

Free Convection Boundary Layer Flow and Heat Transfer of a Nano Fluid over a Moving Plate with Internal Heat Generation

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Abstract
This paper presents considers the extended Blasius Flow problem and Heat transfer characteristics in Nano fluid with internal heat generation by considering free stream parallel to a moving plat plate which has much practical significance. The total transport model employed here includes the effect of Blasius motion and internal heat generation in energy transfer. It is also assumed that the plate moves in same or opposite direction to the free stream. Using the Similarity transformations, the transport equations followed by their Numerical computations. The governing system of highly non-linear ordinary differential equations are solved for three kinds of nanoparticals namely Cooper(Cu), Alumina() and Titania() in the water base fluid with the value of Prandtl Number Pr=6.2 numerically. Initially by the similarity analysis the governing partial differential equations are converted into system of non linear ordinary differential equations. The governing p.d.e's are converted into system of first order o.d.e's and are solved numerically using MATLAB ode45 solver. The effect of various Physical parameters encountered in the problem such as solid volume fraction of three types of nano particles on velocity transfer and Energy transfer are analysed and studied by representing through graphs. The results are compared with earlier published results and are well in agreement.
Keywords: Nano fluid, moving plate, heat transfer, ode solver, solid volume fraction

1. Introduction

A Nanofluid is a fluid containing nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Nanofluids are suspensions of nanoparticles in fluids that show significant enhancement of their properties at modest nanoparticle concentrations. A nanofluid is a dilute suspension of nanometer-size particles and fibers dispersed in a liquid.

The study of flow with heat and mass transfer is a great deal in industrial application such as chemical and drying process, crystal magnetic damping control, chromatography, heat and mass transfer characteristic in Nano fluid for power plants. Further the fluid flow over a continuously Stretching / shrinking surface has numerous important applications in engineering processes such as polymer extrusion, drawing of plastic films and wires, glass fibers, paper production, manufacture of foods, crystal growing liquid films in condensation process. When the fluid is particulates such as emulsions suspension foams and polymer solutions such fluids are nano fluids enhance the thermal conductivity of base fluid enormously which are vary stable. The suspended nano particles can change the transport and thermal property of base fluid. Nano fluid is envisioned to describe a fluid in which nano meter sized particles are suspended in convectional heat transfer of basic fluids. The convectional heat transfer fluids including oil, water and ethylene glycol mixer are poor heat transfer fluids since the thermal conductivity of their fluids play important role on heat transfer coefficient between the heat transfer medium and heat transfer surface. Hence many methods have been taken to improve the thermal conductivity of these fluids by suspending nano or micro sized particles materials in liquid.

The preamble to fluid flow is inception with Sakiadis[3] studied the Boundary layer behavior on continuous solid surface: I. Boundary-layer equations for two dimensional and axisymmetric flow, considered for $\lambda = 0$. Further Eastman et.al[4] studied the Anomalously increased effective thermal conductivity of ethylene glycol-based nanofluids containing copper nanoparticles, Putra et.al[5] studied the Natural convection in nanofluids ,this experimental study show that the thermal conductivity of the base liquid is enhanced by10-50% for small volumetric fraction of nano particles Eastman et.al[6][11][13][14] are reviewed on Thermal condectivity and viscosity of a nanofluid and discussed the preparation of nanofluids , Wen and Ding [7] Formulated the heat transfer application as natural convection in nanofluid also discussed the development of analytical model in nanoparticles, Choi[8] describes a liquid suspension containing ultra-fine particles these are formed by high energy pulsed process from conductive material., the comprehensive references on nano fluids can be found in the book by Das et.al[15] given more details on nanofluids properties and its application and the papers are found in Buongiorno and Hu.[9][17] nanofluid coolants for advanced nuclear power plants, Prashar et.al[10] investigated masurements of nanofluid viscosity and its implications for thermal applications, concluded that the viscosity has to inverse fourfold relative to the inverses in thermal conductivity . After the extensive evaluation of the literature

