Industrial Engineering Letters
ISSN 2224-6096 (Paper) ISSN 2225-0581 (online)
Vol 2, No.8, 2012

Modeling of Production Plan and Scheduling of Manufacturing Process for a Plastic Industry in Nigeria

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Abstract
In this work, production plan was developed for a plastic company with two products that are made up of five parts. The production inputs for each part were analyzed and established. The production constraints in terms of materials available, machine capacities, time and labour were used to develop an integer linear programming model to achieve the objective function. The integer linear program was then used to find the optimum quantities of each part that will yield maximum profit. The developed model was analyzed with TORA optimization solver to obtain results for different constraints. The optimum solution for the LP model gave a monthly production profit of N3,751,922. Further sensitivity analyses performed for different production conditions gave results that are not feasible and the ones that were feasible were not optimal. A decision support system was developed for production planning to help the manager in scheduling decision process. The model is now been used at EagleHeights Plastic Industries Limited for their production planning.

Keywords: Production Plan, Optimum, Linear Program, Constraints, Sensitivity Analysis, TORA

1. Introduction

For many manufacturers the task of meeting the ever rising demand and customer expectations and lowering production costs in an environment of more products, more complexity, more choice and competition is placing great stress on the effectiveness of their planning of activities in the production process. Organizations have already adopted solutions with varying degrees of planning and scheduling capabilities. Yet, operations executive acknowledge that these same systems are becoming out dated, lacking the speed, flexibility and responsiveness to manage their increasing complex production environment.

Plastic industries have grown rapidly over the years in Nigeria and competition among manufacturer is so high that planning and scheduling of resources must be efficient for any of these industries to be profitable and able to survive competition. However, in today's manufacturing scheduling problems, it is of significance to efficiently utilize various resources. Treating set up times separately from processing times allows operations to be performed simultaneously and hence improve resource utilization. This is particularly important in modern production management systems, such as, Just-In-Time (JIT), Optimized Production Technology (OPT), Group Technology (GT), Cellular Manufacturing and time-based competition. The benefits of reducing set up times include; reduced expenses, increased production speed, increased output, reduced lead times, faster charge over, increased competitiveness, increased profitability and customer satisfaction. Planning and scheduling of products, machines, raw materials and labor are paramount to the profitability of any plastic industry in Nigeria and the world at large.

Production planning is one of the most important activities in a production factory. Production planning represents the beating heart of any manufacturing process. According to Veeke and Lodewijks (2005), production planning usually fulfils its functions by determining the required capacities and materials for these orders in quantity and time. Mitchell (1939) discusses the role of production planning department, including routing, dispatching (issuing shop orders) and scheduling. According to Stevenson (2009), in the decision making hierarchy, scheduling decisions are the final step in the transformation process before actual output occurs. Wight (1984) puts the two key problems in the production scheduling as "priorities" and "capacity". In other words, what should be done first? And who should do it? He observes that in manufacturing firms, there are multiple types of scheduling, including the detailed scheduling of shop order that shows when each operation must start and complete. A lot of researchers have done extensive work in developing efficient solution strategies, they include Grossman et al (2002), Maravelias and Sung (2008).

2. A Review of Some Literatures on Production Planning and Scheduling Models

2.1. Production Planning Models

Any planning problem starts with a specification of customer demand that is to be met by the production plan. Excellent general references on production planning are Thomas and McClain (1993), Shapiro (1993), Silver et al. (1998) and Telsang (2001). Production planning problem are one of the most interesting application for optimization tools using mathematical programming. The idea of incorporating uncertainty in mathematical models appears initially with Dantzig, well known as the father of linear programming (Dantzig, 1955). Mula et al. (2006), carried out their research over the following seven categories of production planning: hierarchical production planning, aggregates production planning, material requirement planning, inventory management and supply chain planning. They also identified four modeling approaches: conceptual, analytical, artificial intelligence and simulation models. These modeling approaches were originally defined by Giannoccaro and Pontradolfo(2001).

Hax and Meal (1975) introduced the notion of hierarchical production planning and provide a specific framework for this, whereby there is an optimization model with each level of hierarchy. Each optimization model imposes a constraint on the model at the next level of the hierarchy. Bitran and Triupati (1993), provide a comprehensive survey of hierarchical planning methods and models. Gfrerer and Zapfel (1995), presented a multi-period hierarchical production planning model with two planning levels, that is aggregate and detailed and with uncertain demand. Maybodi and Foote (1995), on the other hand, developed a multi-period model for hierarchical production planning and scheduling with random demand and production failure. Zapfel (1996) presented a hierarchical model that can be incorporated in a manufacturing resource planning (MRP11) system to program the production with demand uncertainty.

Graves (1999) developed linear programming models for production planning under the following context: multiple items with independent demand, multiple shared resources, big bucket time periods and linear costs. Escudero and Kamesam (1995) developed linear programming models for stochastic planning problems and methodology to solve them. They used a production problem process with uncertainty in demand, to characterize a test case. Rota et al (1997) presented a mixed integer linear programming model to address the uncertain nature and complexity of manufacturing environments. Their proposed model includes capacity constraints, firm orders, demand forecasts and supply and subcontracting decisions for a rolling horizon planning processes. Bitran et al (1981, 1982) proposed linear programming models to solve the aggregate production planning problem respectively with a single stage and two stage approach at the product type aggregation level. The objectives were to minimize overall cost, including raw materials cost and inventory costs.

