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Magnetohydrodynamic Flow of Casson Fluid through a Vertical Deformable Porous Stratum

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Abstract
Magnetohydrodynamic (MHD) flow of Casson fluid through a vertical deformable porous stratum is studied. The governing ordinary differential equations of the fluid velocity, the displacement and the heat transfer are solved numerically using shooting technique. The effects of pertinent parameters on the flow velocity, the solid displacement and the heat transfer are discussed through graphs. The skin friction coefficient is displayed through table. It is noticed that the effect of increasing Casson parameter is to increase the skin friction in the deformable stratum.
Keywords: MHD; Casson parameter; deformable layer; shooting technique.

NOMENCLATURE

g	acceleration due to gravity	ϕ^f	the volume fraction components of the fluid phase, $\phi^f + \phi^s = 1$.
K	drag coefficient	μ_f	coefficient of the viscosity
K_0	thermal conductivity	δ	measure of the viscous drag of the outside fluid relative to drag in the porous medium
p	pressure	η	ration of the bulk fluid viscosity to the apparent fluid viscosity in the porous layer
T	temperature	M	Magnetic fluid parameter
T_0	ambient temperature	σ	electrical conductivity
N	buoyancy parameter	B_0	Magnetic field strength
μ_a	apparent viscosity of the fluid in the porous material	τ	shear stress
ρ	density of the fluid	Nu	Nusselt number
β_1	coefficient of linear thermal expansion of the fluid		
β	the Casson parameter		
ϕ^s	volume fraction components of solid phase		

INTRODUCTION

In recent years a great deal of interest has been generated to study heat transfer in fluid flows through porous media because of their extensive applications in engineering, biology and medicine. These include heat exchange between soil and atmosphere, flow of moisture through porous blood vessels can be better understood with theory of flow through deformable porous media. The study of flows through deformable porous materials was initiated by Terzaghi [1] and further developed by Biot [2,3], Atkin and Crane [4] and Kenyon [5] this deformation theory is applied to the study of flows in biological tissue layers and articular cartilage. Dariusz Gawin *et al.* [6] developed coupled heat, water and gas flow in deformable porous media. Barry *et al.* [7] studied fluid flow over a thin deformable porous layer. Flow of Newtonian fluid a channel with deformable porous walls was reported by Ranganatha *et al.* [8]. Sreenadh *et al.* [9] studied MHD Couette flow of a Jeffrey fluid over a deformable porous player.

Gopi Krishna *et al.* [10] developed an entropy generation on viscous fluid in the inclined deformable porous medium. Free convection flow of a Jeffrey fluid through a vertical deformable porous stratum and MHD free surface flow of a Jeffrey fluid over a deformable porous layer was discovered by Sreenadh *et al*. [11, 12]. Hartmann flow over a permeable bed was demonstrated by Rudraiah *et al.* [13]. Sreenadh *et al.* [14] discussed viscous fluid flow in an inclined channel with deformable porous medium. Gopi Krishna *et al.* [15] developed viscous flow and heat transfer in a vertical channel with deformable porous medium. Effect of heat transfer on free surface flow of a Jeffrey fluid over a deformable permeable bed and flow of a Jeffrey fluid between finite deformable porous layers was reported by Sreenadh *et al.* [16, 17]. Asghar *et al.* [18] studied Flow and heat transfer analysis in a deformable channel. Selvi *et al.* [19] investigated viscous flow of Jeffrey fluid in an inclined channel through deformable porous media.

Gopi Krishna *et al.* [20] studied an entropy generation and heat transfer in a Casson fluid flow through a vertical deformable porous stratum. Krishna Murthy [21] discovered MHD Poiseuille flow of a Jeffrey fluid over a deformable porous layer. Syed Tauseef Mohyud-Din *et al.* [22] founded flow of a radioactive Casson fluid through a deformable asymmetric porous channel. Sreenadh *et al.* [23] studied effects of free convection on steady flow through a vertical deformable porous layer with constant heat source. Rudraiah *et al.* [24] studied natural convection through vertical porous stratum.

The present study deals with Magnetohydrodynamic (MHD) flow of Casson fluid through a vertical deformable porous stratum. Here Casson parameter $\beta \to \infty$ this reduces to Newtonian model. The velocity, displacement and temperature are obtained graphically while the skin friction coefficient is calculated numerically.

MATHEMATICAL FORMULATION OF THE PROBLEM

Consider, the steady MHD flow of Casson fluid through a vertical deformable porous stratum as shown in Figure 1. The x - axis is taken midway in the channel and y - axis perpendicular to it. The deformations are assumed to be small and are predominantly in the direction. It is assumed that heat is generated within the fluid by both viscous and Darcy dissipations. The walls are placed at a distance $2b$ and maintained at a constant temperature T_1. A pressure gradient $\frac{\partial p}{\partial x}$ is applied producing an axially directed flow.

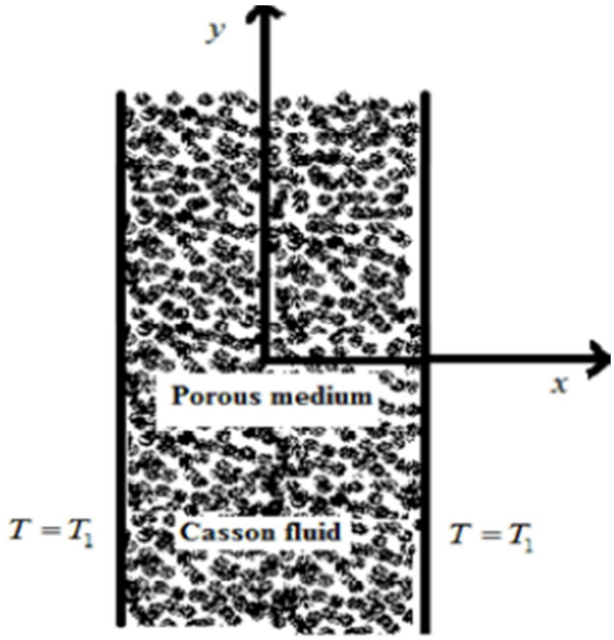

Figure 1. Physical model of the problem

The governing equations for the flow velocity, the displacement and the heat transfer are shown below

$$2\mu_a\left(1+\frac{1}{\beta}\right)\frac{\partial^2 v}{\partial y^2}-\phi^f\frac{\partial p}{\partial x}-\sigma B_0^2 v-Kv+\rho g\beta_1\left(T-T_0\right)=0 \tag{1}$$

$$\mu\frac{\partial^2 u}{\partial y^2}-\phi^s\frac{\partial p}{\partial x}+Kv=0 \tag{2}$$

$$\frac{\partial^2 T}{\partial y^2}+\frac{2\mu_a}{K_0}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial v}{\partial y}\right)^2+\frac{K}{K_0}v^2=0 \tag{3}$$

The boundary conditions are

$$\left.\begin{array}{l}\frac{dv}{dy}=0,\frac{du}{dy}=0,\frac{dT}{dy}=0\,at\;y=0\\ v=0,u=0,T=T_1\,at\;y=b\end{array}\right\} \tag{4}$$

NON- DIMENSIONALIZATION OF THE FLOW QUANTITIES

We introduce the following non-dimensional quantities are

$$\left.\begin{aligned} &y^* = \frac{y}{b},\ y^* = \frac{x}{b},\ v^* = \frac{2\mu_a v}{\rho g \beta_1 b^2 \left(T_1 - T_0\right)} \\ &u^* = \frac{\mu u}{\rho g \beta_1 b^2 \left(T_1 - T_0\right)}, \theta = \frac{T - T_0}{T_1 - T_0}, p^* = \frac{p}{\rho g \beta_1 b \left(T_1 - T_0\right)} \end{aligned}\right\} \tag{5}$$

In view of the above dimensionless quantities, the equations (1)-(4) take the following form. The asterisks (*) are neglected here after

$$\left(1+\frac{1}{\beta}\right)\frac{d^2 v}{dy^2} - \phi^f G - \delta\eta v - M^2 v + \theta = 0 \tag{6}$$

$$\frac{d^2 u}{dy^2} - \phi^s \frac{dp}{dx} + \delta\eta v = 0 \tag{7}$$

$$\frac{d^2\theta}{dy^2} + N\left(1+\frac{1}{\beta}\right)\left(\frac{dv}{dy}\right)^2 + N\delta\eta v^2 = 0 \tag{8}$$