Buongiorno[11][22] has shown that the higher heat transfer coefficients in nanofluids connot be explained. Weidmen et.al[12] studied the effect of transpiration on self similar boundary layer flow over a moving surface, he showed that second solution is not easily realizable. Trisaksri and Wongwises[13] made a Critical review of heat transfer characteristics of nanofluids, Wang and Mujumdar[14][17][18] studied the Heat transfer characteristics of nano fluids a review, Tzou[16] studied the Thermal stability of nanofluids in natural convection and concluded that the higher turbulence triggered by the nano particles prompted higher heat transfer coefficients then the effect of enhanced thermal conductivity. Tiwari and Das[19] studied the Heat transfer Augmentation in a two sided lid-driven differentially heated square cavity utilizing nanofluids, considered the uniform free stream parallel to a fixed or moving flat plate. Minsta et.al[20] has given new Temperature dependent thermal conductivity data for water based nano-fluids. Kakac[21] made a review of convective heat transfer enhancement with nanofluids.Nield and Kuznetsov[23][24][37] studied the Chaenge-Minkowycz problem for natural convective boundary-layer flow in a porous medium saturated by a nanofluid, Ishaket,al[25] studied the flow and heat transfer charactoristics of a moving flat plate in a parallel stream with constant surface heat flux and showed the variation and comparision of f''(0) with previous resuls. Shukla et.al[26] investigated the thermal performance of cylindrical heat pipe using nano fluids. Nada et.al[27] [31]studied the effect of variable thermal conductivity and variable viscosity on heat transfer in water – Al_2O_3 nanofluid confessed in an enclose. Kuznetsov and Nield[28] studied the Natural concective boundary –layer flow of nanofluid past a vertical plate. Khan and Pop[29] studied the Boundary layer flow of a nanofluid past a stretching sheet . Wang and Leon[30] given the broad range of current and future application of nanofluid. Bachok et.al[32][33] investigated the boundary layer flow of nanofluids over a moving surface in a nanofluid. Khan and Aziz[34] investigated natural convective flow of ananofluid over a vertical plate with uniform heat flux.The forced and free convection of nanofluid past vertical and horizontal surface is studied by Sarkar[35] and has made a critical review on convective heat transfer correlations of nanofluids. Ahmed et.al[36] studied the Blasius and Sakiadis problems in nanofluids. Rohini et.al[38] studied the Boundary layer flow over a moving surface in a nanofluid beneath a uniform free stream and nanofluid saturated in porous medium has investigated by Bachok[39] also studied a flow and heat transfer over a rotating porous disk in a nanofluids. On observing above literature no researcher has been considered the effect of heat source/sink on heat transfer analysis of nanofluid containing copper, alumina and titania. Hence in this paper we studied the effect of heat source/sink on heat transfer of nanofluid where as more industrial application.

2. Mathematical formulation of the problem:

Consider a steady two-dimensional laminar boundary layer flow of nanofluid over a stretching sheet coinciding with the plane y = 0 and flow being confined to y > 0. The flow is generated, due to the stretching of the sheet, caused by the simultaneous application of two equal and opposite forces along the x – axis. Keeping the origin fixed, the sheet is then stretched with a velocity $u = U_w$. The stretching sheet maintained at constant temperature T_w and the value is assumed to be greater then the ambient temperature T_∞ . The pressure gradient and external forces are neglected. The fluid is water based nanofluid contained different types of solid particles, say copper (Cu), Aluminum(Al_2O_3)and TiO_2.The nanofluid is assumes to be incompressible an flow is assumed to be laminar. It is also assumed that base fluid (i,e water) and the nonoparticles are in thermal equilibrium and no slip occurs between them. The governing boundary layer equations for this investigation are based on the balance law of mass, linear momentum and energy equation. Taking the above investigation into consideration, The study boundary layer equations governing the flow and Heat transfer for a nanofluid in the presence of internal Heat generation/absorption can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{UQ}{(\rho C_p)_{nf}}(T - T_\infty) \tag{3}$$

The associated boundary conditions are

$$\left.\begin{array}{l} u = U_w,\ v = 0, T = T_w\ at\ y = 0 \\ \quad u \to U_\infty,\ T \to T_\infty\ as\ y \to \infty \end{array}\right\} \tag{4}$$

Where u and v are the velocity components along x and y-direction respectively. T is the temperature of the nanofluid. T_∞ is the temperature of the nanofluid far away from the sheet, Q is the temperature dependent volumetric rate of heat source when Q>0 and heat sink when Q<0, dealing with the situation of exothermic and endothermic chemical reaction respectively,The thermo physical properties of the nanofluids are given in the Table1(see Oztop and Abu-Nada [14]).

Table 1
Thermo physical properties of regular fluid and nanoparticles

Physical properties	Regular fluid(Water)	Cu	Al_2O_3	TiO_2
C_p (J/kg K)	4179	385	765	686.2
$\rho(kg/m^3)$	997.1	8933	3970	4250
$\kappa(W/mk)$	0.613	400	46	8.9538

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \tag{5}$$

Where ϕ is the volume fraction of the nanoparticles. Thermal diffusivity of the nanofluid is

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \tag{6}$$

Where Heat capacitance $C_p$of the nanofluid is obtained as

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \tag{7}$$

And the thermal conductivity of the nanofluidk_{nf} for the spherical nano particle can be written as Maxwell[39]

$$\frac{k_{nf}}{k_f} = \frac{(k_s+2k_f)-2\phi((k_f-k_s)}{(k_s+2k_f)+\phi((k_f-k_s)} \tag{8}$$