2.2 Production Scheduling Models.

The topic of production scheduling has been extensively researched over the years .When using a mathematical approach for solving industrial scheduling problem, different techniques such as constraint programming (CP), mixed integer linear programming(MILP), mixed integer non linear programming (MINLP),or even a hybrid of CP and MILP formulations can be used, depending on the type of problem MILP models are typically solved by brand and bound algorithms, and tend to solve general scheduling problems more efficiently than CP formulations (Leung,2009).

In scheduling, it is necessary to consider the setup time and cost. Allahverdi et al (1999) presented a research on scheduling problems with separate setup times and cost. Reklaitis (2000) performed a comprehensive review of scheduling problems with consideration to sequence-dependent transition between products. Allahverdi and Soroush (2006) presented a research about the importance of reducing setup times and costs and they emphasized on advantages of considering setup costs and setup times. Doganis and Sarimveis (2008) have presented optimal scheduling model based on mixed integer linear programming (MILP). Their model involved set up times and setup cost. The scheduling was done on a single machine. Doganis and Sarimveis (2007) presented an optimal scheduling problem based on mixed integer linear programming for yogurt packing lines that consist of multiple parallel machines in which different products could not be produced synchronously by two machines. Sadi-Nezhad and Darian (2010) worked on production scheduling for products on different machines with setup cost and time. They developed a production scheduling model based on mixed integer linear programming with set up cost and time. They went further to develop a decision support system for production scheduling in order to help the manager in decision processing. Castro et al addressed the short term scheduling problem of a three parallel production line polymer compounding plant. The mixed integer linear programming model they proposed was based on a resource task network (RTN) discrete time formulation.

2.3. Integration of Production Planning and Scheduling

Production planning and scheduling belong to different decision making levels in process operation, they are also closely related since the result of planning problem is the production target of scheduling problem. A lot of researchers have proposed production planning methods that incorporate scheduling sub models (Basset et al (1996); Grossman et al (2002). Maravelias and Sung (2008) and Shah (2005) reviewed the integration of medium term production planning and long term scheduling. They discussed different modeling approaches for the integration of production planning with scheduling and went further to discuss the main solution strategies developed to solve the integrated models effectively. Lin et al (2002) presented a three level integrated model for medium term multi-stage production scheduling.

Yan et al (2007) hierarchically solved an integrated model. They first solved the production planning problem in the presence of aggregate capacity constraints to get the production amount and then use tabu -search to ensure feasibility at the lower level. Papageorgiou and Pantelids (1996) proposed an integrated planning and scheduling model where each higher level time period is made up of cycles. Eridirk-Dogan and Grossman (2006) proposed an integrated planning and scheduling model for scheduling continuous task on a single machine. They used iteration method to solve the resulting model. Joly et al (2002) proposed an integrated model for a refinery. The planning problem defined refinery topology and operating points, while the scheduling problem managed crude oil unloading from pipe lines, transferring to storage tanks and charging into units. The integrated model was solved using the branch and bound method.

In production planning and scheduling, it is very necessary to consider various uncertainties which affect the production processes. In the real world, there are many forms of uncertainties that affect production processes. Galbraith (1973) defined uncertainty as the difference between the amount information required to perform a task and amount of information already possessed. Ho (1989) categorized uncertainty into two groups: (i) environmental uncertainty and (ii) system uncertainty. Environmental uncertainty includes uncertainties beyond the production processes such as demand uncertainty and supply uncertainty. System uncertainty is related to uncertainties within the production processes, such as operation yield production lead time, failure of production system, quality and changes to product structure uncertainties etc. Honkomp et al (1999), Sand and Engell (2004) discussed hierarchical approaches that employ rolling horizon method to address problems under uncertainty. In this study, various uncertainties affecting production processes are considered in solving the problem of production planning and scheduling.

3. Materials and Methods

The research methodology adopted in this work is a case study of an existing production system in a plastic manufacturing company in order to investigate and improve its production plan for maximum profit. The data for this research were obtained from production information of the products made available by the management of Eagleheights Plastic Industries limited. The data obtained from this company were analyzed using linear programming (LP), with an objective of maximizing profit through optimal production of these items. The model to be generated through this approach can be applied in other production planning and scheduling processes. The simulation and solution technique to this model will base on the Linear programming solutions and Sensitivity Analysis method using TORA optimization software (Taha, 2007).

3.1. Development of the Production Plan/Scheduling Model

In this study linear programming technique is used in production planning and scheduling of five parts of two particular plastic products on three different machines. The parts are the cover, the bowl (bucket), the handle of 4-litre plastic paint bucket, cover for Ice cream bowl and the Ice cream bowl. The aim here is to develop a plan and schedule that will give the quantities of these parts. The linear programming model developed for the production plan of these products was then analyzed with a TORA operations research tool.

Therefore in formulating an integer linear programming model for the products, which are; a four liter paint bucket (K1) and an ice cream container (K2), we shall describe a bit of the production scenario or process. These products comprise of K1: a handle, a cover and a body, K2: a cover, and a container. They are produced on three machines of different capacities; Machine-3 which has the highest capacity is used in producing only the body of the paint bucket, Machine-2 which is next in capacity is used to produce the paint cover and the ice cover, while Machine-1 with the least capacity is used to produce the bucket handle and the ice cream container.