Where

$$\delta = \frac{Kb^2}{\mu_f},\ \eta = \frac{\mu_f}{2\mu_a},\ N = \frac{\rho g^2 \beta_1^2 b^4 \left(T_1 - T_0\right)}{2K_0 \mu_a}, M^2 = \frac{\sigma B_0^2 b^2}{2\mu_a},\ G = \frac{dp}{dx} \text{ is the pressure gradient}$$

The boundary conditions are

$$\left.\begin{aligned} &\frac{dv}{dy} = 0, \frac{du}{dy} = 0, \frac{d\theta}{dy} = 0 \quad at \quad y = 0 \\ &v = 0, u = 0, \theta = 1 \qquad at \quad y = 1 \end{aligned}\right\} \tag{9}$$

Skin friction Coefficient

The skin friction in the velocity field at the wall in non-dimensional form is given by

$$\tau = \left(1+\frac{1}{\beta}\right)\left(\frac{dv}{dy}\right)_{at\ y=1} \tag{10}$$

Rate of Heat Transfer

The rate of heat transfer at the wall is given by

$$Nu = -\left(\frac{d\theta}{dy}\right)_{at\ y=1} \tag{11}$$

RESULTS AND DISCUSSION

The present study deals with MHD flow of Casson fluid through a vertical deformable porous stratum. The coupled non linear governing equations are solved numerically using Runge-Kutta fourth order along with shooting technique. The effects of pertinent parameters on the velocity, the displacement and the temperature are displayed graphically. The skin friction coefficient is calculated numerically and is shown in Table-1.

The effects of magnetic parameter on velocity $v(y)$, displacement $u(y)$ and temperature $\theta(y)$ are shown in Figures 2, 3 and 4. We noticed the velocity, displacement and temperature are decrease with increasing values of magnetic field parameter M . This reduction causes the Lorentz force associated with the magnetic field increases and it produces more resistance to the transport phenomena in the vertical porous stratum. The influence of Casson parameter on velocity $v(y)$, displacement $u(y)$ and temperature $\theta(y)$ are displayed in Figures 5, 6 and 7. We have seen that the velocity, displacement and temperature are decrease with increasing values of Casson parameter β . This fact that the higher values of Casson parameter results the decrease in yield stress (the fluid behaves as a Newtonian fluid when the Casson parameter becomes large).

The variation of fluid velocity $v(y)$, displacement $u(y)$ and temperature $\theta(y)$ are examined with the effect of pressure gradient G is shown in Figures 8-10. We examine that it is reduces for higher values of pressure gradient. The impact of volume fractional of the fluid ϕ^f and viscosity parameter η on velocity $v(y)$, displacement $u(y)$ and temperature $\theta(y)$ are sketched in Figures 11-16. This shows that velocity and temperature are decrease for higher values of volume fraction of the fluid and the opposite nature in displacement.

The effect of buoyancy parameter N on fluid velocity $v(y)$, displacement $u(y)$ and temperature $\theta(y)$ are shown in Figures 17-19. We have seen that it is enhance with increasing values of N. The effect of drag coefficient δ on fluid velocity $v(y)$, displacement $u(y)$ and temperature $\theta(y)$ are shown in Figures 20-22. We noticed that the velocity and temperature reduce with increasing drag coefficient δ and the opposite behavior in solid displacement. This fact that increasing viscosity parameter $\left(\mu_f/2\mu_a\right)$, gives rise to an increase in the velocity in the porous layer. This shows that the increase in the drag enhances the displacement of solid particles in the deformable porous layer.

The magnitude of the skin friction coefficient is evaluated numerically for equation (10) for distinct values of buoyancy parameter N and is shown in Table1. We have seen that the skin friction coefficient at the vertical wall $y=1$ enhances with higher values of N. The same behavior is noticed in the case of vertical undeformable porous layer Rudraiah *et al.* [24]. Higher skin friction is observed for a given buoyancy force for a non-Newtonian Casson parameter when compared with Newtonian fluid.

