# Optimal Analysis of Packaging Products of MAHEU Plant in Intafact Beverages Limited Using GPALS and MATLAB Optimization Software 

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#### Abstract

This work focused on the optimization of the two packaging products; Supershake and Chibuku made up of three and two parts respectively. Copolymer polypropylene and white or colored batch materials are the two raw materials needed to produce the two packaging products. The manufacturing plan was developed for the organization. The production inputs of $1.11,6.67,15.78,2.47$ and 7.70 units were generated as the objective function coefficients; 308 hours per month for day shift and 364 hours per month for night shift were established. Production time of 10 seconds, 20 seconds, 12 second, 10 seconds and 12 seconds per unit of the five parts were established. The manufacturing constraints in terms of machine capacities, material availability, time and labour were extensively used to develop an integer linear programming model to obtain the optimum quantities of each part that will yield the maximum profit. The developed model was analyzed with GPALS and MATLAB optimization solver to obtain results for the linear programming model which gave a monthly production net profit of N3,751,932. A decision support system was developed for the manufacturing planning to assist the management of Maheu plant in Intafact Beverages Limited in decision making. The model is now being used in the manufacturing plan of the company and also recommended for application in organizations with similar production inputs.


Keywords: Manufacturing plan, Production inputs, Manufacturing constraints, Optimization, Profit and Decision making

## 1. Introduction

For many manufacturers, the task of meeting the ever rising demand and customer expectations and lowering manufacturing costs in an environment of more products, more complexity, more choice and competition is placing great stress on the effectiveness of their planning of activities in the manufacturing processes. Organizations have already adopted solutions with varying degrees of planning and scheduling capabilities. Yet, operations executive acknowledge that these same systems are becoming outdated, lacking the speed, flexibility and responsiveness to manage their increasing complex manufacturing environment.

It is known that as the business grows, its operations expand its product range and adopt a global manufacturing supply chain. They must therefore, seek alternatives to the current ways of doing things if they are to remain competitive and responsive to customers' needs. Beverage and plastic industries have grown rapidly over the years in Nigeria and competition among manufacturers is so high that planning and scheduling of resources must be efficient for any of these industries to be profitable and able to survive competition.
The decision to manufacture multiple products on common resources results in the need for change over and set up activities, representing costly disruptions to manufacturing processes. Therefore, set-up reduction is an important feature of the continuous improvement program of any beverage and plastic manufacturing industry. It is even more critical if the industry expects to respond to changes like shortened lead times, smaller lot sizes and higher quality standards. In beverage and plastic production, packaging plastic product of different types and colors require different set up times. However, in today's manufacturing scheduling problems, it is of significance to efficiently utilize various resources. Treating set up times separately from processing times allows operations to be performed simultaneously and hence improve resource utilization. This is particularly important in modern manufacturing management systems, such as, Just-In-Time (JIT), Optimized Production Technology (OPT), Group Technology, (GT), Cellular Manufacturing and time-based competition. The benefits of reducing set up times include; reduced expenses, increased production speed, increased output, reduced lead times, faster change over, increased competitiveness, increased profitability and customer satisfaction. Planning and scheduling of products, machines, raw materials and labor are paramount to the profitability of any beverage and plastic industry in Nigeria and the world at large. The development of an optimal production plan for MAHEU plant is important at this juncture as the model developed can be applied to other organizations with similar production input and service organizations.

The aim of the study is to develop an optimal manufacturing plan for packaging products of MAHEU plant in Intafact Beverages Limited.

Production planning is one of the most important activities in a production industry. This represents the heartbeat of any manufacturing process. Its purpose is to minimize production time and costs, efficiently organize the use of resources and maximize efficiency in the work place. Production planning incorporates a multiplicity of production elements, ranging from the everyday activities of staff to the ability to realize accurate delivery time for the customer. The production planning usually fulfils its functions by determining the required capacities and materials for these orders in quantity and time [1]. The role of production planning department, including routing, dispatching (issuing shop orders) and scheduling. Production scheduling pertains to establishing the timing of the use of equipment, facilities and human activities in an organization. In the decision making hierarchy, scheduling decisions are the final step in the transformation process before actual output occurs [2]. The two key problems in the production scheduling are "priorities" and "capacity" [3]. In other words, what should be done first? And who should do it? He observes that in manufacturing firms, there are multiple types of scheduling, including the detailed scheduling of shop order that shows when each operation must start and complete. Detailed scheduling was defined as the actual assignment of starting and/or completion of duties to operations or groups of operations to show when these must be done if the manufacturing order is to be completed on time [4]. The scheduling personnel determine which specific worker and machine will does which task.