The materials involved in the production of these products parts are the same and they are, Co-polymer Poly Propylene, PPCP (material M) and White or Colored Batch (material n) mixed at a definite proportion. The Linear programming model to be developed at this juncture is to help in a month optimum production plan for these products with an objective to maximize profit; hence we formulate the LP as follows;

3.2. Definition of Variables and Parameters

Alternative Variables:

X_{ij} = No of part i produced at shift j

i = 5 part of 2 products

j = 2 shifts of Day (11hrs) and Night (13hrs)

Model variables and parameters:

$P_T(K_1, K_2)$ = Total profit from sales of product K_1 and K_2

$$As\ K_1 = x_1 + x_2 + x_3$$

$$K_2 = x_4 + x_5$$

$$i \Rightarrow \begin{array}{l} 1 = Handle\ of\ Paint\ bucket \\ 2 = Cover\ of\ Paint\ bucket \\ 3 = Body\ of\ Paint\ bucket \\ 4 = Cover\ of\ Ice\ cream\ container \\ 5 = Body\ of\ Ice\ cream\ container \end{array}$$

S_i = Selling price of unit part of i

C_i = Unit cost of production of part i (material)

F_c = Fixed costs (salary, power, maintenance, etc.)

d_i = Demand of part i per month

T_j = Total available machine time (hours) for shift j

t_i = Required production time (hours) for unit of part i

M_i = Weight of material m (kg) needed to produce a unit of part i

n_i = Weight of material n (kg) needed to produce a unit of part i

A_m = Available quantity of material M (kg)

A_n = Available quantity of material n (kg)

$\forall i$ = For all values of i

$\forall j$ = For all values of j

Objective Function:

$$Max\ P_T(K_1, K_2) = \left\{\sum\sum (S_i - C_i)X_{ij}\right\} - F_c \qquad (1)$$

Subject to:

$$\left.\begin{array}{c} \sum_{i=1}^{5} t_i x_{ij} \geq T_j \quad \forall j \\ \sum_{j=2}^{2} x_{ij} \geq d_i \quad \forall i \\ \sum_{i=1}^{5}\sum_{j=1}^{2} M_i x_{ij} \leq A_m \\ \sum_{i=1}^{5}\sum_{j=1}^{2} n_i x_{ij} \leq A_n \\ t_i \sum_{j=1}^{2} x_{ij} \leq (24 \times 28 \times 1) \quad \forall_{ij} \\ x_{11} + x_{12} = x_{21} + x_{22} \\ x_{21} + x_{22} = x_{31} + x_{32} \\ x_{41} + x_{42} = x_{51} + x_{52} \\ x_{ij} \geq 0 \end{array}\right\} \quad (2)$$

On expansion of the Linear Programming model of equations (1) and (2) we have:

$$Max\ P_T(K_1, K_2) = (S_1 - C_1)[x_{11} + x_{12}] + (S_2 - C_2)[x_{21} + x_{22}] + (S_3 - C_3)[x_{31} + x_{32}] + (S_4 - C_4)[x_{41} + x_{42}] + (S_5 - C_5)[x_{51} + x_{52}] - F_c \quad (3)$$

S.t:

$$t_1x_{11} + t_2x_{21} + t_3x_{31} + t_4x_{41} + t_5x_{51} \geq T_1 \quad (4)$$
$$t_1x_{12} + t_2x_{22} + t_3x_{32} + t_4x_{42} + t_5x_{52} \geq T_2 \quad (5)$$
$$x_{11} + x_{12} \geq d_1 \quad (6)$$
$$x_{21} + x_{22} \geq d_2 \quad (7)$$
$$x_{31} + x_{32} \geq d_3 \quad (8)$$
$$x_{41} + x_{42} \geq d_4 \quad (9)$$
$$x_{51} + x_{52} \geq d_5 \quad (10)$$
$$M_1(x_{11} + x_{12}) + M_2(x_{21} + x_{22}) + M_3(x_{31} + x_{32}) + M_4(x_{41} + x_{42}) + M_5(x_{51} + x_{52}) \leq A_m \quad (11)$$
$$n_1(x_{11} + x_{12}) + n_2(x_{21} + x_{22}) + n_3(x_{31} + x_{32}) + n_4(x_{41} + x_{42}) + n_5(x_{51} + x_{52}) \leq A_n \quad (12)$$
$$t_1(x_{11} + x_{12}) \leq 24 \times 28 \times 1 \quad (13)$$
$$t_2(x_{21} + x_{22}) \leq 24 \times 28 \times 1 \quad (14)$$
$$t_3(x_{31} + x_{32}) \leq 24 \times 28 \times 1 \quad (15)$$
$$t_4(x_{41} + x_{42}) \leq 24 \times 28 \times 1 \quad (16)$$
$$t_5(x_{51} + x_{52}) \leq 24 \times 28 \times 1 \quad (17)$$
$$x_{11} + x_{12} - x_{21} - x_{22} = 0 \quad (18)$$
$$x_{21} + x_{22} - x_{31} - x_{32} = 0 \quad (19)$$
$$x_{41} + x_{42} - x_{51} - x_{52} = 0 \quad (20)$$
$$x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}, x_{41}, x_{42}, x_{51}, x_{52} \geq 0$$

For our computation using Tora software, we have our variables to be:

$x_{11} = x_1, x_{12} = x_2, x_{21} = x_3, x_{22} = x_4, x_{31} = x_5, x_{32} = x_6, x_{41} = x_7, x_{42} = x_8, x_{51} = x_9, x_{52} = x_{10}$

