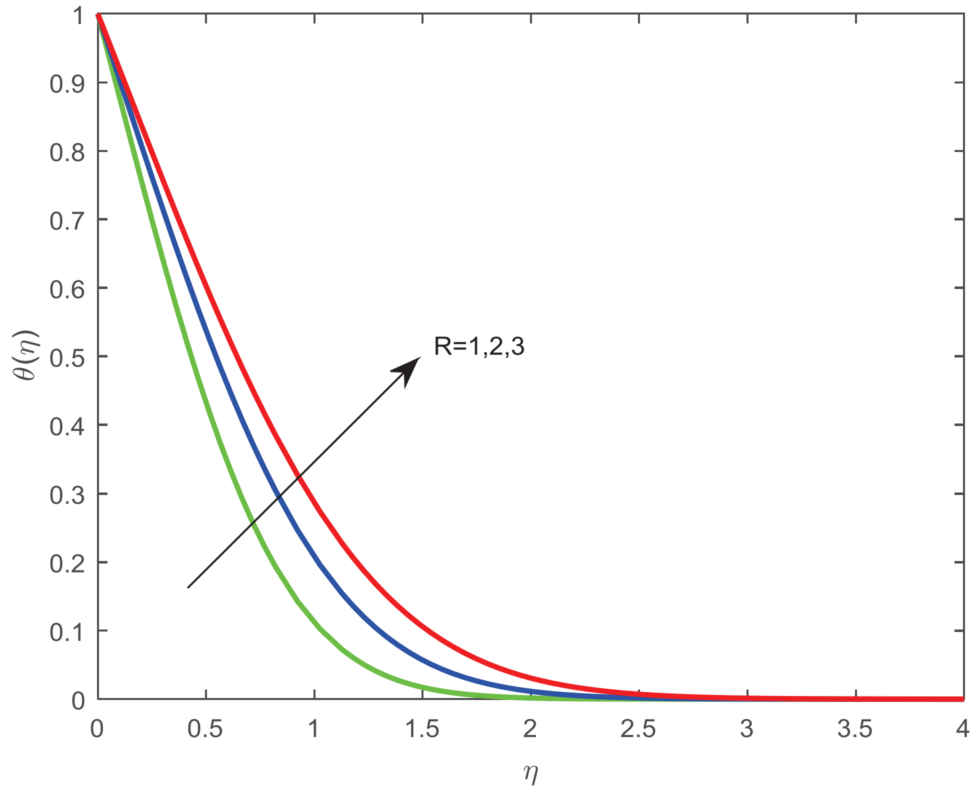

Fig.3 Temperature field for different values of radiation parameter

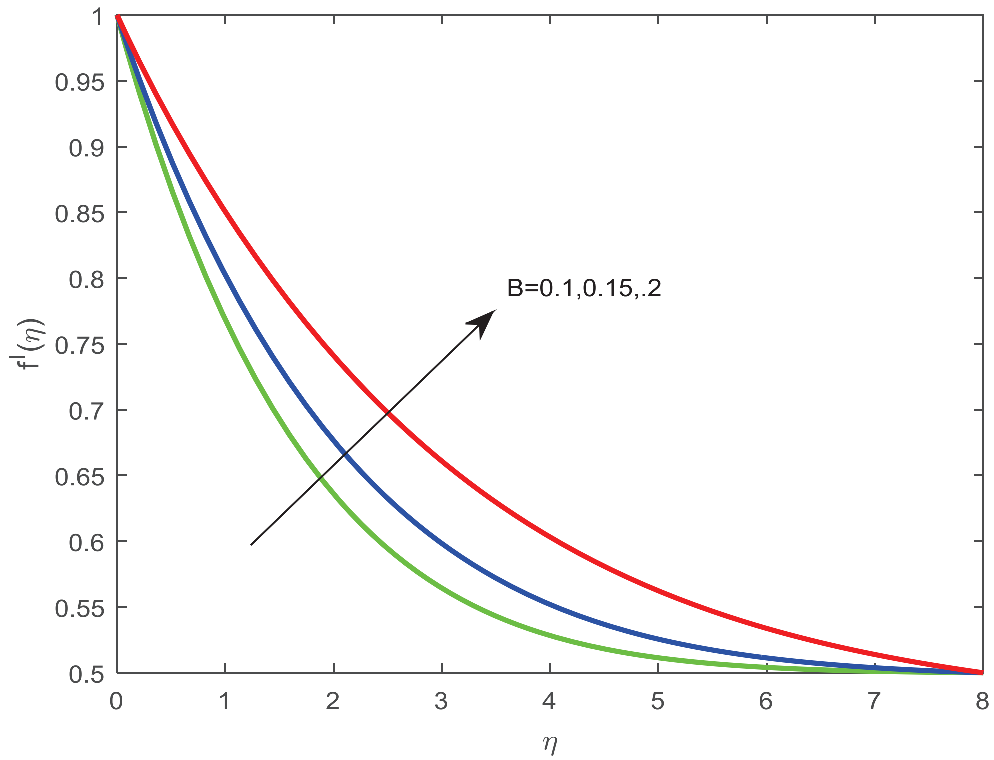

Fig.4 Velocity field for different values of magnetic parameter

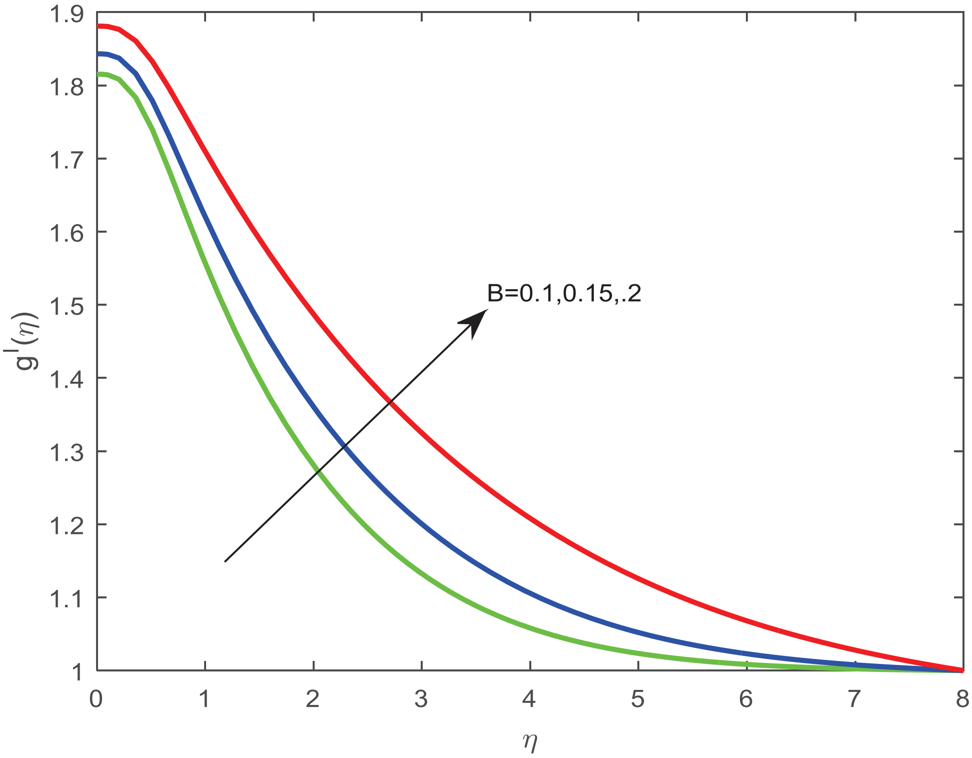

Fig.5 Induced magneticfiled for different values of magnetic parameter

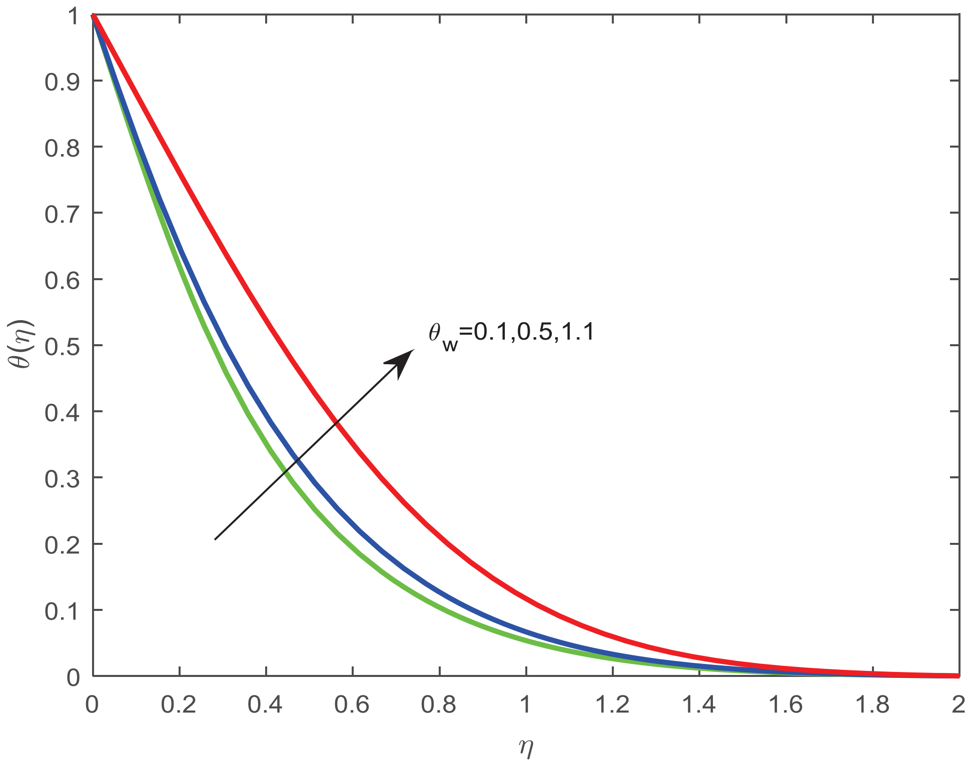

Fig.6 Temperature field for different values of temperature parameter

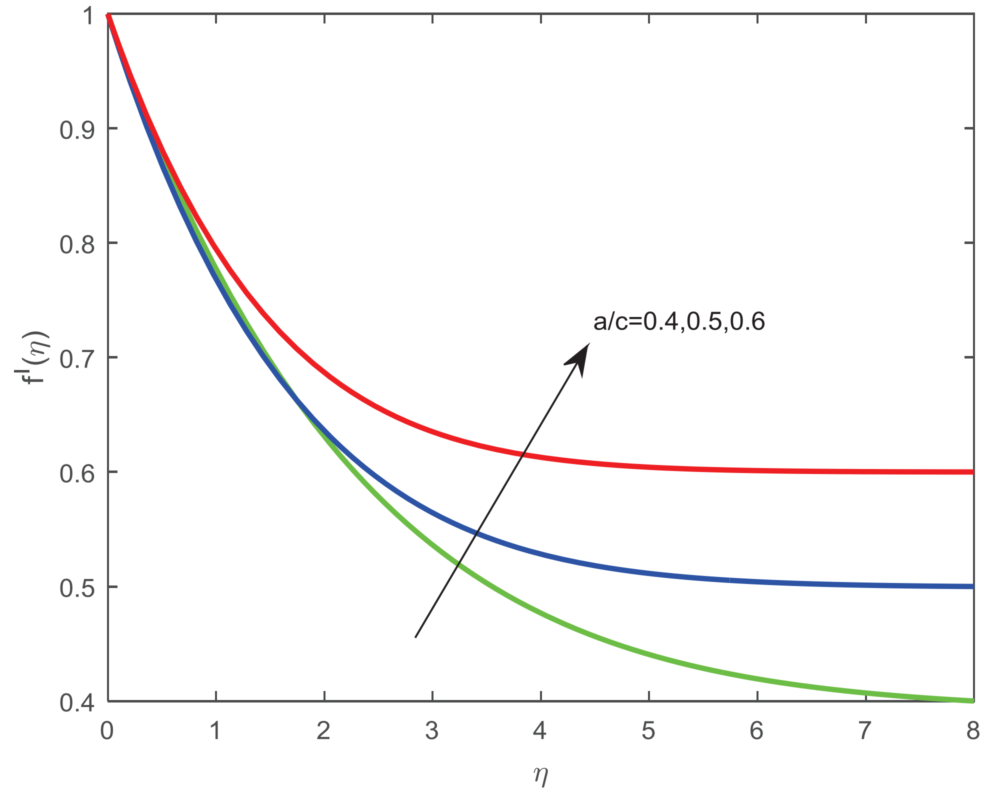