Production planning and scheduling belongs to different decision making levels in process operations. They are also closely related since the result of planning problem is the production target of scheduling problem. It is necessary to develop methodologies that can effectively integrate production planning and scheduling. A lot of researchers have done extensive work in developing efficient solution strategies [5, 6].

### 1.1 Manufacturing Planning Models

Any planning problem starts with a specification of customer demand that is to be met by the production plan. Production planning problem are one of the most interesting application for optimization tools using mathematical programming. The idea of incorporating uncertainty in mathematical models appears initially with Dantzig, well known as the father of linear programming [7].

An exhaustive model for production planning under uncertainty was received [8]. They carried out their research over the following seven categories of production planning: hierarchical production planning, aggregate production planning, material requirement planning, inventory management and supply chain planning. They also identified four modeling approaches: conceptual, analytical, artificial intelligence and simulation models. These modeling approaches were originally defined [9]. Mula et al concluded that the analytical modeling approach, in particular stochastic programming was the most frequency encountered [10].

Hax and Meal introduced the notion of hierarchical production planning and provide a specific framework for this, whereby there is an optimization model with each level of hierarchy [11]. Each optimization model imposes a constraint on the model at the next level of the hierarchy. Bitran and Triupati, provided a comprehensive survey of hierarchical planning methods and models [12]. While Gfrere and Zapfel,presented a multi-period hierarchical production planning model with two planning models, that is aggregate and detailed and with uncertain demand [13]. On the other hand, Maybodi and Foote developed a multi-period for hierarchical production planning and scheduling with random demand and production failure [14]. Zapfel, presented hierarchical model that can be incorporated in a manufacturing resource planning (MRP) system to program the production with demand uncertainty [15].

Graves, developed linear programming model for production planning under the following context: multiple items with independent demands. Multiple shared resources, big bucket time periods and linear costs [16]. Fandal and Stammend, present an optimization model for a supply chain composed of the following process: procurement, production, distribution and recycling [17]. The model is a multi-product, multi-echelon, multicountry, dynamic application with the objective of maximizing global after tax profit. Vila et al, discuss a strategic production planning problem with an application to the lumber industry [18]. The context included the possibility to choose between the production and a planning horizon which is divided into four periods, each corresponding to one session. The objective is global after tax profit maximization. Ouhimmou et al., developed an optimization model to support tactical decisions concerning procurement contracts, inventory levels and demand allocation and outsourcing policies [19]. The overall objective is total cost maximization.

Billington et al, studied the interaction among lead times, lot sizing and capacity constraints in a production process with complex bill of materials and uncertainty in demand and lead times [20]. Escudero et al, analyzed different modeling approaches for the production and capacity problem using stochastic programming [21]. Escudero and Kamesam, develop linear programming models for stochastic planning problems and methodology to solve them [22]. They used a production problem with uncertainty in demand, to characterize a test case. Rota et al, presented a mixed linear programming model to address the uncertain nature and complexity of manufacturing environments [23]. Their proposed model includes; capacity constraints, firm orders, subcontracting decisions, demand forecasts and supply for a rolling planning processes.

Aggregate production planning is not left out in literature. According to Axsater, the purpose of the aggregate plan is to ensure that long term considerations are not ignored when making long term decisions [24]. According to Lee and Khumawala, aggregate production planning is related to how it will determine aggregate planning production levels, inventory and work force size [25]. Production planning has serious impact on the cost of production. Gianesi, pointed out the impact of the planning process on direct and indirect costs, on delivery speed, on delivery reliability and on flexibility [26]. Bitran et al. proposed linear programming models to solve the aggregate production planning problem respectively with a single stage and two stage approach at the product type aggregation level [27]. The objectives were to minimize overall cost, including raw materials cost and inventory and backorder goals per period.