3.3. Model Analysis Using TORA Software

The data obtained from the company went through some traditional computational analyses and the results will now be placed as the coefficients of the variables in equations (3) to (20).Our objective function equation now becomes:

$$Max\ P_T(K_1, K_2) = 1.11[x_{11} + x_{12}] + 6.67[x_{21} + x_{22}] + 15.78[x_{31} + x_{32}] + 2.47[x_{41} + x_{42}] + 7.70[x_{51} + x_{52}] - 1{,}146{,}250 \quad (21)$$

$$\therefore Max\ P_T(K_1, K_2) = [1.11x_{11} + 1.11x_{12}] + [6.67x_{21} + 6.67x_{22}] + [15.78x_{31} + 15.78x_{32}] + [2.47x_{41} + 2.47x_{42}] + [7.70x_{51} + 7.70x_{52}] - 1{,}146{,}250 \quad (22)$$

Equation (22) is our optimum profit equation and will now be analyzed with TORA. The software input window is given as in Fig. 1;

4. Sensitivity Analysis of Data and Result Discussion

Some data, necessary for your mathematical model is inherently uncertain. Consider profit per item, for example, which is approximately from estimates of the fluctuating costs of raw materials, expected sales volumes, labour costs, etc. What one wants to know from sensitivity analysis is which data has a significant impact on the results: then concentration will be on getting accurate data for those items, or at least running through several scenarios with various values of the crucial data in place to get an idea of the range of possible outcomes.

There are several ways to approach sensitivity analysis. If the model is small enough to solve quickly one can use a brute force approach; simply change the initial data and solve the model again to observe what results are got. This can be done many times as needed. At the opposite extreme, if the model is very large and takes a long time to solve, apply the formal methods of classical sensitivity analysis. This method rely on the relationship between the initial tableau and any later tableau (in particular the optimum tableau) to quickly update the optimum solution when changes are made to the coefficients of the original tableau.

Our original Linear Programming model is depicted in equations (3) to (20) with its input shown in Fig. 1, gives its optimal solution at iteration 13, the summary of this optimum iteration simplex tableau is shown in Fig. 2.

The optimum solution of this LP model shows a profit [z (max)] of N4,898,182. The net profit of this solution is obtained when the fixed cost is subtracted, which gives N3,751,932. The solution suggests a scheduling and production plan as follows; X_9 which is X_{51}, in the solution means that the Ice cream body should be produced to a maximum quantity of 203,636 pieces. X_6 which is X_{32}, in the solution means that the Paint bucket body should be produced to a maximum quantity of 120,000 pieces. X_2 which is X_{12}, in the solution means that the Paint bucket handle should be produced also to a maximum quantity of 120,000 pieces. X_4 which is X_{22}, in the solution means that the Paint bucket cover should be produced also to a maximum quantity of 120,000 pieces. X_7 which is X_{41}, in the same optimum solution means that the Ice cream cover is to be produced to a maximum quantity of 203,636 pieces.

Finally, these indicate that for a gross profit of N4,898,182 (i.e. net profit of N3,751,932) to be achieved during a month production plan, all parts that make up the Paint bucket should be produced only during the night shifts. While those of Ice cream container should be produced only during the day shifts but more than the stipulated demand of 90,000 pieces given in the model, though the model gave room for inventory.

4.1. Post-Optimal Analysis

Series of sensitivity analysis in the form of post-optimal analysis were conducted on the original LP model and the following inferences were drawn.

When the inequality signs of equations (4) and (5) were changed from ≥ to ≤, the optimum solution was obtained at the simplex tableau of iteration 7 with a profit [z (max)] of -N31,683,172. The optimum solution is not feasible as the value of z (max) is negative and some of the artificial variables were still found in the solution making up for some of the variables especially those of the Paint bucket. The summary of the optimum solution simplex tableau is in Fig. 3.

Another change on the LP model was made on the right-hand-side of equations (11) and (12) where the total available material resources was reduced from 40,000kg of PPCP to 30,000kg, and 250kg of master batch to 200kg. in the same effect the inequality signs of these equations were changed from ≥ to ≤. The optimum solution for this model occurred at the simplex tableau of the 14th iteration with a profit [z (max)] of N4,003,276. The summary of the optimum solution's simplex tableau is in Fig. 4. This LP model suggested the same production plan as that of the original LP model, but due to the reduction in the quantity of material resources, the production quantity of Ice cream container was reduced to 115,642 to accommodate this change in constraints.

In next sensitivity analysis, we tried to stop any form of inventory by trying not to exceed demand, hence changing the inequality signs of equations (6) to (10) from ≥ to ≤. The optimum solution simplex tableau of this model occurred at the 8th iteration tableau with a profit [z (max)] of N3,945,900. The net profit for this optimum solution is then N2,799,650. The optimum solution tableau of this model is shown in Fig. 5. The model's optimum solution suggested a production plan and scheduling as follows; the production of 120,000 pieces of Paint bucket cover, X_3 which is X_{21}, only during day (morning) shifts. That of 120,000 pieces of Paint bucket handle, X_1 which is X_{11}, to take effect also during the day shifts. While 119,821 pieces of the Paint bucket body, X_5 which is X_{31}, should at morning shifts and a complementary quantity of 179 pieces of the same part should be produced at night shifts. It also suggested the production of Ice cream body, X_{10} which is X_{52}, to a maximum quantity of 110,000 pieces. The complementary Ice cream cover, X_7 which is X_{41}, will be produced to a quantity of 110.000 pieces also, hence inventory is unavoidable.

Finally, it is apparent that the original LP model produced the optimum solution that will give the best production plan and the maximum profit, hence it is recommended for application.