The thermal expansion coefficient of the nanofluid can be determined by

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s \tag{9}$$

Also the effective dynamic viscosity of the nanofluid is given by Brinkman[40]as

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{10}$$

Here the subscripts nf, f and s represents the thermophysical properties of the nanofluids, base fluid and the nano-solid particles respectively

By using the similarity transformation

$$\eta = \left(\frac{U}{\nu_f x}\right)^{1/2} y\,, u = Uf'(\eta), v = \frac{y}{2x}Uf'(\eta) - (\nu_f U)^{1/2}\frac{f(\eta)}{2\sqrt{x}}\theta(\eta) = \frac{(T-T_\infty)}{(T_w-T_\infty)} \tag{11}$$

The fundamental equations of the boundary layer (1)-(3) ar transformed to ordinary differential ones that are locally valid as follows:

$$f''' + \frac{(1-\phi)^{2.5}}{2}\left\{1-\phi+\phi\frac{\rho_s}{\rho_f}\right\}ff'' = 0 \tag{12}$$

$$\theta'' + \frac{Pr\left\{1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right\}}{\frac{k_{nf}}{k_f}}\left(\frac{f\theta'}{2} + H\theta\right) = 0 \tag{13}$$

In view of (5), The boundary conditions (4) turn into

$$\left.\begin{array}{l} f(0) = 0, f'(0) = \lambda,\ \theta(0) = 1 \\ f'(\eta) \to 1-\lambda,\ \theta(\eta) \to 0\ as\ \eta \to \infty \end{array}\right\} \tag{14}$$

Her the prime denotes differentiation with respect to η, $H = \frac{Q}{\rho C}$ is the Heat source parameter, $Pr = \frac{\nu_f}{\alpha_f}$ is the Prandlt number,

Another important characteristics of the present investigation are the skin friction coefficient and the of the Nusselt number which are defined as follows

$$C_f = -\frac{\mu_{nf}}{\rho_f}\left(\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{1}{(1-\phi)^{2.5}}{Re_x}^{-1/2}f''(0) \tag{15}$$

And

$$Nu_x = \frac{xk_{nf}}{k_f(T_w-T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\frac{k_{nf}}{k_f}{Re_x}^{-1/2}\theta'(0), \tag{16}$$

Respectively . Here $Re_x = \frac{u_w x}{\nu_f}$ is the local Reynolds number.

3. Numerical Solution

The boundary value problem of the equation (12) and (13)subjected to boundary conditions (14) is solves using shooting techniques by converting these equations into an equivalent initial value problems. Using an inbuilt ode45 solver in MATLAB software we obtain the solution of momentum and heat transfer equations with an accuracy 10^{-5} . Numerical values of Nusselt's number and Skin friction are calculated for particular set of governing parameters. Our results are in good agreement with earlier results in the limiting cases.

4. Result and discussion:

Here we discuss the effect of Heat source/sink on temperature profile for copper- water system, Alumina-water system and Titania –water system. The effect of Heat source/sink parameter on temperature profile for copper-water system is shown in Fig 1which is plotted for various values of H(Heat source/sink parameter). for H <0 shows the effect of Heat sink and H>0 shows the effect of Heat source. Figure 1(a) shows the effect of Heat source/sink on temperature profile for clear fluid i.e$\varphi = 0$, it depicts that temperature profile increases with increase in the Parametric values of H and decreases with decreasing values of H which shows that thermal boundary layer thickness is thickening . The same effect observed when $\varphi \neq 0$ which has shown in figure 1(b) and 1(c). Fig2(a,b,c) shows the effect of Heat source/sink parameter on temperature profile for Alumima-water system which shows the same effect as in case of Cu-water system.2(a) is for clear fluid and 2(b,c) are non-clear fluid.Fig3(a,b,c) shows the effect of Heat source/sink parameter on temperature profile for Titania-water system which shows the same effect as in case of Cu-water system.2(a) is for clear fluid and 2(b,c) are non clear fluid.Figure 4(a,b,c) shows the effect of velocity ratio parameter λ on temperature profile.Fig 4(a) shows the effect of λ on temperature profile for Cu-water system, 4(b) Alumina water system and 4(c) for Titania-water system.It is observed that temperature profile decreases with increase in the valueof . the same effect observed in Alumina-water system and Titania – water system. Fig 5(a,b,c) represents the effect of nano-particle volume fraction φ for Cu-water system, in Alumina-water system and Titania-water system respectively. It is observed that temperature profile is decreasing asymptotically with increasing value of φ.

Table(1) shows the Thermo physical properties of regular fluid and nanoparticles and Table (2) shows the accurate representation numerical data for the values of $f''(0)$ *and* $\theta'(0)$.