## 2. Methodology

The research methodology adopted in this work is a case of an existing production system in a packaging manufacturing company in order to investigate and improve its manufacturing plan for maximum profit. The model to be generated through this approach can be applied in planning and scheduling processes. The simulation and solution technique to this model will be based on Sensitivity Analysis method using GPALS and MATLAB optimization software.
2.1. Model Formulation: In order to formulate an integer linear programming model for the products, which are; a Super shake $\left(k_{1}\right)$, and Chibuku $\left(k_{2}\right)$, we shall describe a bit of the production scenario or process.

These products consist of $k_{1}$ : crate of super shake, super shake cover and super shake body, $k_{2}$ : chibuku crate and container. They are produced on three different machines of different capacities; Machine 1 which has the least capacity is used in producing super shake cover and Chibuku crates.

Machine 2 which is the next in capacity to the 1 st is used to produce the super shake crate and the Chibuku plastic body containers.

Machine 3 which has the highest capacity is used in producing only the body of the super shake containers.
The materials involved in the production of these products parts are the same and they are (1) PPCP (material M), co-polymer poly propylene and (2) white or colored Batch (Material N) mixed at a different proportion.

The linear programming model to be developed at this juncture is to help in a monthly optimum manufacturing plan for these products with an objective to maximize profit; hence we formulate the LP as follows.

[^0]$\mathrm{P}_{\mathrm{T}}$ optimum profit $\left(\mathrm{K}_{1}, \mathrm{~K}_{2}\right)=\left\{\Sigma\left(\mathrm{S}_{\mathrm{i}-} \mathrm{C}_{\mathrm{i}}\right) \mathrm{X}_{\mathrm{ij}}\right\}-\mathrm{Fc}$
Subject to:
\[

$$
\begin{gather*}
\sum_{i=1}^{5} t_{i} x_{i j} \geq T_{j} \quad \forall_{j} \\
\sum_{j=2}^{2} x_{i j} \geq d_{i} \quad \forall i \\
\sum_{i=1}^{5} \sum_{j=i}^{2} M_{i x i j} \leq \quad A_{m} \\
\sum_{t=1}^{5} \sum_{j=1}^{2} n_{i} x_{i j} \leq A_{n} \\
t_{i} \sum_{j=1}^{2} x_{i j} \leq(24 \times 28 \times 1) \quad \forall_{i j}  \tag{2}\\
x_{11}+x_{12}=x_{21}+x_{22} \\
x_{21}+x_{22}=x_{31}+x_{32} \\
x_{41}+x_{42}=x_{51}+x_{52} \\
x_{i j} \geq 0
\end{gather*}
$$
\]

On expansion of the linear programming model of equation (1) and (2) [28], we have;
$\operatorname{Max} P_{T}\left(K_{1}, K_{2}\right)$

$$
\begin{align*}
= & \left(S_{1}-C_{1}\right)\left[X_{11}+X_{12}\right]+\left(S_{2}-C_{2}\right)\left[X_{21}+X_{22}\right] \\
& \left(S_{3}-C_{3}\right)\left[X_{31}+X_{32}\right]+\left(S_{4}-C_{4}\right)\left[X_{42}+X_{42}\right] \\
& +\left(S_{5}-C_{5}\right)\left[X_{51}+X_{52}\right]-F_{C} \tag{3}
\end{align*}
$$

Subject to:

$$
\begin{align*}
& t_{1} x_{11}+t_{2} x_{21}+t_{3} x_{31}+t_{4} x_{41}+t_{5} x_{51} \geq T_{1}  \tag{4}\\
& t_{1} x_{12}+t_{2} t_{22}+t_{3} t_{32}+t_{4} x_{42}+t_{5} x_{52} \geq T_{2}  \tag{5}\\
& x_{11}+x_{12} \geq d_{1}  \tag{6}\\
& x_{21}+x_{22} \geq d_{2}  \tag{7}\\
& x_{31}+x_{32} \geq d_{3}  \tag{8}\\
& x_{41}+x_{42} \geq d_{4}  \tag{9}\\
& x_{51}+x_{52} \geq d_{5}  \tag{10}\\
& M_{1}\left(x_{11}+x_{12}\right)+M_{2}\left(x_{21}+x_{22}\right)+M_{3}\left(x_{31}+x_{32}\right)+M_{4}\left(x_{41}+x_{42}\right) \\
& +M_{5}\left(x_{51}+x_{52}\right) \leq A_{n}
\end{align*}
$$