Fig.7 Velocity field for different values of stretching ratio parameter

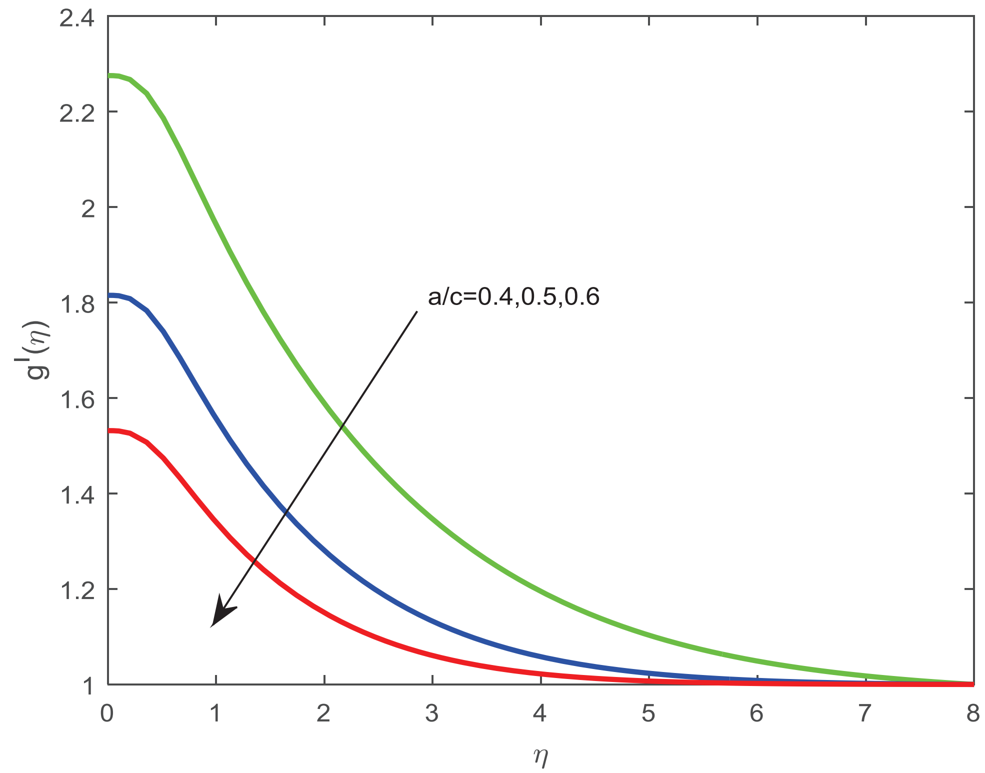

Fig.8 Induced magneticfield for different values of stretching ratio parameter

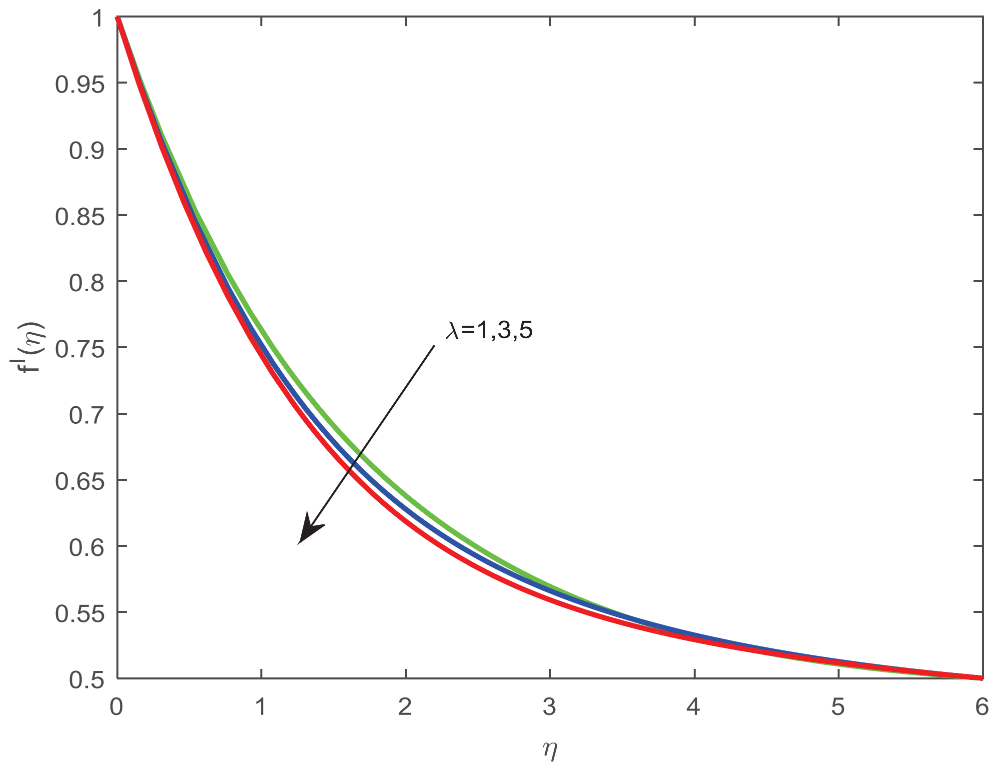

Fig.9 Velocity field for different values of reciprocal Prandtl number

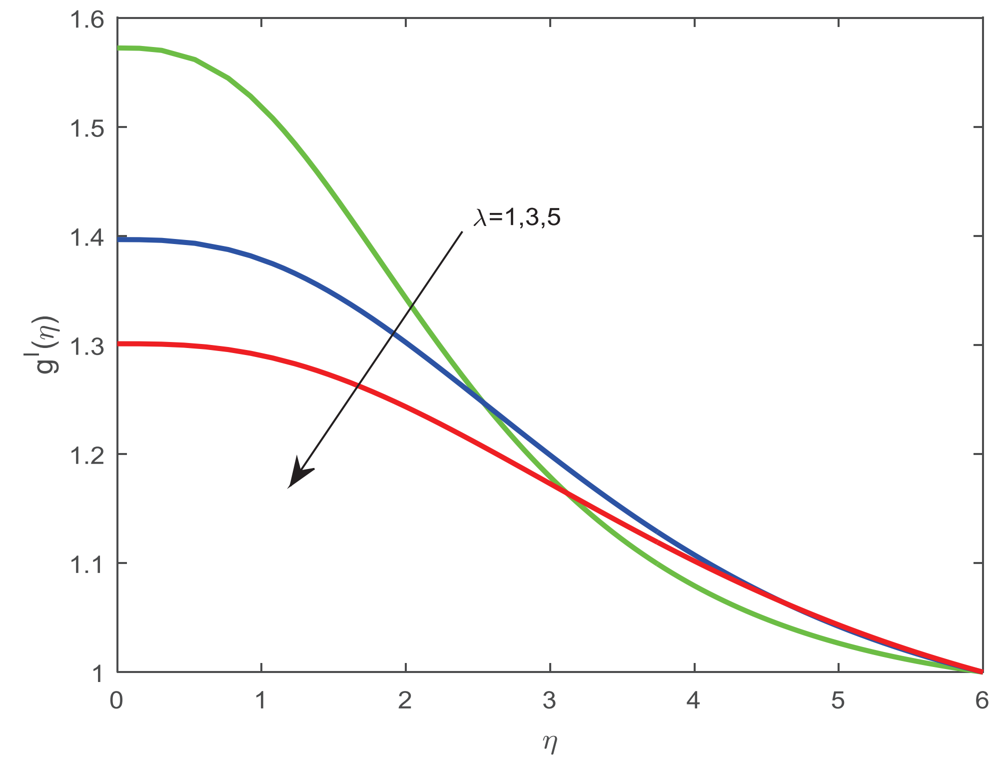

Fig.10 Induced magneticfield for different values of reciprocal Prandtl number

Table 1 Variation in friction factor (C_f) and local Nusselt number (Nu_x)

β	R	B	a/c	λ	$\mathrm{Re}_x^{1/2}\, C_f$	$\mathrm{Re}_x^{-1/2}\, Nu_x$