5. Conclusions

Here we have presented heat transfer analysis of continuous moving plate in a nanofluid .it is assumed that the plate moves in the same or opposite direction to the free stream. The governing system of non-linear ordinary differential equations are solved numerically using shooting technique with Runge-Kutta method (ode45 MATLAB solver) for three types of nano particles namely Copper, Alumina and Titania with water of $P_r = 6.2$ has the base fluid. The effects of Heat source/sink, velocity ratio parameter λ and nano particle volume fraction . Nano fluids are capable of changing temperature profile on the boundary layer and these fluids work as key factor for heat transfer enhancement. Numerical values of skin friction coefficients were obtained for various nano particles and compared with other existing results.

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Table-2

				Cu-water		Al2O3		TiO2	
Pr	λ	H	ϕ	f''(0)	θ'(0)	f''(0)	θ'(0)	f''(0)	θ'(0)
6.2	-0.5	-0.5	0	0.39785	-0.08843	0.39785	-0.08843	0.39785	-0.45423
			0.1	0.46737	-0.08891	0.39736	-0.23196	0.40164	-0.44849
			0.2	0.4846	-0.08855	0.38031	-0.22784	0.38694	-0.44053
		-0.1	0	0.39785	-0.0692	0.39785	-0.0692	0.39785	-0.25001
			0.1	0.46737	-0.07007	0.39736	-0.1516	0.40164	-0.24856
			0.2	0.4846	-0.06999	0.38031	-0.14815	0.38694	-0.24268
		-0.05	0	0.39785	-0.06659	0.39785	-0.06659	0.39785	-0.21745
			0.1	0.46737	-0.06752	0.39736	-0.13971	0.40164	-0.21679
			0.2	0.4846	-0.06748	0.38031	-0.13633	0.38694	-0.21115
		0	0	0.39785	-0.06394	0.39785	-0.0639	0.39785	-0.18242
			0.1	0.46737	-0.06492	0.39736	-0.12725	0.40164	-0.18266
			0.2	0.4846	-0.06492	0.38031	-0.12395	0.38694	-0.17724
		0.1	0	0.39785	-0.05846	0.39785	-0.05846	0.39785	-0.10287
			0.1	0.46737	-0.05956	0.39736	-0.10036	0.40164	-0.10536
			0.2	0.4846	-0.05965	0.38031	-0.09718	0.38694	-0.10025

Values of $f''(0)$ and $\theta'(0)$ for different nano fluids

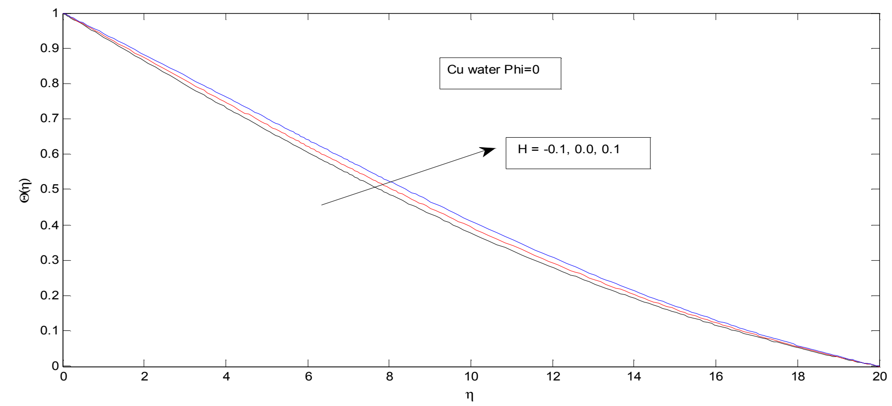


Fig.1(a). Temperature profile for Cu water for various values of H with $\varphi = 0, \lambda = -0.3$ and Pr=6.2

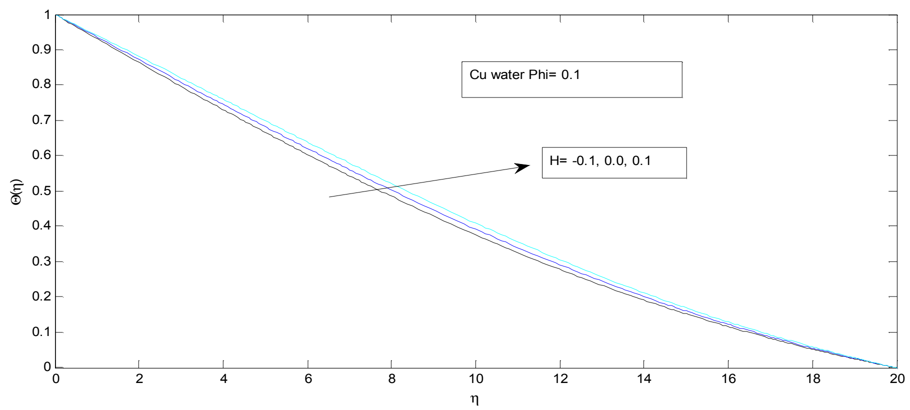


Fig.1(b). Temperature profile for Cu water for various values of H with $\varphi = 0.1, \lambda = -0.3$ and Pr=6.2