Period under consideration is 24 hrs per day for 28days in a month
$n_{1}\left(x_{11}+x_{12}\right)+n_{2}\left(x_{21}+x_{22}\right)+n_{3}\left(x_{31}+x_{32}\right)+n_{4}\left(x_{41}+x_{42}\right)+n_{5}\left(x_{51}+x_{52}\right) \leq A_{n}$
$t_{1}\left(x_{11}+x_{12}\right) \leq 24 \times 28 \times 1$
$t_{2}\left(x_{21}+x_{22}\right) \leq 24 \times 28 \times 1$
$t_{3}\left(x_{31}+x_{32}\right) \leq 24 \times 28 \times 1$
$t_{4}\left(x_{41}+x_{42}\right) \leq 24 \times 28 \times 1$
$t_{5}\left(x_{51}+x_{52}\right) \leq 24 \times 28 \times 1$
$x_{11}+x_{12}-x_{21}-x_{22}=0$
$x_{21}+x_{22}-x_{31-} x_{32}=0$
$x_{41}+x_{42}-x_{51}-x_{52}=0$
$x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}, x_{41}, x_{42}, x_{51}, x_{52} \geq 0$
For our computation using GPALS and MATLAB Software, we have our variable to be:

$$
\begin{aligned}
& x_{11}=x_{1} \\
& x_{12}=x_{2} \\
& x_{21}=x_{3} \\
& x_{22}=x_{4} \\
& x_{31}=x_{5} \\
& x_{32}=x_{6} \\
& x_{41}=x_{7}
\end{aligned}
$$

$$
\begin{aligned}
& x_{42}=x_{8} \\
& x_{51}=x_{9} \\
& x_{52}=x_{10}
\end{aligned}
$$

### 2.2. Determination of Values of Parameters (Coefficients)

In order to solve the linear programming model of equations (3) to (20), we need to evaluate the coefficient of the independent variables to enable us make our input into the GPALS and MATLAB software for analysis. These coefficients (parameters) were obtained based in the data (information) given by the management of Intafact Beverages Limited (MAHEU plant) as follows:
The selling price of the product $K_{1}=\# 90$
The selling price of the product $\mathrm{K}_{2}=\mathrm{N} 23$
The mixing ratio of material M (PPCP) to material n (master batch) was obtained from the organization as 0.150 kg ( 150 g ) of m , therefore the distributions for the five parts are given below;
Ratio of master batch to PPCP:
$150 \mathrm{~g}: 25000 \mathrm{~g}$, which

## yields $\longrightarrow 1 \mathrm{~g}: 166.7 \mathrm{~g}$

Therefore, a unit of super shake crate which weighs 10 g is made up; 0.06 g (master batch): 9.94 g (PPCP)
A unit of super shake cover which weighs 60 g is made up of 0.36 g (master batch): 9.64 g (PPCP)
A unit of Chibuku crate which weighs 10 g is made up of 0.06 g (master batch): 9.94 g (PPCP).
A unit of super shake body which weighs 142 g is made up of; 0.85 g (master batch): 9.94 g (PPCP)
A unit of chibuku container/ body which weighs 31 g is made up of ;
0.19 g (master batch): 30.81 g (PPCP)

The costs of these materials as given by the company are as follows;
$15,000 \mathrm{~kg}(15000000 \mathrm{~g})$ of PPCP is $=\mathrm{N} 4,650,000.00$
$25 \mathrm{~kg}(25000 \mathrm{~g})$ of master batch is $=\mathrm{N} 22,000.00$
From these costs, we can now get the cost of materials for a unit of part ithus;

$$
\begin{aligned}
c_{1} & =c_{1 m}+c_{1 n} \\
& =3.08+0.05= \\
c_{2} & =c_{2 m}+c_{2 n} \\
& =18.49+0.32=\sharp 18.81 \\
c_{3} & =c_{3 m}+c_{3 n} \\
& =43.76+0.75=\sharp 44.57
\end{aligned}
$$