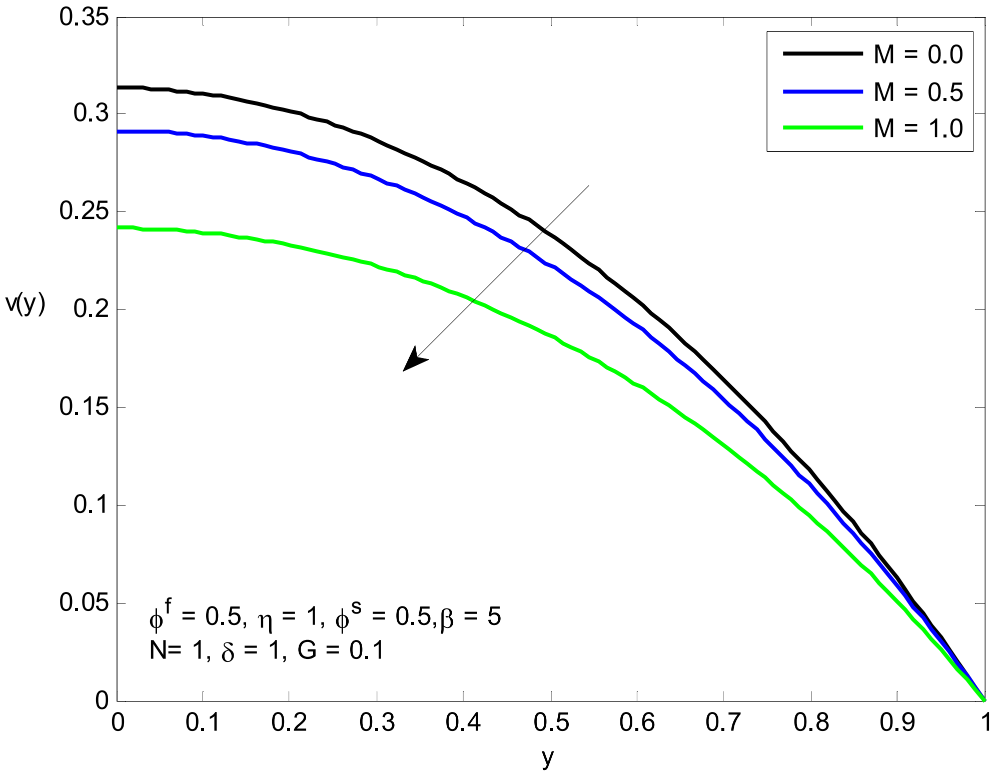

Figure 2. The velocity $v(y)$ for different values of M

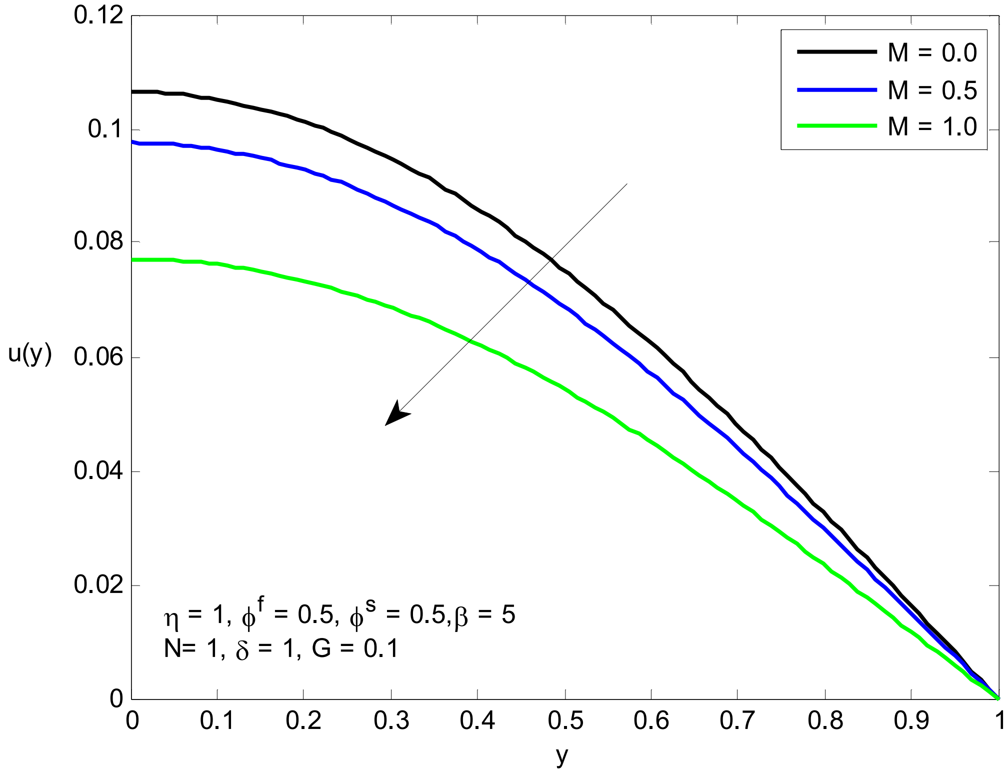

Figure 3. The displacement $u(y)$ for different values of M

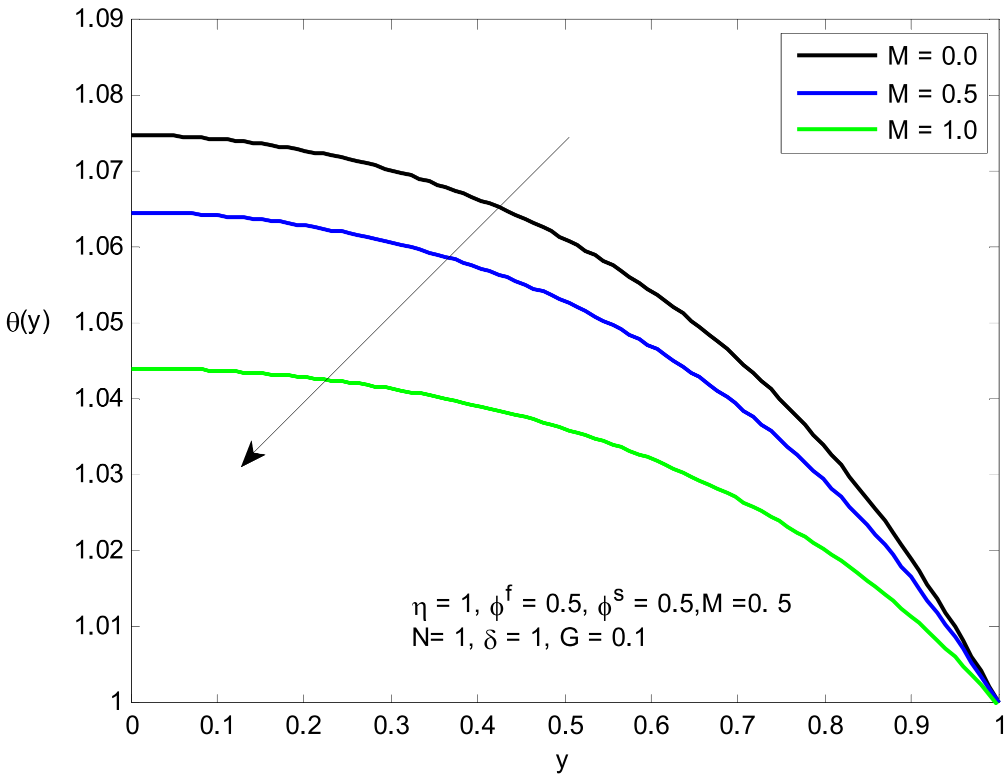

Figure 4. The temperature $\theta(y)$ for different values of M

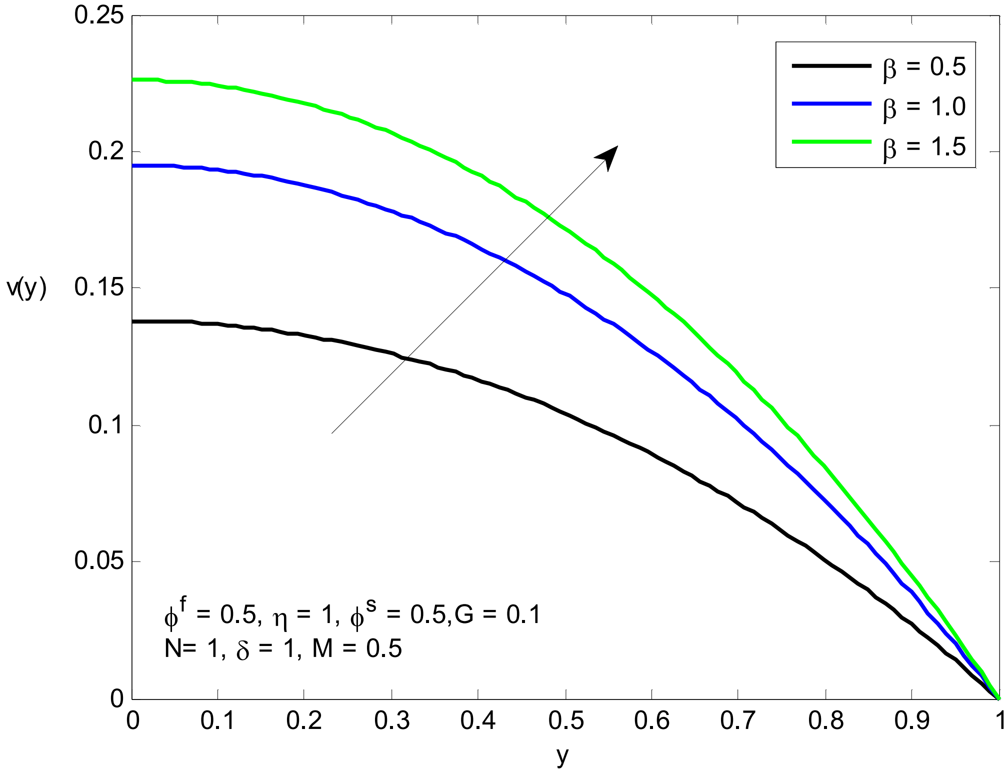

Figure 5. The velocity $v(y)$ for different values of β