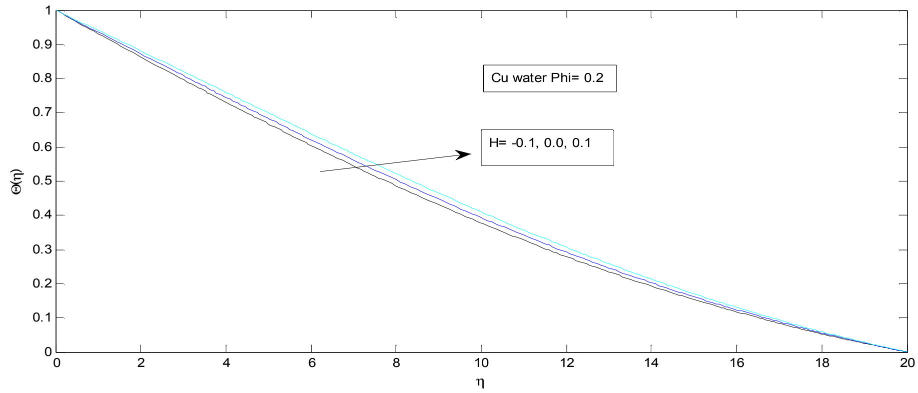


Fig.1(c). Temperature profile for Cu water for various values of H with $\varphi = 0.2, \lambda = -0.3$ and Pr=6.2

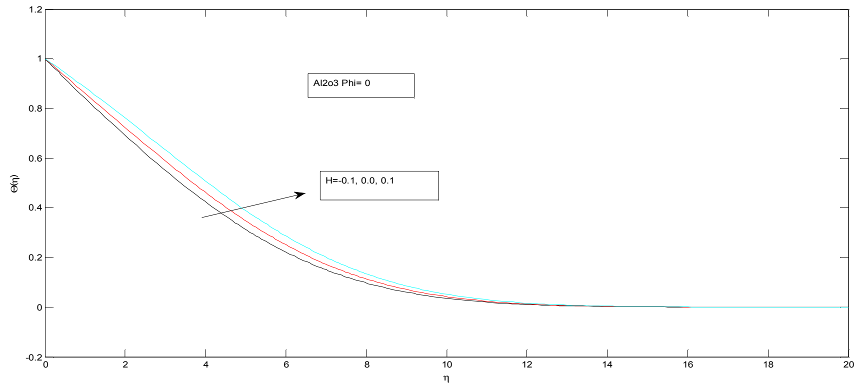


Fig.2(a). Temperature profile for Al_2O_3 water for various values of H with $\varphi = 0, \lambda = -0.3$ and Pr=6.2

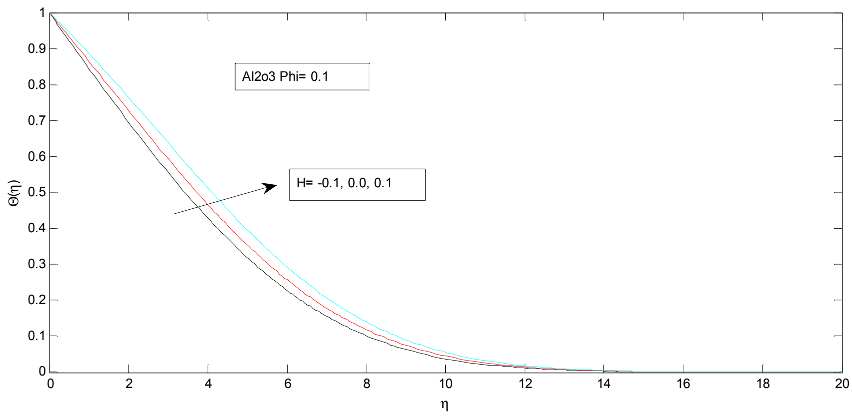


Fig.2(b). Temperature profile for Al_2O_3 water for various values of H with $\varphi = 0.1, \lambda = -0.3$ and Pr=6.2