Therefore the unit material cost of $k_{1}$ is;

$$
\begin{aligned}
c_{1}+c_{2}+c_{3} & =\sharp 66.51 \\
\text { Also; } c_{4} & =c_{4 m}+c_{4 m} \\
& =3.08+0.05= \pm 3.13 \\
c_{5} & =c_{5 m}+c_{5 n} \\
& =9.55+0.17=\ldots 9.72
\end{aligned}
$$

Therefore, the ratio of material cost of $k_{2}$ is

$$
c_{4}+c_{5}=\# 12.85
$$

Based on the ratio of material cost of each part $i$, we calculate the selling price of unit of part $i$ with respect to selling price of $k_{1}$ and $k_{2}$;
For $k_{1}$ the ratio is $1: 6.01: 14.22$
For $k_{1}$ the ratio is $=1: 3.11$
Hence; $\quad s_{1}=\frac{1}{21.23} \times \frac{90}{1}=\# 4.24$
$s_{2}=6.01 \times 4.24=\mathrm{N} 25.48$
$s_{3}=14.22 \times 4.24=\mathrm{N} 60.29$
$s_{4}=\frac{1}{4.11} \times \frac{23}{1}=\$ 5.60$
$s_{5}=3.11 \times 5.60=\# 17.42$
So, we now have the objective function coefficients arithmetically obtained as;

$$
\begin{aligned}
& s_{1}-c_{1}=1.11 \\
& s_{2}-c_{2}=6.67
\end{aligned}
$$

$$
\begin{aligned}
s_{3}-c_{3} & =15.78 \\
s_{4}-c_{4} & =2.47 \\
s_{5}-c_{5} & =7.70
\end{aligned}
$$

The total available day shift time $T_{1}=11 \times 24=308 \mathrm{hrs} /$ month . The total available day shift time $T_{2}=13 \times 24=364 \mathrm{hrs} /$ month .
Production times for various units of parts $\mathrm{i}\left(\mathrm{t}_{\mathrm{i}}\right)$ are;

$$
\begin{aligned}
& t_{1}=0.0028 \mathrm{hrs} \times 60 \times 60=10 \text { secs } . \\
& t_{2}=0.0056 \mathrm{hrs} \times 60 \times 60=20 \text { secs } \\
& t_{3}=0.0033 \mathrm{hrs} \times 60 \times 60=12 \text { secs } . \\
& t_{4}=0.0028 \mathrm{hrs} \times 60 \times 60=10 \text { secs } . \\
& t_{5}=0.003 \mathrm{hrs} \times 60 \times 60=12 \text { secs } .
\end{aligned}
$$

For the needed material consumption in kg , we have;

$$
\begin{aligned}
m_{1} & =9.94 \times 10^{-3} \mathrm{~kg}(9.94 \mathrm{~g}) \\
m_{2} & =141.15 \times 10^{-3} \mathrm{~kg}(141.15 \mathrm{~g}) \\
m_{3} & =59.69 \times 10^{-3} \mathrm{~kg}(59.69 \mathrm{~g}) \\
m_{4} & =9.94 \times 10^{-3} \mathrm{~kg}(9.94 \mathrm{~g}) \\
m_{5} & =30.81 \times 10^{-3} \mathrm{~kg}(30.81 \mathrm{~g}) \\
n_{1} & =0.06 \times 10^{-3} \mathrm{~kg}(0.06 \mathrm{~g}) \\
n_{2} & =0.36 \times 10^{-3} \mathrm{~kg}(0.36 \mathrm{~g}) \\
n_{3} & =0.85 \times 10^{-3} \mathrm{~kg}(0.85 \mathrm{~g}) \\
n_{4} & =0.06 \times 10^{-3} \mathrm{~kg}(0.06 \mathrm{~g}) \\
n_{5} & =0.19 \times 10^{-3} \mathrm{~kg}(0.19 \mathrm{~g})
\end{aligned}
$$