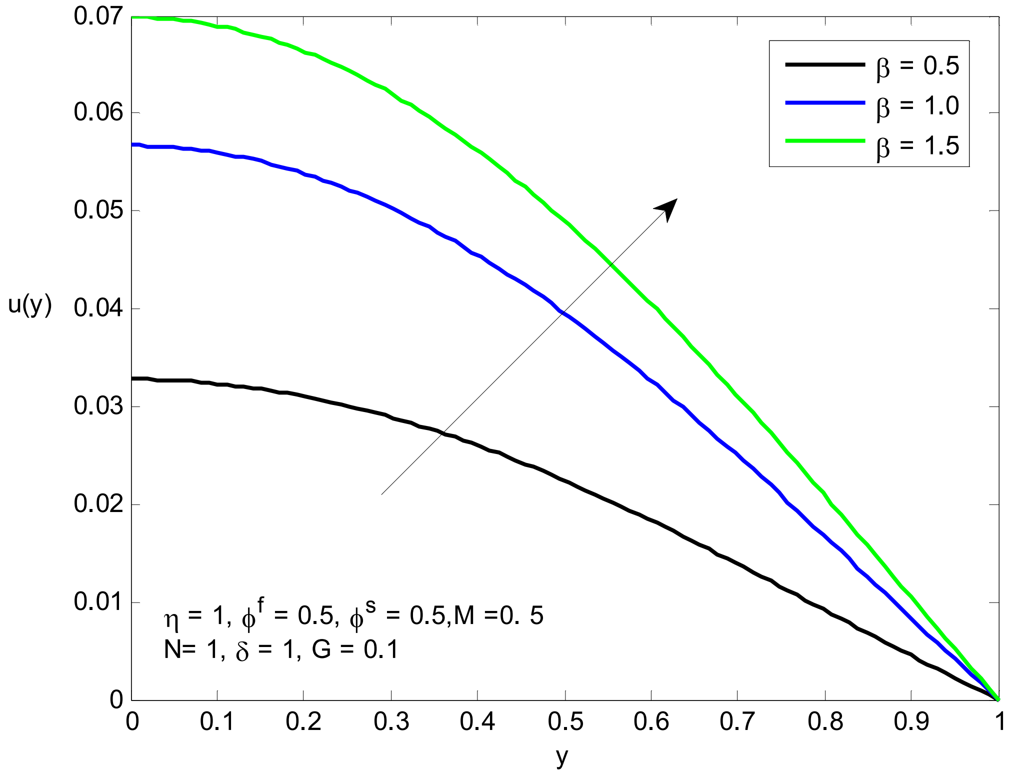

Figure 6. The displacement $u(y)$ for different values of β

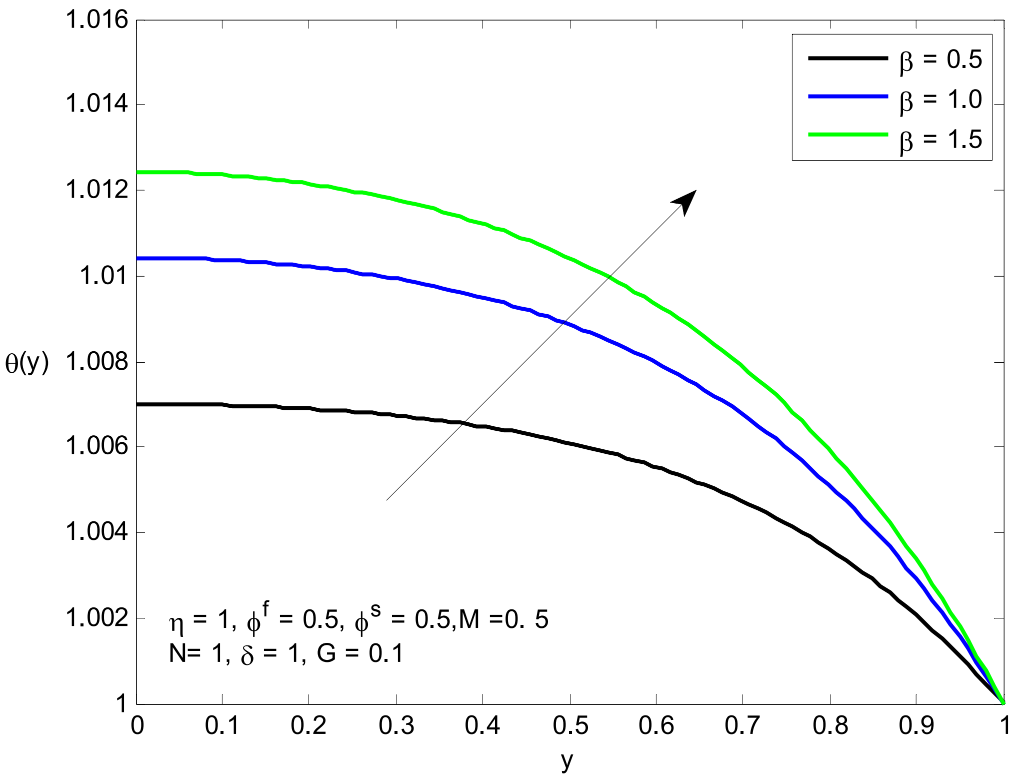

Figure 7. The temperature $\theta(y)$ for different values of β

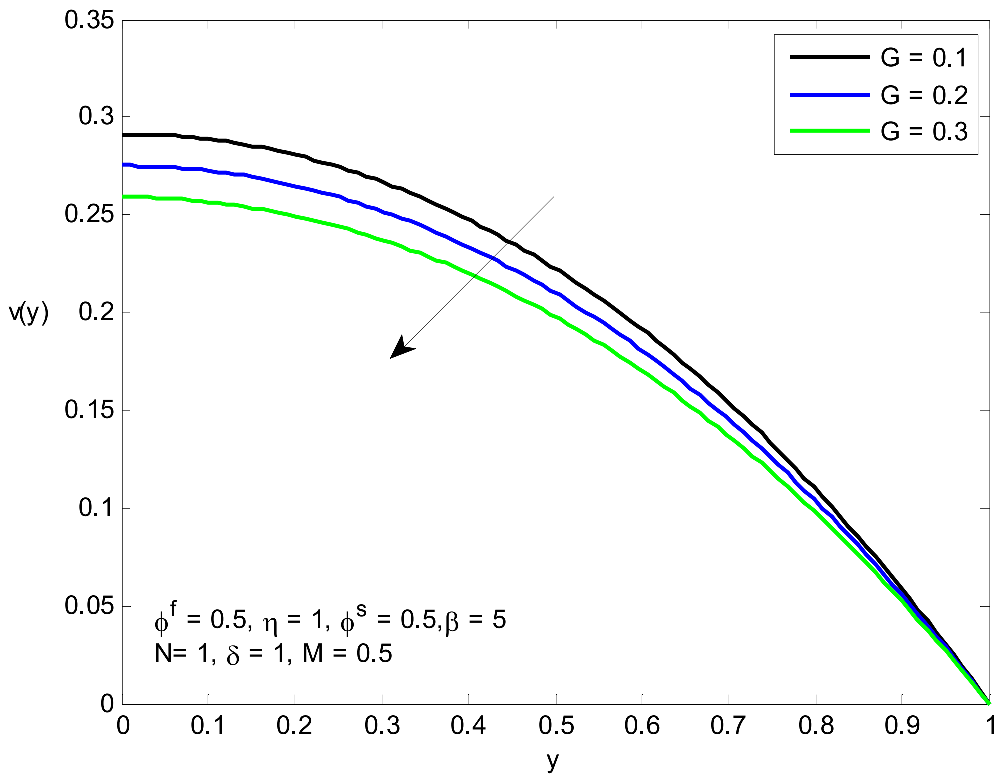

Figure 8. The velocity $v(y)$ for different values of G

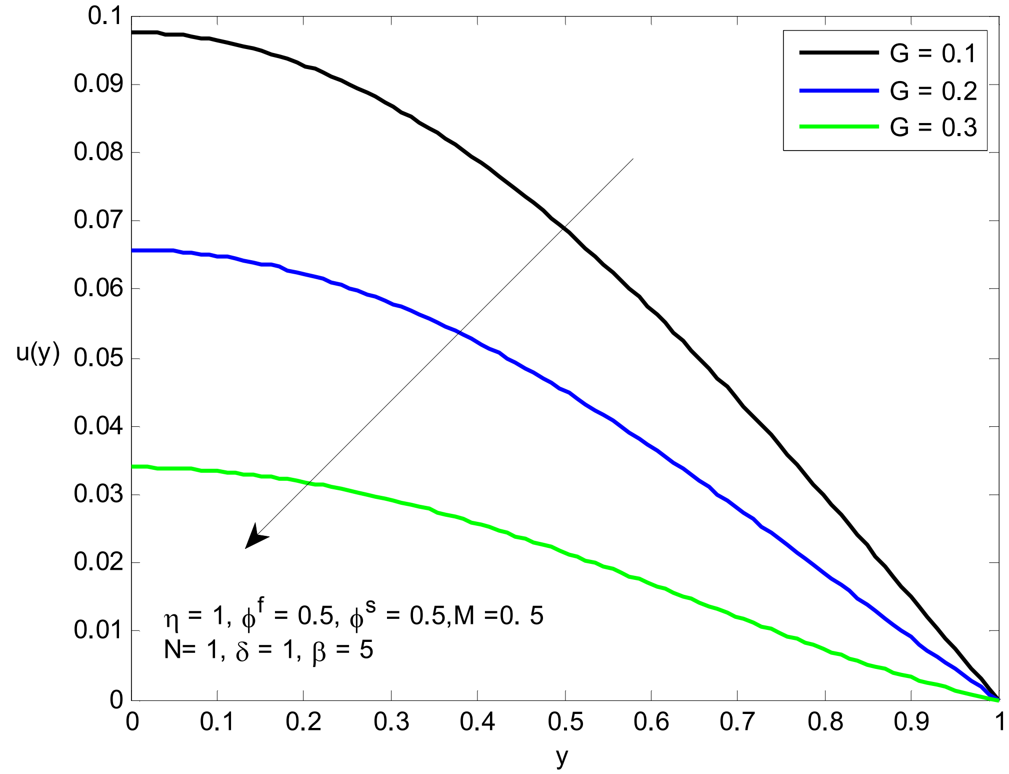

Figure 9. The displacement $u(y)$ for different values of G

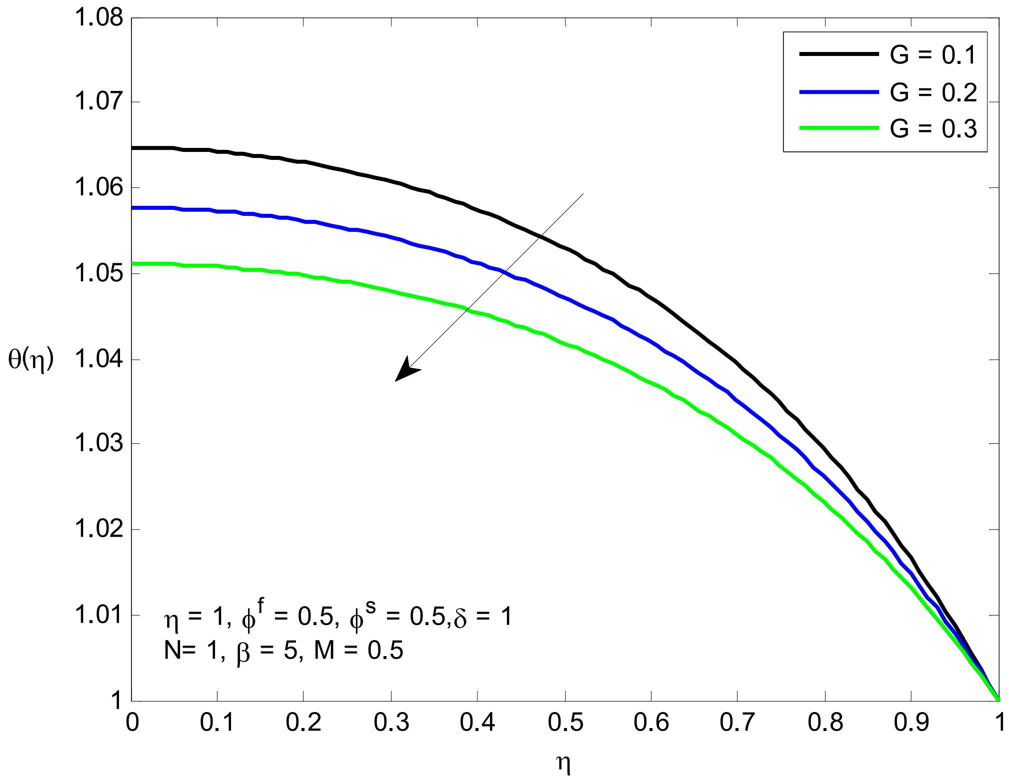