5. Conclusion

The monthly production plan for Eagle height industry was developed in this work. The factory has five (5) separate items and two (2) products were produced in three (3) different injection machines. Material requirement, unit production time and sequence dependency were considered. In order to articulate properly the problem, the model was formulated as integer linear programming. The model was solved by TORA optimization solver software. It gave the products quantity which should be produced in each machine during each shift and expected profit if the optimum production plan for the month is adhered to. The results were presented in the figures 2 to 5. The Linear Programming model developed here could be used for other different applications of multiple products or service operations with more than one type of service at a time.

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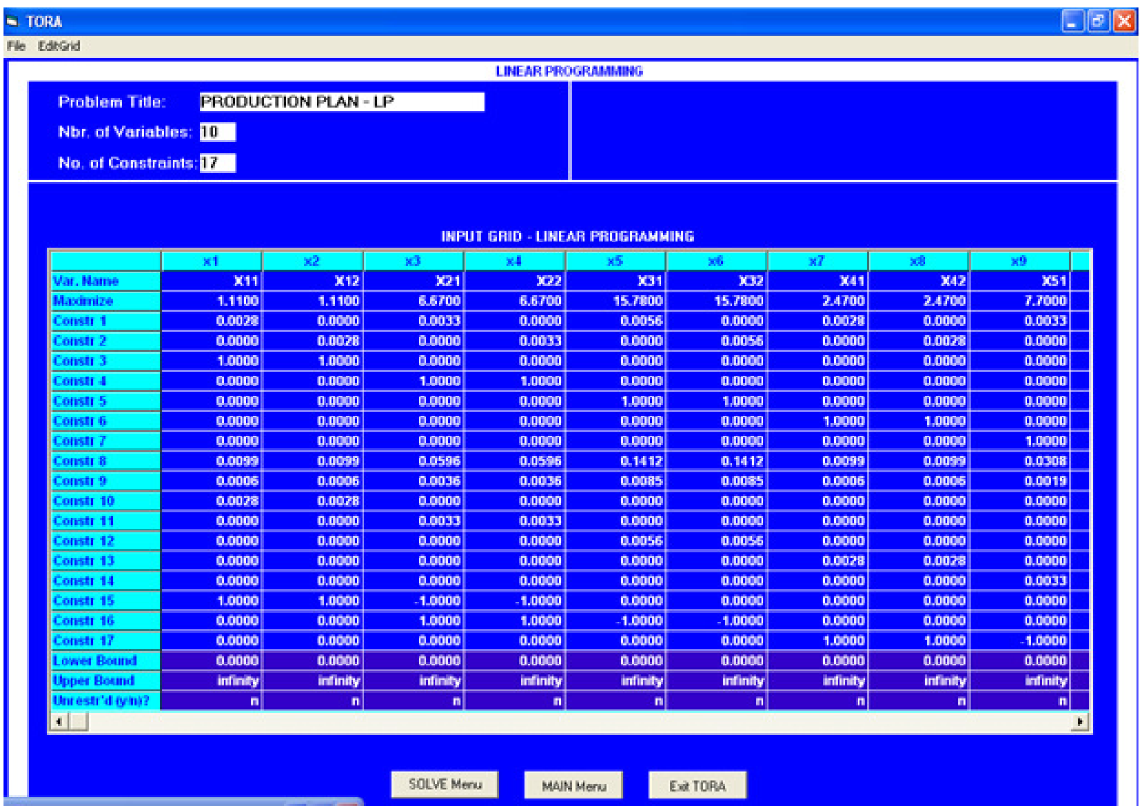
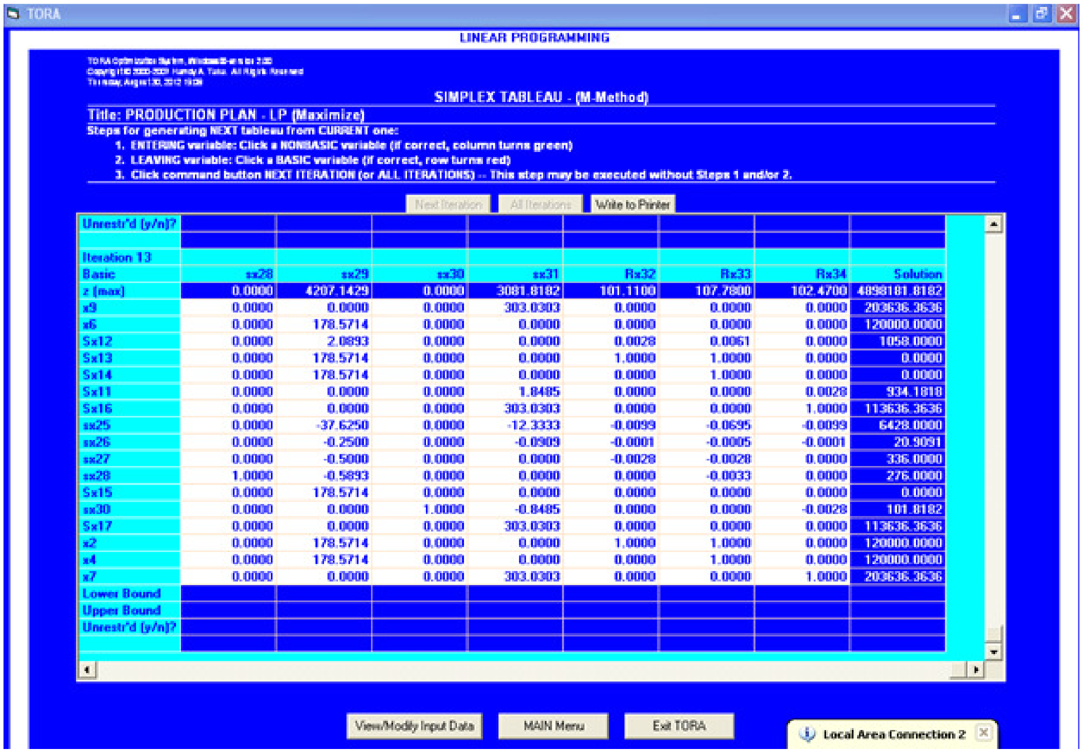