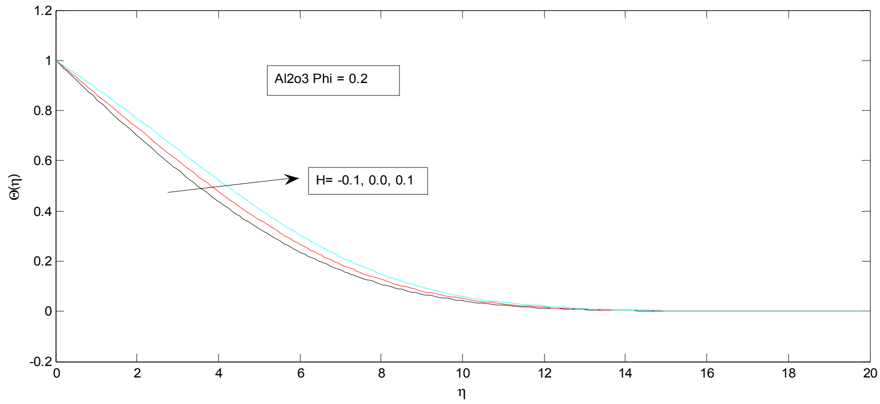


Fig2(c). Temperature profile for Al_2O_3 water for various values of H with $\varphi = 0.2, \lambda = -0.3$ and Pr=6.2

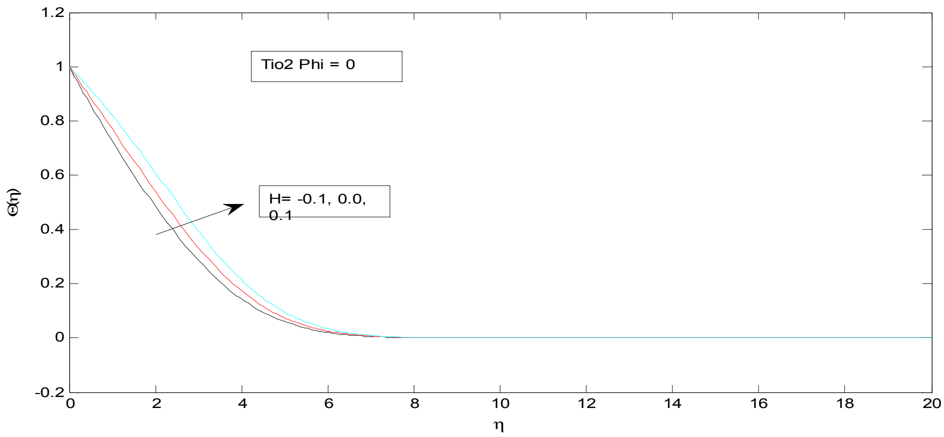
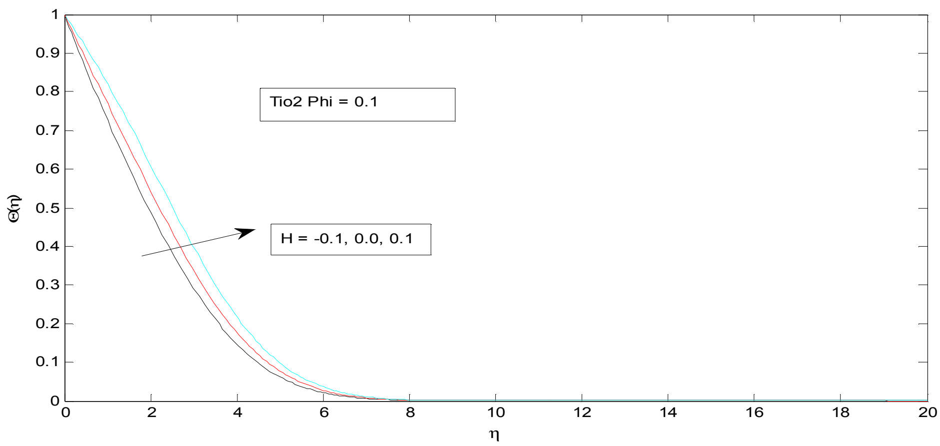


Fig.3(a). Temperature profile for TiO_2 water for various values of H with $\varphi = 0, \lambda = -0.3$ and Pr=6.2**Fig.**



3(b). Temperature profile for TiO_2 water for various values of H with $\varphi = 0.1, \lambda = -0.3$ and Pr=6.2