Figure 10. The temperature $\theta(y)$ for different values of G

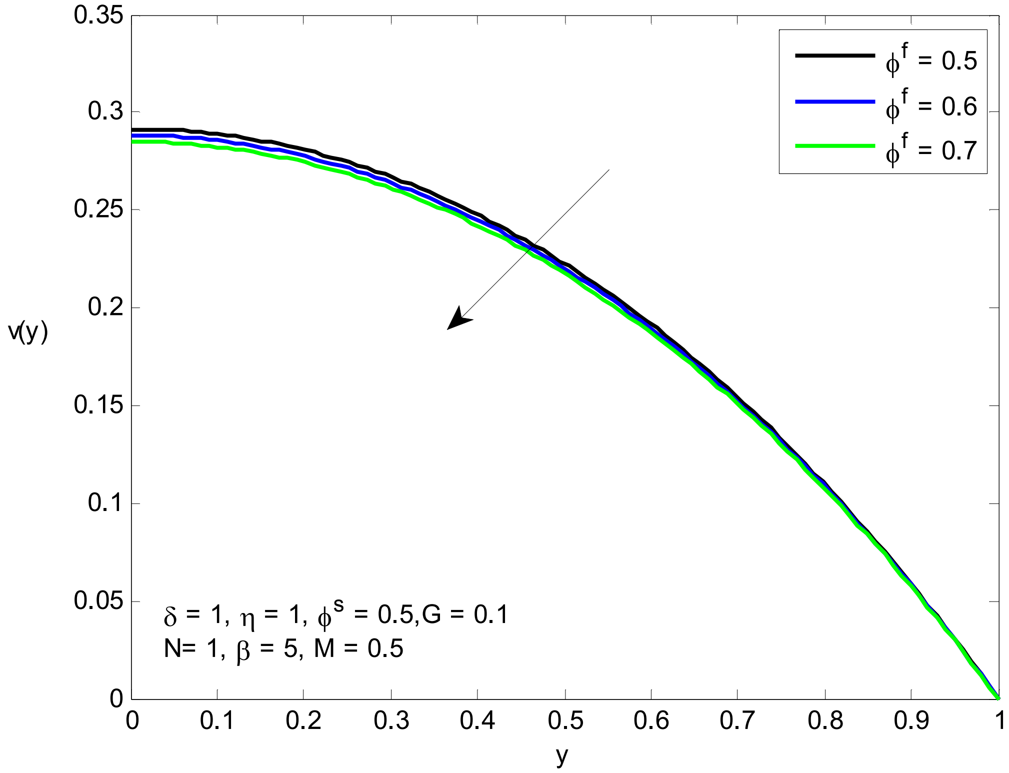

Figure 11. The velocity $v(y)$ for different values of ϕ^f

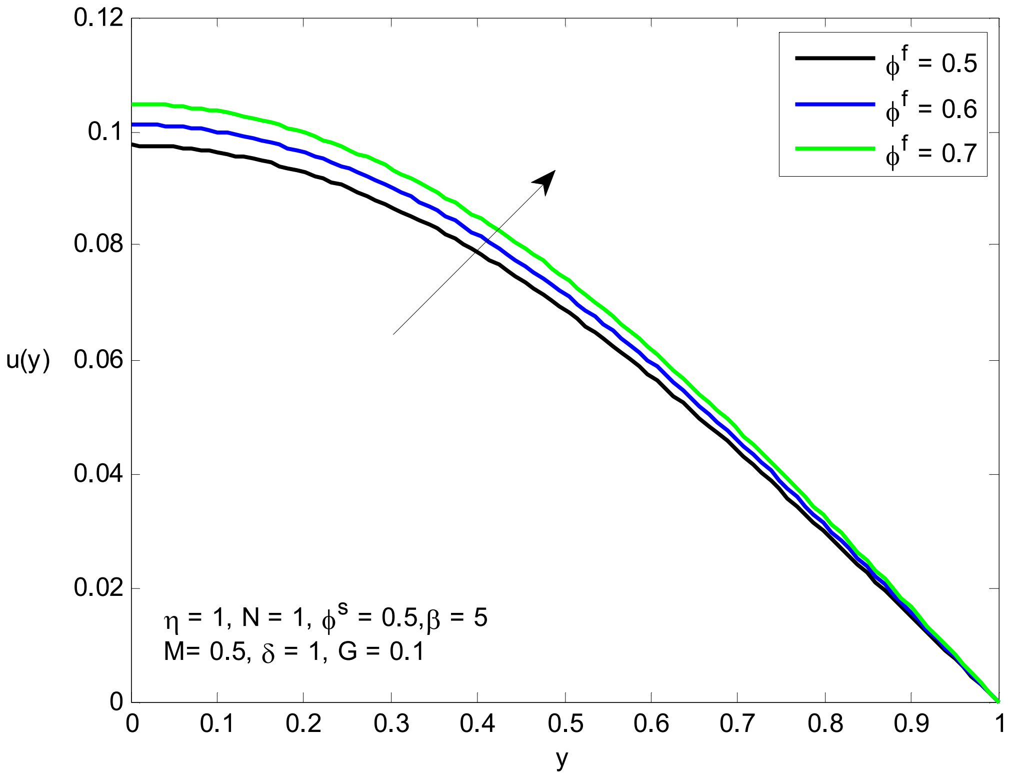

Figure 12. The displacement $u(y)$ for different values of ϕ^f

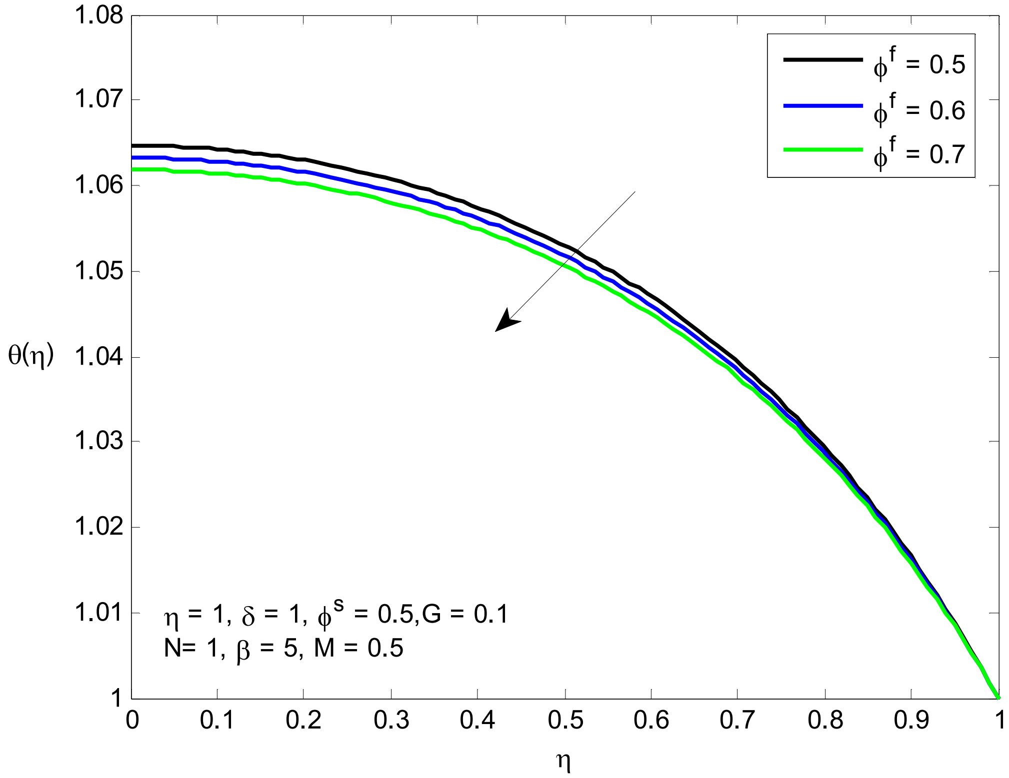

Figure 13. The temperature $\theta(y)$ for different values of ϕ^{f}

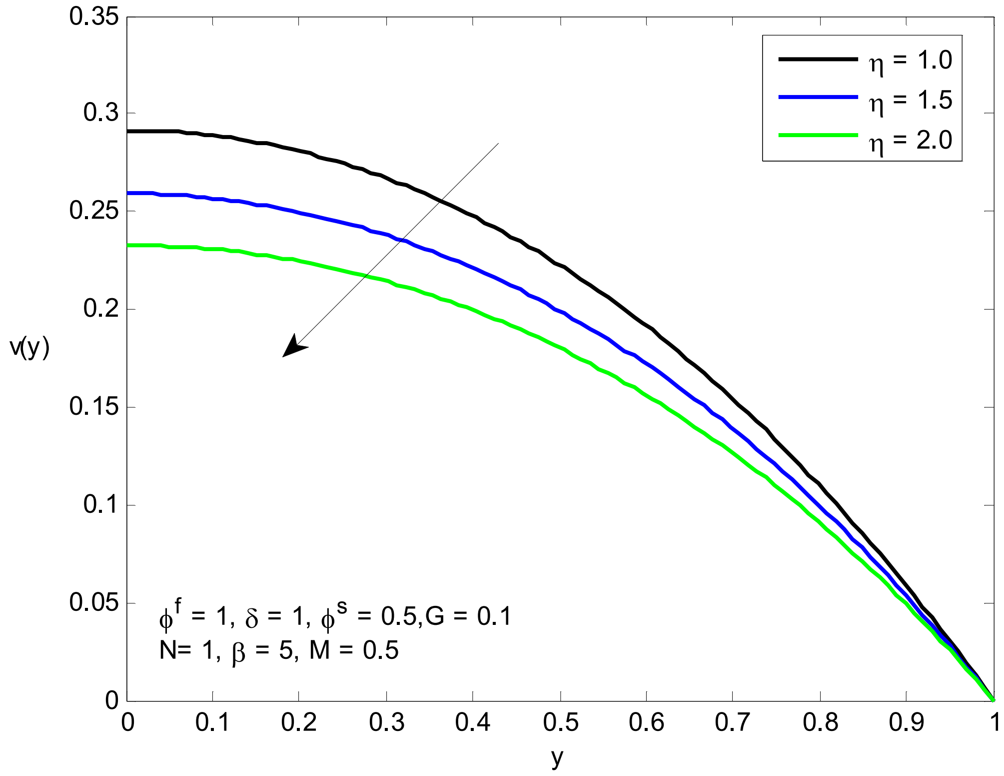