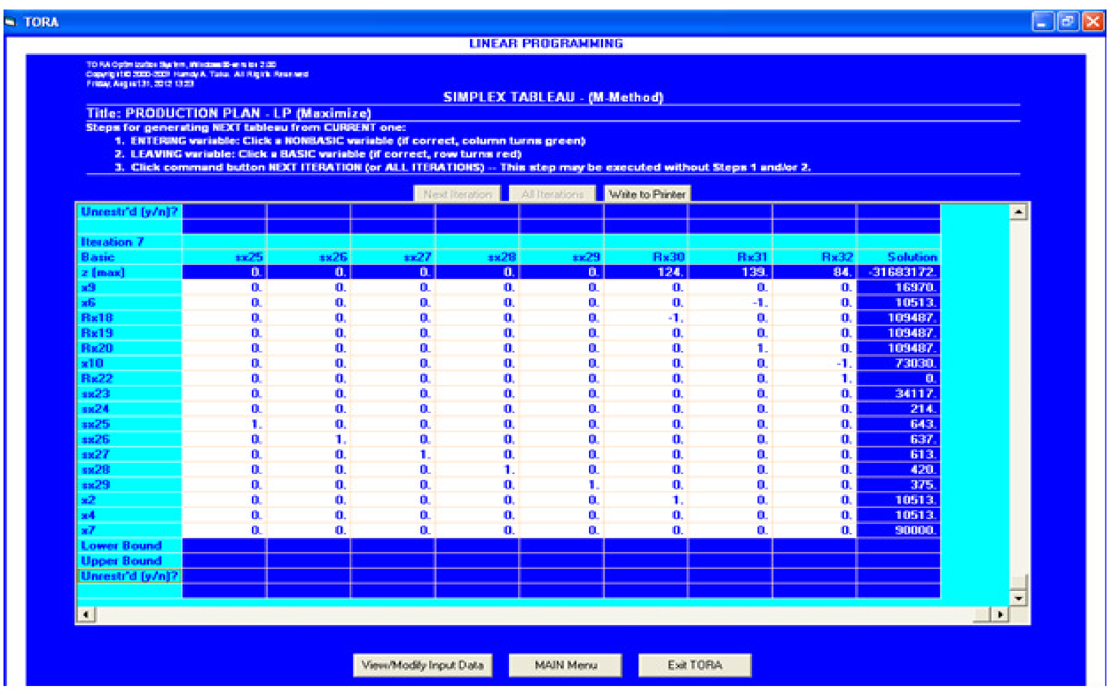
	x1	x2	x3	x4	x5	x6	x7	x8	x9
Var. Name	X11	X12	X21	X22	X31	X32	X41	X42	X51
Maximize	1.1100	1.1100	6.6700	6.6700	15.7800	15.7800	2.4700	2.4700	7.7000
Constr 1	0.0028	0.0000	0.0033	0.0000	0.0056	0.0000	0.0028	0.0000	0.0033
Constr 2	0.0000	0.0028	0.0000	0.0033	0.0000	0.0056	0.0000	0.0028	0.0000
Constr 3	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Constr 4	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Constr 5	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000
Constr 6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000
Constr 7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
Constr 8	0.0099	0.0099	0.0596	0.0596	0.1412	0.1412	0.0099	0.0099	0.0308
Constr 9	0.0006	0.0006	0.0036	0.0036	0.0085	0.0085	0.0006	0.0006	0.0019
Constr 10	0.0028	0.0028	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Constr 11	0.0000	0.0000	0.0033	0.0033	0.0000	0.0000	0.0000	0.0000	0.0000
Constr 12	0.0000	0.0000	0.0000	0.0000	0.0056	0.0056	0.0000	0.0000	0.0000
Constr 13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0028	0.0028	0.0000
Constr 14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0033
Constr 15	1.0000	1.0000	-1.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Constr 16	0.0000	0.0000	1.0000	1.0000	-1.0000	-1.0000	0.0000	0.0000	0.0000
Constr 17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	-1.0000
Lower Bound	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n