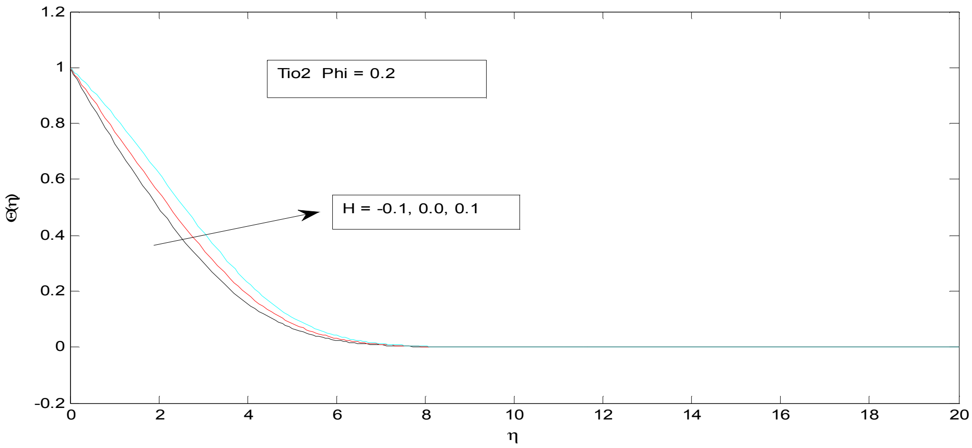


Fig.3(c). Temperature profile for TiO_2 water for various values of H with $\varphi = 0.2, \lambda = -0.3$ and Pr=6.2

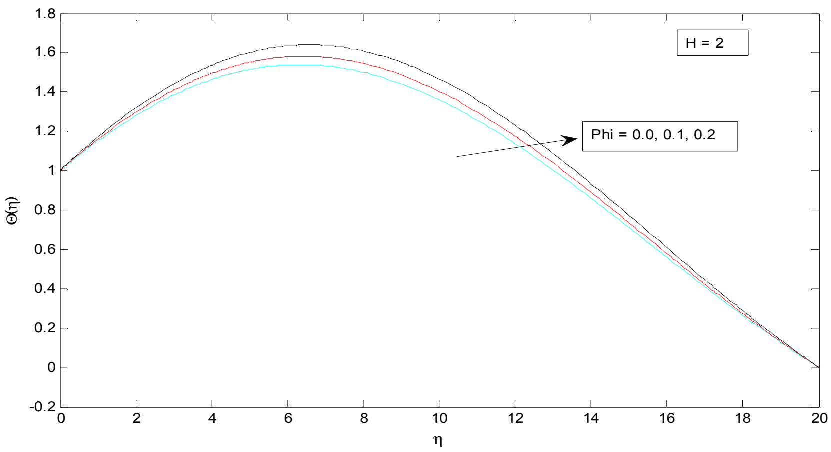


Fig.4(a). Temperature profile for Cu-water for various values of φ with $H = 2, \lambda = -0.3$ and Pr=6.2

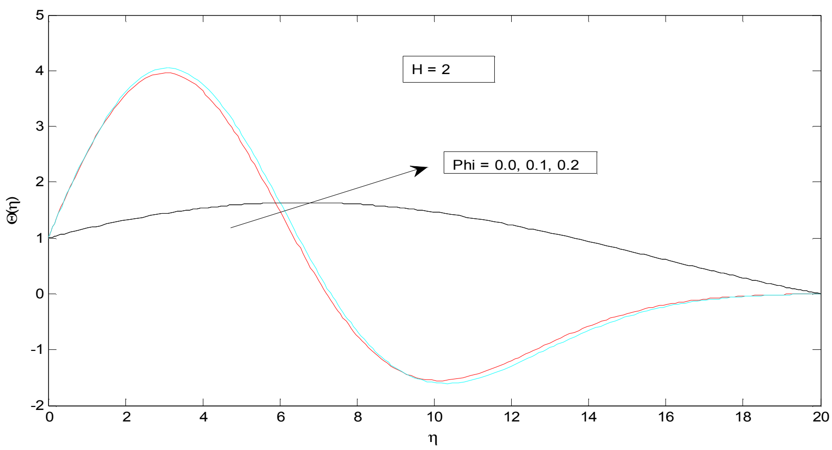


Fig.4(b). Temperature profile for Al_2O_3-water for various values of φ with $H = 2, \lambda = -0.3$ and Pr=6.2

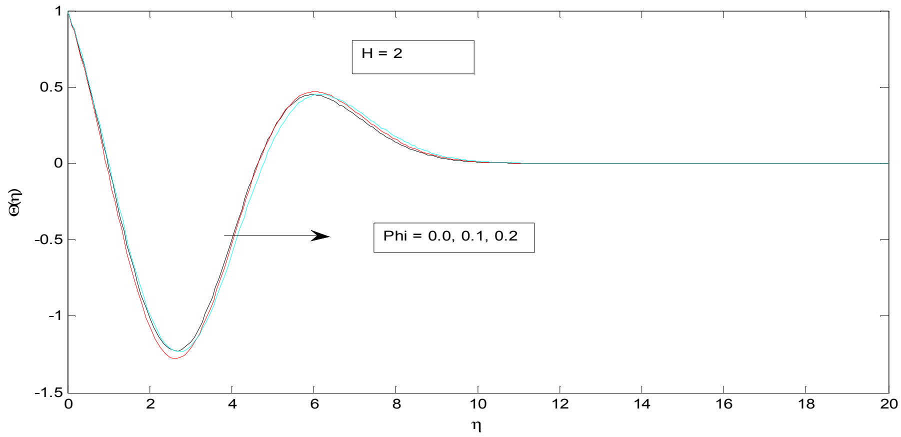


Fig.4(c). Temperature profile for TiO_2-water for various values of φ with $H = 2, \lambda = -0.3$ and Pr=6.2