Figure 14. The velocity $v(y)$ for different values of η

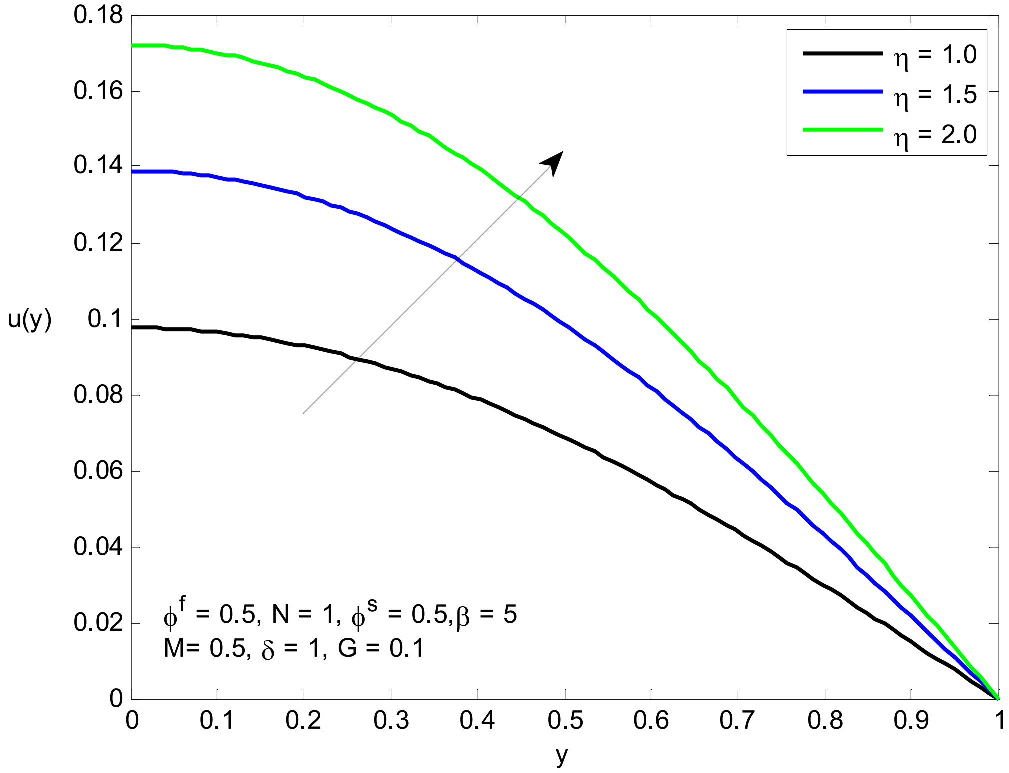

Figure 15. The displacement $u(y)$ for different values of η

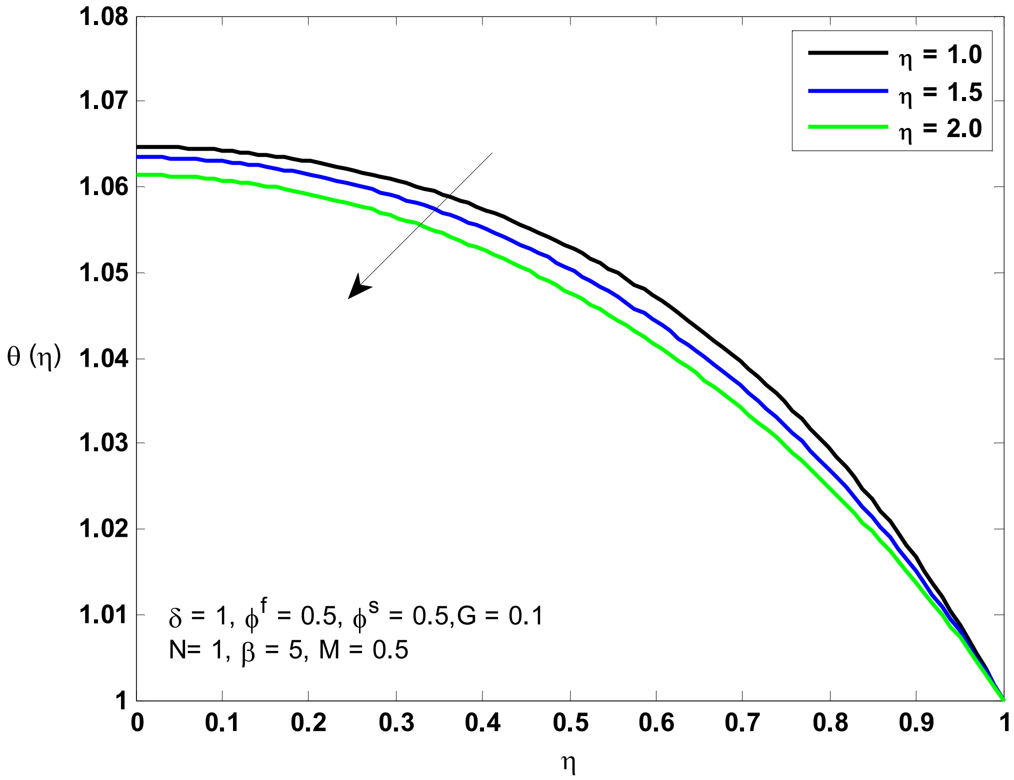

Figure 16. The temperature $\theta(y)$ for different values of η

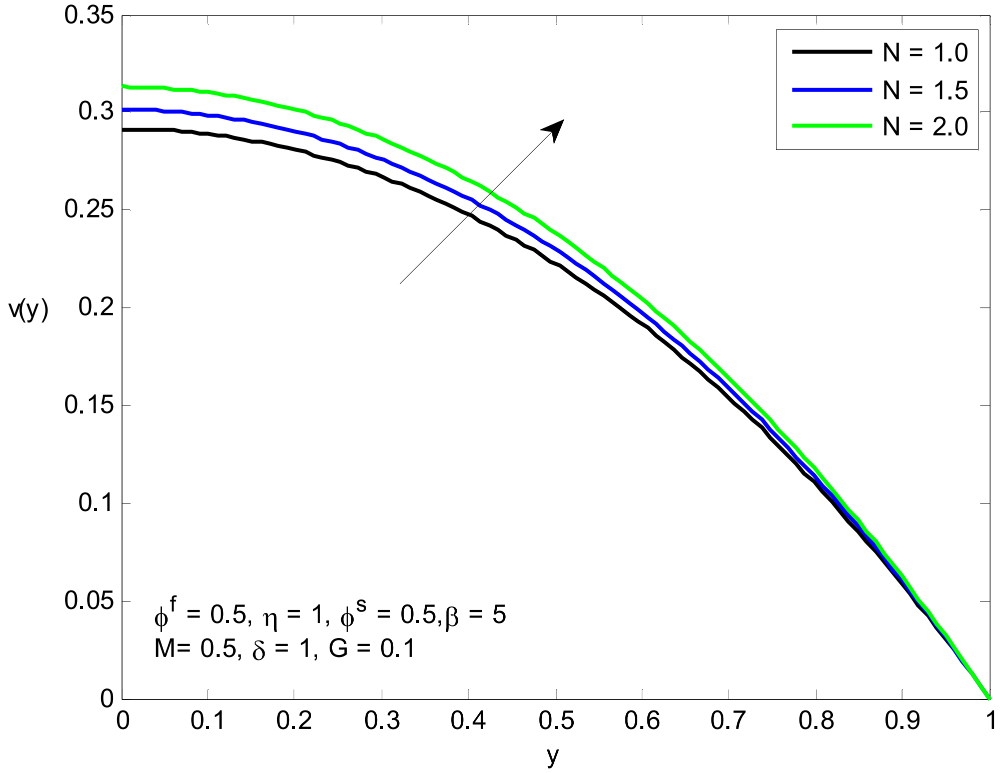

Figure 17. The velocity $v(y)$ for different values of N

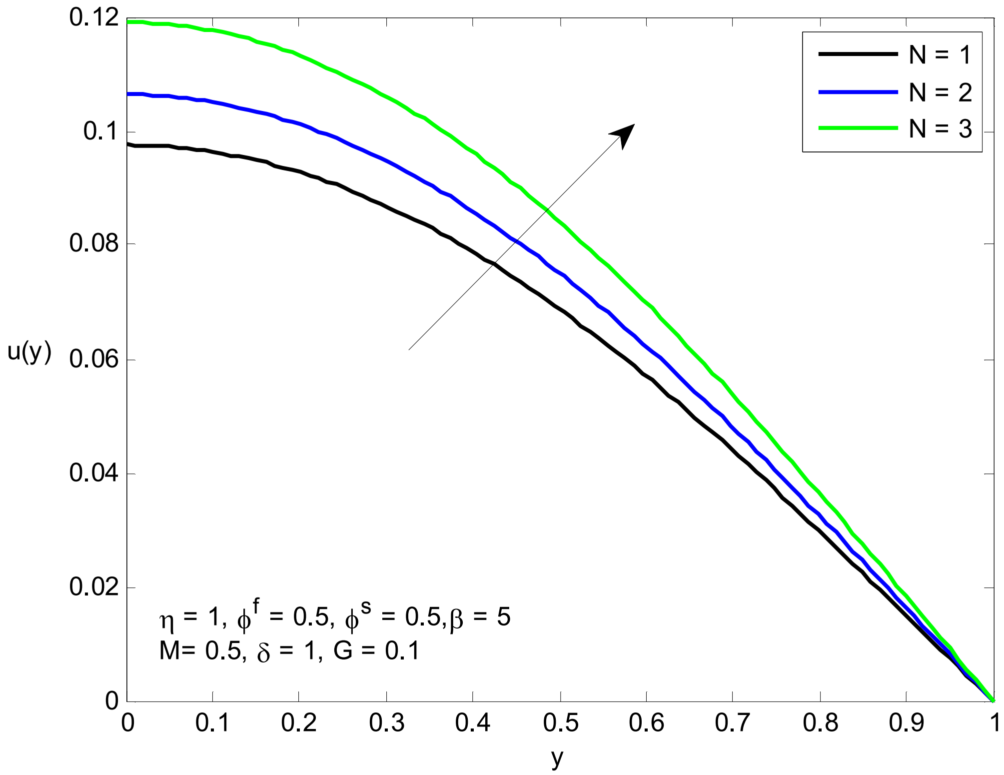