Fig. 1: A Typical TORA LP Window



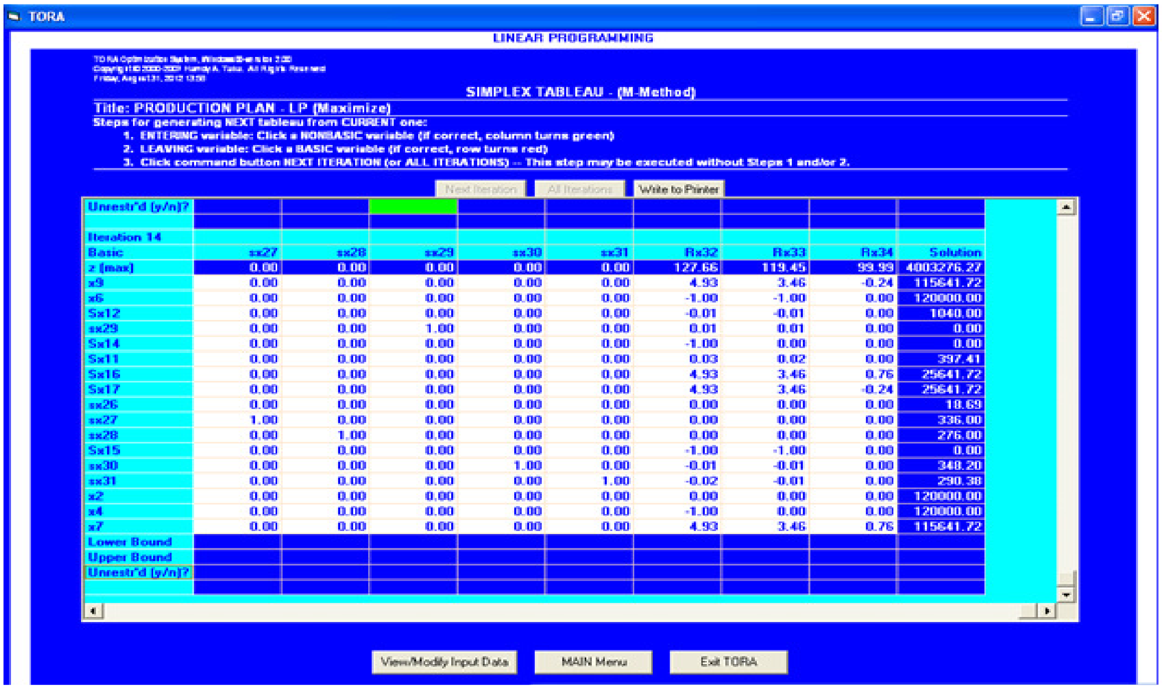
Iteration 13								
Basic	sx28	sx29	sx30	sx31	Rx32	Rx33	Rx34	Solution
z (max)	0.0000	4207.1429	0.0000	3081.8182	101.1100	107.7800	102.4700	4898181.8182
x9	0.0000	0.0000	0.0000	303.0303	0.0000	0.0000	0.0000	203636.3636
x6	0.0000	178.5714	0.0000	0.0000	0.0000	0.0000	0.0000	120000.0000
Sx12	0.0000	2.0893	0.0000	0.0000	0.0028	0.0061	0.0000	1058.0000
Sx13	0.0000	178.5714	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000
Sx14	0.0000	178.5714	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
Sx11	0.0000	0.0000	0.0000	1.8485	0.0000	0.0000	0.0028	934.1818
Sx16	0.0000	0.0000	0.0000	303.0303	0.0000	0.0000	1.0000	113636.3636
sx25	0.0000	-37.6250	0.0000	-12.3333	-0.0099	-0.0695	-0.0099	6428.0000
sx26	0.0000	-0.2500	0.0000	-0.0909	-0.0001	-0.0005	-0.0001	20.9091
sx27	0.0000	-0.5000	0.0000	0.0000	-0.0028	-0.0028	0.0000	336.0000
sx28	1.0000	-0.5893	0.0000	0.0000	0.0000	-0.0033	0.0000	276.0000
Sx15	0.0000	178.5714	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
sx30	0.0000	0.0000	1.0000	-0.8485	0.0000	0.0000	-0.0028	101.8182
Sx17	0.0000	0.0000	0.0000	303.0303	0.0000	0.0000	0.0000	113636.3636
x2	0.0000	178.5714	0.0000	0.0000	1.0000	1.0000	0.0000	120000.0000
x4	0.0000	178.5714	0.0000	0.0000	0.0000	1.0000	0.0000	120000.0000
x7	0.0000	0.0000	0.0000	303.0303	0.0000	0.0000	1.0000	203636.3636
Lower Bound								
Upper Bound								
Unrestr'd (y/n)?								

Fig. 2: Summary of Simplex Tableau of Iteration 13 for Optimum of the Original LP Model