Estimated demand for super shake containers

$$
d_{k i}=120,000
$$

Estimated demand for Chibuku containers

$$
d_{k_{2}}=90,000
$$

Fixed cost obtained from MAHEU PLANT is:
Machine costs per month;
Machine $1=\$ 86,000$
Machine $2=\mathrm{N} 190,000$
Machine $3=\$ 226,000$
Power cost per month;
National power supply $=\$ 120,000$
AGO/Diesel = N138,250
Labour/Manpower = $\mathrm{N} 246,000$
Machine Maintenance $=\$ 140,000$
Total Cost $=\mathrm{N}, 146,250$

## 3. Results and Discussion

### 3.1. Data Analysis Using GPALS Software:

The data obtained above through computation will now be placed as the coefficients in equations (3) to (20), and the objective function now becomes:
$\operatorname{Max} P_{T}\left(K_{1}, K_{2}\right)$
$=\left(S_{1}-C_{1}\right)\left[x_{11}+x_{12}\right]+\left(S_{2}-C_{2}\right)\left[x_{21}+x_{22}\right]+\left(S_{3}-C_{3}\right)\left[x_{31}+x_{32}\right]+\left(S_{4}-C_{4}\right)\left[x_{41}+\right.$
$\left.x_{42}\right]+\left(S_{5}-C_{5}\right)\left[x_{51}+x_{52}\right]-f_{c}$
$\longrightarrow \operatorname{Max} P_{T}\left(K_{1}, K_{2}\right)$

$$
=1.11\left[x_{11}+x_{12}\right]+6.67\left[x_{21}+x_{22}\right]+15.78\left[x_{31}+x_{32}\right]+2.47\left[x_{41}+x_{42}\right]+
$$

$7.70\left[x_{51}+x_{52}\right]-f_{c}$
Where $F_{c}=$ Fixed cost
$\square \operatorname{Max}_{T}\left(K_{1}, K_{2}\right)$

$$
\begin{align*}
& \quad=\left[1.11 x_{11}+1.11 x_{12}\right]+\left[6.67 x_{21}+6.67 x_{22}\right]+\left[15.78 x_{31}+15.78 x_{32}\right]+ \\
& {\left[2.47 x_{41}+2.47 x_{42}\right]+\left[7.70 x_{51}+7.70 x_{52}\right]-1,146,250} \tag{21b}
\end{align*}
$$

Equation (21b) is our optimum profit equation and will now be analyzed using GPALS. The software input window is given as:


Figure 1: A Typical GPALS LP Window 1
A typical input into a GPALS software is exhibited above with the original data (coefficients) shown at respective columns and rows. By clicking the solve menu button prompts then the next step to the solution.

From figure 1, it can be seen that after the input of the objective function coefficients and the constraints that has been generated from the data obtained from the organization under study, choosing the objective type (maximization) and clicking solve; four million, eight hundred and ninety eighty thousand, one hundred and eighty two naira ( $\mathrm{N} 4,898,182$ ) was obtained as the gross optimum profit.

The net profit of this solution will be obtained when the fixed cost, one million, one hundred and forty six thousand, two hundred and fifty naira ( $\mathrm{N} 1,146,250$ ) is subtracted from the gross optimum profit, which gave three million, seven hundred and fifty one thousand, nine hundred and thirty two naira ( $\mathrm{N} 3,751,932$ ).

The solution suggests a manufacturing plan and scheduling as follows; X9 which is X 51 in the solutions means that the chibuku body should be produced to a maximum quantity of 203, 636 pieces.
$\mathrm{X}_{3}$ which is $\mathrm{X}_{21}$, in the solution means that the supershake pack should be produced to a maximum quantity of 120,000 pieces. $\mathrm{X}_{6}$ which is $\mathrm{X}_{32}$, in the solution means that the supershake containers should be produced to a maximum quantity of 120,000 pieces.
$\mathrm{X}_{4}$ which is $\mathrm{X}_{22}$, in the solution means that the supershake cover should be produced also to a maximum quantity of 120,000 pieces. $X_{7}$ which is $X_{41}$ in the same optimum solution means that the chibuku pack is to be produced to a maximum quantity of 203, 636 pieces.