Figure 18. The displacement $u(y)$ for different values of N

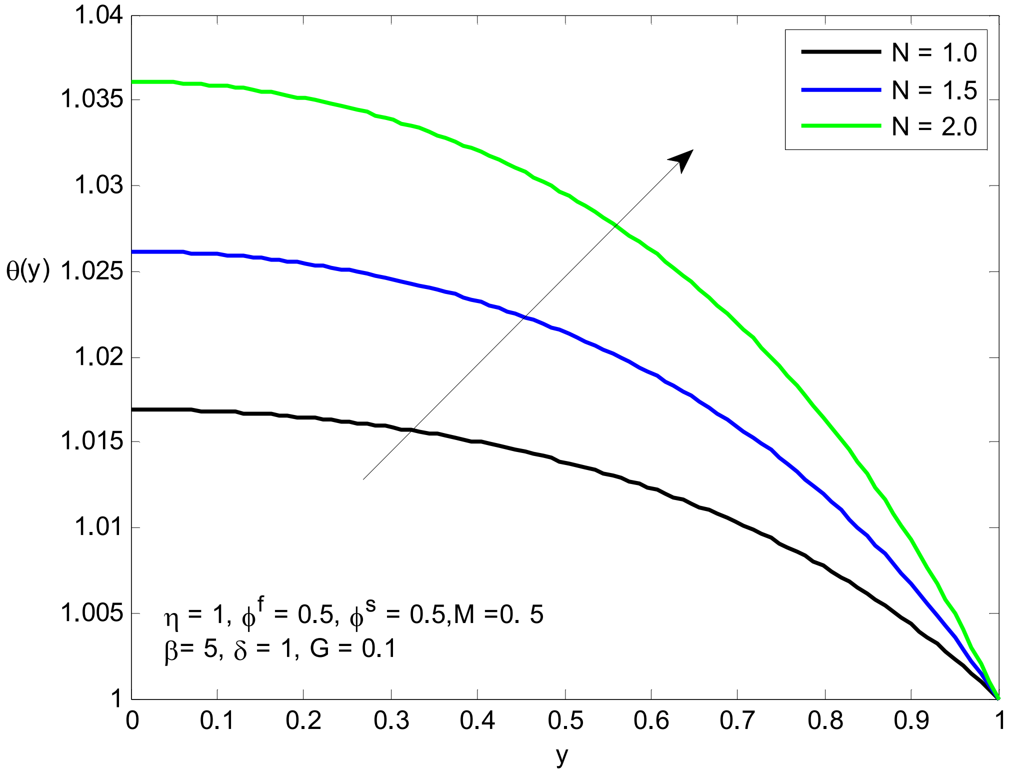

Figure 19. The temperature $\theta(y)$ for different values of N

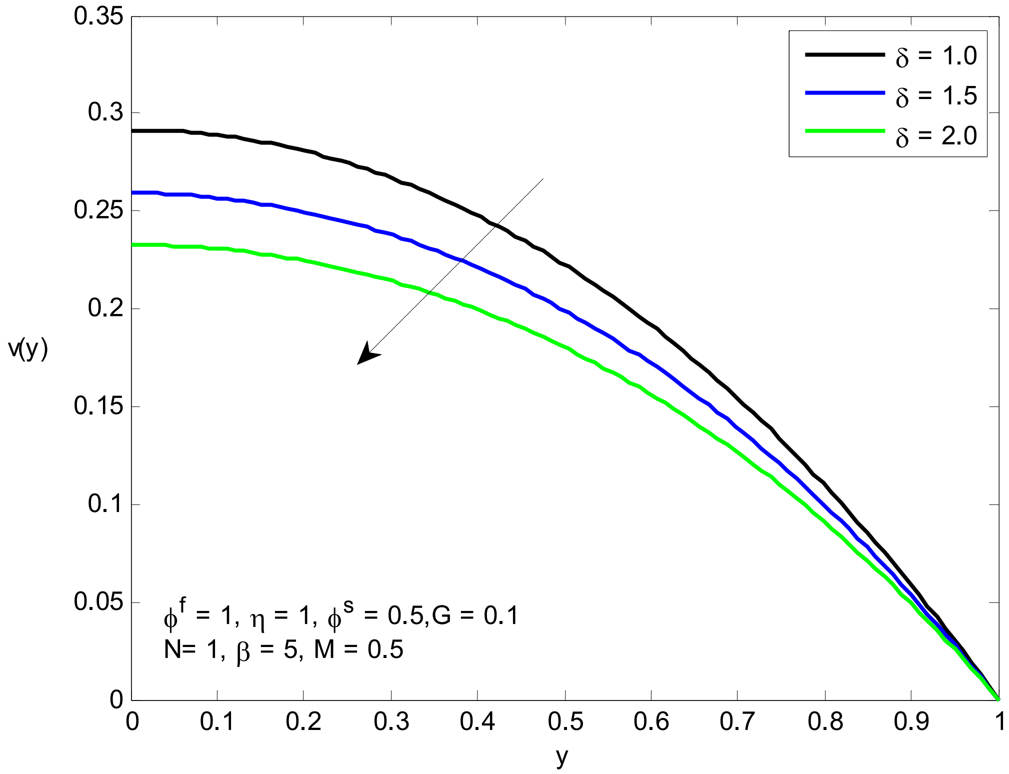

Figure 20. The velocity $v(y)$ for different values of δ

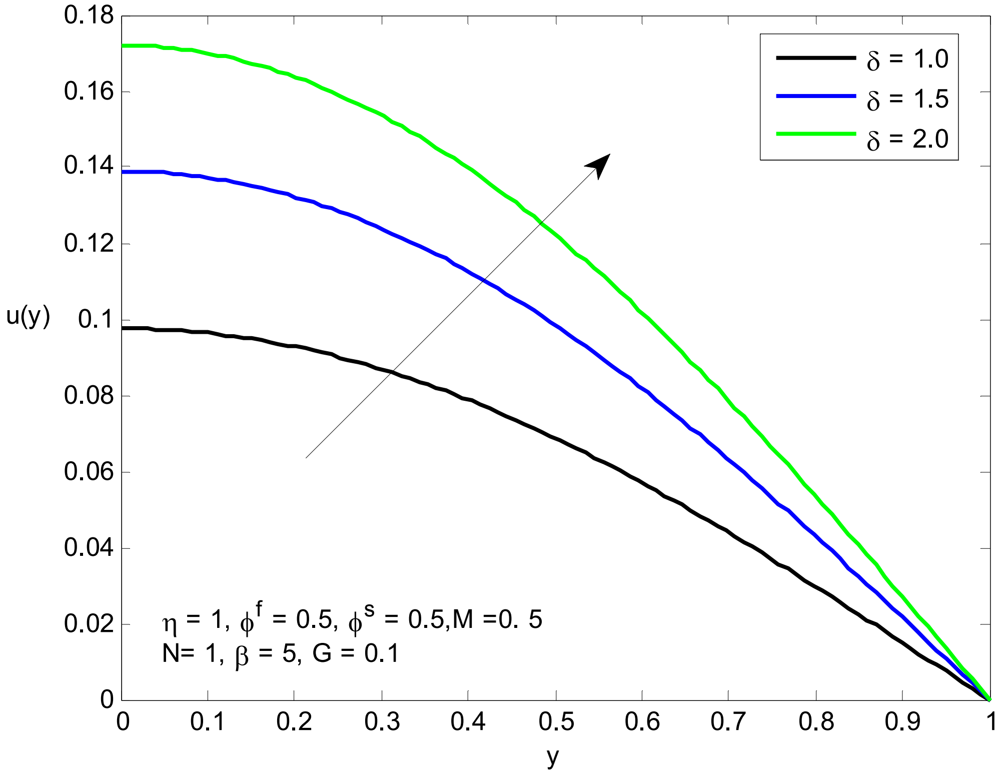

Figure 21. The displacement $u(y)$ for different values of δ

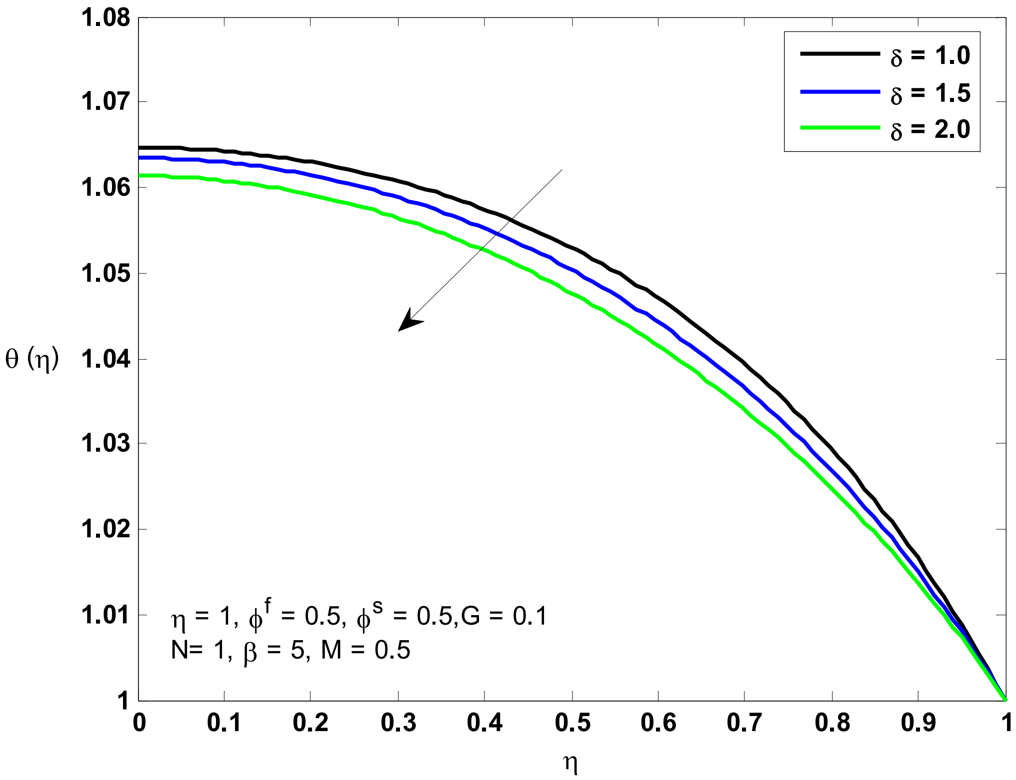

Figure 22. The temperature $\theta(y)$ for different values of δ

Table 1. The Skin friction coefficient τ at $y = 1$ for different values of N

S.No.		$N=1$	$N=2$	$N=3$
1	Rudraiah *et al.*[24] (undeformable porous layer)	0.8056	0.8497	0.8937
2	Present work (deformable porous layer $M = 0, \beta \to \infty$)	0.7252	0.7268	0.7285
3	Present work with ($M = 0, \beta \to \infty$)	1.0660	1.3006	1.5352

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