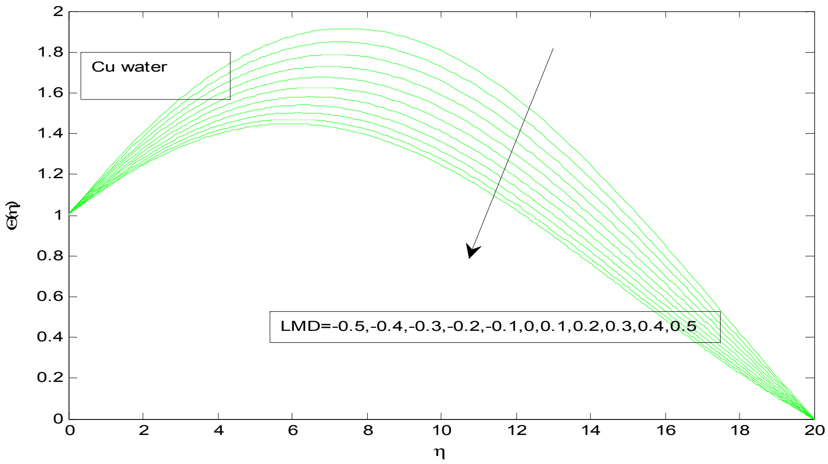


Fig.5(a). Temperature profile for Cu-water for various values of λ with $H = 2, \varphi = 0.1$ and Pr=6.2

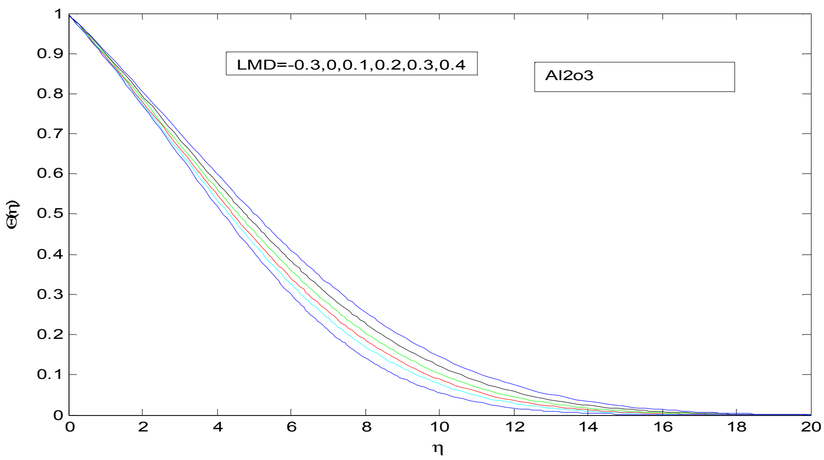


Fig.5(b). Temperature profile forAl_2O_3-water for various values of λ with $H = 2, \varphi = 0.1$ and Pr=6.2

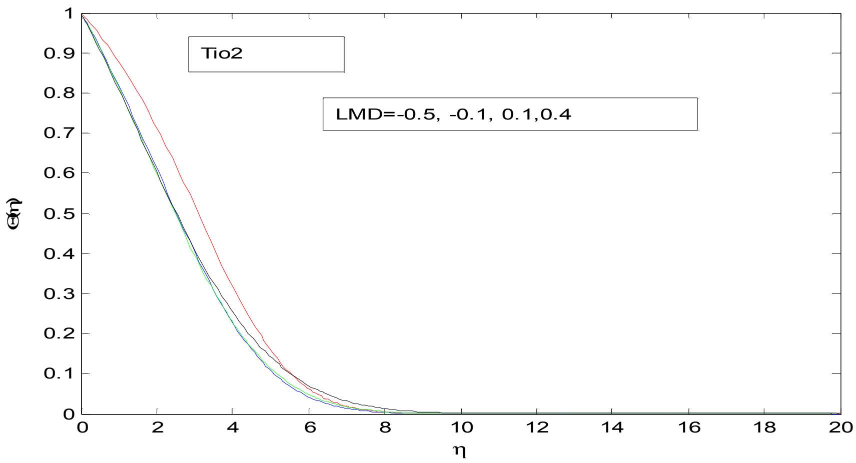


Fig.5(c). Temperature profile for TiO_2-water for various values of λ with $H = 2, \varphi = 0.1$ and Pr=6.2