Furthermore, this indicates that for a gross profit of $\mathrm{N} 4,898,182$ to be achieved during a monthly manufacturing plan, all parts that make up the supershake should be produced only during the night shift while the chibuku should be produced only during the day shift but more than the stipulated demand of 90,000 pieces given by the model, though the model gave room for inventory.

Another change on the model made on the right-hand side of the equations (11) and (12) where the total available material resources was reduced from $40,000 \mathrm{~kg}$ of PPCP to $30,000 \mathrm{~kg}$ and 250 kg of master batch to 200 kg . In the effect, the inequality signs of these equations changed from $\geq$ to $\leq$ (greater than or equal to, to less than or equal to).

The optimum solution for the model gave four million, three thousand, two hundred and seventy six naira only ( $\mathrm{N} 4,003,276$ ).
3.2. Sensitivity Analysis Using GPALS Optimization Solver


Figure 2: A Typical GPALS Software Window 2
In figure 2, the model gave an optimum profit of ( $\mathrm{N} 4,003,276$ ), the value was so as a result of reduction in the raw materials (PPCP and master batches). The model suggested the same manufacturing plan as that of the original LP model, but as a result of reduction in the quantity of the material resources, the production of the Chibuku containers was reduced to 115,642 pieces to accommodate the change in constraints.
Table1: Detailed Linear Program Result in GPALS

| Title: | Linear program |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Number of variables: 10. |  |  |  |  |
| Number of constraints: 17. |  |  |  |  |
| Maximization of objective function. |  |  |  |  |
| Primary objective function value: $4.89818181818348 \mathrm{E}+06$ |  |  |  |  |
| Dual objective function value: $4.89818181818182 \mathrm{E}+06$ |  |  |  |  |
| No. | Name | Value | Reduced Cost | Description |
| 1 | x1 | 93045.75 | 0 |  |
| 2 | x2 | 26954.25 | 0 |  |
| 3 | x3 | 76515.46 | 0 |  |
| 4 | x4 | 43484.54 | 0 |  |
| 5 | x5 | 117061.4 | 0 |  |
| 6 | x6 | 2938.638 | 0 |  |
| 7 | x7 | 192890.1 | 0 |  |
| 8 | x8 | 10746.22 | 0 |  |
| 9 | x9 | 194735.6 | 0 |  |
| 10 | x10 | 8900.76 | 0 |  |
| No. | Constraint | Dual Value | Activity (A*x) | Relaxation \|b- A*x| |
| 1 | CONST1 | 0 | 2380.665 | 2072.665 |
| 2 | CONST2 | 0 | 294.8892 | 69.11084 |
| 3 | CONST3 | -1.291351 | 120000 | 0 |
| 4 | CONST4 | 1.425769 | 120000 | 0 |
| 5 | CONST5 | 1.84663 | 120000 | 0 |
| 6 | CONST6 | 0 | 203636.4 | 113636.4 |
| 7 | CONST7 | 0 | 203636.4 | 113636.4 |
| 8 | CONST8 | 0 | 33585.78 | 6414.218 |
| 9 | CONST9 | 0 | 203.3091 | 46.69091 |
| 10 | CONST10 | 0 | 336 | 336 |
| 11 | CONST11 | 0 | 396 | 276 |
| 12 | CONST12 | -4560.901 | 672 | 0 |
| 13 | CONST13 | 0 | 570.1818 | 101.8182 |
| 14 | CONST14 | -3081.818 | 672 | 0 |
| 15 | CONST15 | 0.1813512 | 0 | 0 |
| 16 | CONST16 | -7.914418 | 0 | 0 |
| 17 | CONST17 | -2.47 | 0 | 0 |

Table 2: Detailed Linear Program Result in GPALS


The difference between the detailed linear programmed result and sensitivity analysis is not farfetched; sensitivity analysis tends to address the minor and major changes if there is a change in the original data. The detailed linear program result cannot be totally presented before the management since the data and figures may not be trusted unless it undergoes sensitivity analysis.