0.5					-0.303296	1.209661
1.0					-0.372816	1.198239
1.5					-0.409458	1.192435
	1				-0.309131	1.208331
	2				-0.309131	0.946552
	3				-0.309131	0.800788
		0.5			-0.302736	1.209787
		1.0			-0.249572	1.219391
		1.5			-0.180341	1.232002
			0.4		-0.278463	1.203306
			0.5		-0.302736	1.209787
			0.6		-0.275401	1.215931
				0.5	-0.321107	1.207280
				1.0	-0.340453	1.203828
				1.5	-0.351822	1.201666

Table 2 Comparison of the skin friction coefficient when $R = 0$.

a/c	Ali et al. (2011)	Present Results
0.1	-0.9694	-0.96935
0.2	-0.9181	-0.91820
0.5	-0.6673	-0.66737
2	2.0175	2.01749
3	4.7293	4.72934

4. Conclusions

This study presents a stagnation-point flow of a casson fluid towards a stretching sheet with nonlinear thermal radiation and induced magneticfield effects. The nonlinear partial differential equations are transformed into a set of nonlinear ordinary differential equations using self-suitable transformations, which are then solved numerically using Runge-Kutta based shooting technique. The conclusions are as follows:

- Magneticfiled parameter have tendency to enhance the heat transfer rate.
- Nonlinear thermal radiation enhances the thermal boundary layer thickness.
- Stretching ratio parameter helps to control the flow.
- Reciprocal magnetic Prandtl number have tendency to enhance the induced magneticfield profiles.
- Radiation parameter does not shown significant difference in the friction factor. But it helps to boost the temperature profiles of the flow.

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