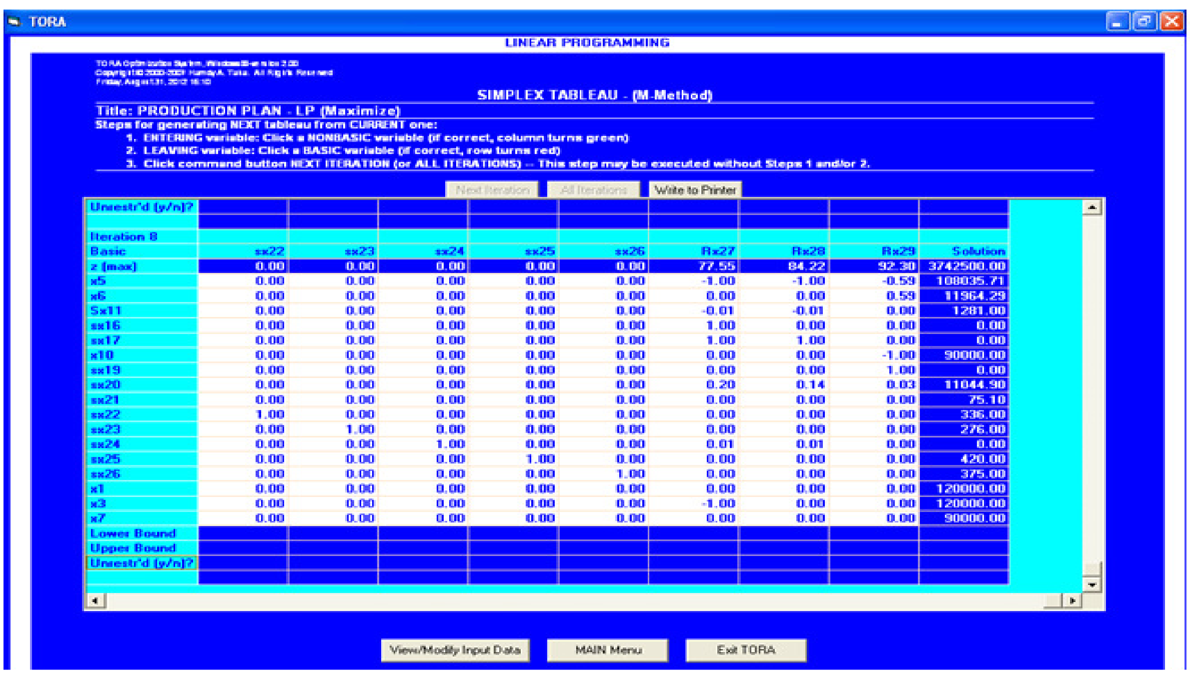
Iteration 7									
Basic	sx25	sx26	sx27	sx28	sx29	Rx30	Rx31	Rx32	Solution
z (max)	0.	0.	0.	0.	0.	124.	139.	84.	-31683172.
x9	0.	0.	0.	0.	0.	0.	0.	0.	16970.
x6	0.	0.	0.	0.	0.	0.	-1.	0.	10513.
Rx18	0.	0.	0.	0.	0.	-1.	0.	0.	109487.
Rx19	0.	0.	0.	0.	0.	0.	0.	0.	109487.
Rx20	0.	0.	0.	0.	0.	0.	1.	0.	109487.
x10	0.	0.	0.	0.	0.	0.	0.	-1.	73030.
Rx22	0.	0.	0.	0.	0.	0.	0.	1.	0.
sx23	0.	0.	0.	0.	0.	0.	0.	0.	34117.
sx24	0.	0.	0.	0.	0.	0.	0.	0.	214.
sx25	1.	0.	0.	0.	0.	0.	0.	0.	643.
sx26	0.	1.	0.	0.	0.	0.	0.	0.	637.
sx27	0.	0.	1.	0.	0.	0.	0.	0.	613.
sx28	0.	0.	0.	1.	0.	0.	0.	0.	420.
sx29	0.	0.	0.	0.	1.	0.	0.	0.	375.
x2	0.	0.	0.	0.	0.	1.	0.	0.	10513.
x4	0.	0.	0.	0.	0.	0.	0.	0.	10513.
x7	0.	0.	0.	0.	0.	0.	0.	0.	90000.
Lower Bound									
Upper Bound									
Unrestr'd (y/n)?									

Fig. 3: Summary of Simplex Tableau of Iteration 7 for Optimum of the First Post-Optimal Analysis



Iteration 14									
Basic	sx27	sx28	sx29	sx30	sx31	Rx32	Rx33	Rx34	Solution
z (max)	0.00	0.00	0.00	0.00	0.00	127.66	119.45	99.99	4003276.27
x9	0.00	0.00	0.00	0.00	0.00	4.93	3.46	-0.24	115641.72
x6	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	0.00	120000.00
Sx12	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	1040.00
sx29	0.00	0.00	1.00	0.00	0.00	0.01	0.01	0.00	0.00
Sx14	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00
Sx11	0.00	0.00	0.00	0.00	0.00	0.03	0.02	0.00	397.41
Sx16	0.00	0.00	0.00	0.00	0.00	4.93	3.46	0.76	25641.72
Sx17	0.00	0.00	0.00	0.00	0.00	4.93	3.46	-0.24	25641.72
sx26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.69
sx27	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	336.00
sx28	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	276.00
Sx15	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	0.00	0.00
sx30	0.00	0.00	0.00	1.00	0.00	-0.01	-0.01	0.00	348.20
sx31	0.00	0.00	0.00	0.00	1.00	-0.02	-0.01	0.00	290.38
x2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	120000.00
x4	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	120000.00
x7	0.00	0.00	0.00	0.00	0.00	4.93	3.46	0.76	115641.72
Lower Bound									
Upper Bound									
Unrestr'd (y/n)?									

Fig. 4: Summary of Simplex Tableau of Iteration 14 for Optimum of the Second Post-Optimal Analysis



Iteration 8									
Basic	sx22	sx23	sx24	sx25	sx26	Rx27	Rx28	Rx29	Solution
z (max)	0.00	0.00	0.00	0.00	0.00	77.55	84.22	92.30	3742500.00
x5	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	-0.59	108035.71
x6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	11964.29
Sx11	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	1281.00
sx16	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
sx17	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00
x10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	90000.00
sx19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
sx20	0.00	0.00	0.00	0.00	0.00	0.20	0.14	0.03	11044.90
sx21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	75.10
sx22	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	336.00
sx23	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	276.00
sx24	0.00	0.00	1.00	0.00	0.00	0.01	0.01	0.00	0.00
sx25	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	420.00
sx26	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	375.00
x1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	120000.00
x3	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	120000.00
x7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	90000.00
Lower Bound									
Upper Bound									
Unrestr'd (y/n)?									

Fig. 5: Summary of Simplex Tableau of Iteration 8 for Optimum of the Third Post-Optimal Analysis