### 3.3. Data Analysis Using MATLAB Optimization Solver



Figure 3: A typical MATLAB solver interface Graphical Representations of the Results Generated By MATLAB Optimization Solver after Sixteen Iterations


Figure 4: Graph of objective function coefficient against number of variables
Figure 4, shows the graph shows bar of current points against the ten variables that constitute the objective function coefficients.

Total Function Evaluations: 187


Figure 5: Graph showing number of iterations
Figure 5, shows the total number of iterations that were carried out to obtain the optimum result.


Figure 6: Graph of current function values against iterations
Figure 6, shows the function values against the number of iterations. The value $-3.75193 \mathrm{e}+006$ which is $3,751,934$ is the optimum profit gotten before -4 at the $y$-axis. This value ( $3,751,934$ Naira) is the optimum net profit.


Figure 7: Graph of step size against iterations
Figure 7, shows the step sizes of the 16 iterations it took to achieve the optimum result. The average step size is $5.908 \mathrm{e}-005$
4. Conclusion: The monthly manufacturing plan for Maheu plant in Intafact Beverages Limited was developed in this work. In order to articulate properly the problem, the model was formulated as integer linear programming. The model was solved by GPALS and MATLAB Optimization solver software. It gave the products quantity which should be produced in each machine during each shift and expected profit if the optimum manufacturing plan for the month is adhered to. The results were presented in the discussions. The linear programming model developed here could be used for other different applications of services and operations with more than one type of service at a time or multiple products.

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[^0]:    2.1.1. Definition of Variable and Parameters (Alternative Variable):
    $\mathrm{X}_{\mathrm{ij}}=$ No of parts i produced at shift j
    $\mathrm{i}=5$ parts of 2 products.
    $\mathrm{j}=2$ shifts of Day ( 11 hrs ) and Night (13hrs)
    Model variables and parameters;
    $P_{T}\left(K_{1}, K_{2}\right)=$ Total profit from sales of product $k_{1}$ and $k_{2}$
    As; $k_{1}=x_{1}+x_{2}+x_{3}$
    $k_{2}=x_{4}+x_{5}$
    $\mathrm{i} \quad \Longrightarrow \mathrm{i}_{1} \Longrightarrow$ crate of super shake.
    $\mathrm{i}_{2} \Longleftrightarrow$ super shake cover.
    $\mathrm{i}_{3} \longrightarrow$ super shake body.
    $\mathrm{i}_{4} \longmapsto$ Chibuku crate
    $\mathrm{i}_{5} \longrightarrow$ Chibuku body
    $\mathrm{S}_{\mathrm{i}} \longrightarrow$ selling price of unit of i .
    $\mathrm{C}_{\mathrm{i}} \Longrightarrow$ unit cost of manufaturing of part i(material)
    $\mathrm{F}_{\mathrm{c}} \Longleftrightarrow$ fixed costs (salary/overtime, power, maintenance etc)
    $\mathrm{d}_{\mathrm{i}} \quad \Longrightarrow$ Demand of part i per month.
    $\mathrm{t}_{\mathrm{j}} \Longleftrightarrow$ Total available machine time (hours) for shift j
    $\mathrm{t}_{\mathrm{i}} \Longrightarrow$ Required production time (hours) for unit part i
    $\mathrm{x}_{\mathrm{ij}} \longrightarrow$ number of variables at each shift day and night.
    $\mathrm{M}_{\mathrm{i}} \Longleftrightarrow$ weight of material $\mathrm{m}(\mathrm{kg})$ needed to produce a unit of part i
    $\mathrm{N}_{\mathrm{i}} \longrightarrow$ weight of material $\mathrm{n}(\mathrm{kg})$ needed to produce unit price i
    $\mathrm{A}_{\mathrm{m}} \Longrightarrow$ Available quantity of material $\mathrm{M}(\mathrm{kg})$
    $\mathrm{A}_{\mathrm{n}} \Longleftrightarrow$ Available quantity of material $\mathrm{n}(\mathrm{kg})$
    $\forall i \Longleftrightarrow$ For all values of $i$
    $\forall j \Longrightarrow$ for all values of j
    Objective function:

