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Application of Dynamic Programming Model to Production Planning, in an Animal Feedmills.

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Abstract
The problem of this study is that of determining the quantity of products to produce and inventory level to carry from one period to the other, with the objective of minimizing the total costs of production and the annual inventory, while at the same time meeting the customer's demand. A mathematical model was formulated for a multi-product problem using Dynamic Programming approach. The model was solved using the solution procedure proposed by Wagner and Whitin. The results show that the minimum total cost will be achieved with production in periods 1, 2, and 4. While demand for period 3 are satisfied with inventory from period 2. The total cost of this plan is N225, 704, 210.00, which is N6, 155, 765.00 less than the existing plan.
Keywords: Inventory, Model, Periods, Plan, Cost.

1. Introduction

It is a known fact that the most measurable goal of a profit oriented organizations is to make profits and achieve a sustainable growth for the future amid such uncertainty and stiff competition. And it is not just a matter of producing and selling, but making decisions on setting production schedules and meeting it, satisfying customer's demands, boosting the company's image, maintaining the quality of products and keeping inventory cost as low as possible.

Teslang (2000), defined Production management as the process of decision making related to production processes so that the resulting goods or services are produced according to specification in the amounts and by the schedule demanded and at minimum cost. In making decision on production planning, the production quantity for each period as well as that of each product is stated. Some key reasons for developing this kind of plan include, supply the finished products as specified in the sales programme on or before the due date; ensure a balance or economic utilization of labour, plant and capital in the most efficient way possible.

A general dynamic programming model can be easily formulated for a single dimension process from the principle of optimality. The programming situation involves a certain quantity of economic resources (space, finance, people, and equipment) which can be allocated to a number of different activities (Michael, 2005). Dynamic programming is handy in solving a problem with multi-stage problem, a particular situation in which there is appreciable variation in average monthly demand and availability of raw materials among the different periods under consideration (Hamdy, 2007).

An inventory is the quantity of commodity that a business must maintain to ensure smooth operation, with the goal of minimizing the total cost of inventory (Lucey, 2002). Typically, holding costs are estimated to cost approximately 15-35% of the material's actual value per year (Charles, 2005). The primary factors that drive this up include additional rent needed, great insurance premiums to protect inventory, opportunity costs, and the cost of capital to finance inventory.

One of the problem often encountered in production planning in industries with large product demand, is production planning requirement. The problem is that of determining the quantity to be produced and the inventory quantity to be carried, such that the demand of each period will be met at minimum total production cost. The above problem has characteristics of dynamic programming problem as stated by Hillier and Lieberman (2001).

Specifically Wagner-Whitin dynamic programming inventory model solution procedure was adopted in this study and it is characterised by three types of equations (Michael, 2005), namely; Initial conditions, a recursive relation and an optimal value function.

Wagner and Whitin (1958) used dynamic economic lot size model as a guide to formulating a model that will handle both the production rate and inventory levels simultaneously. The decision variables are the production rate and inventory levels. Because the number of combinations, in general, can be as large as the product of the number of possible values of the respective variables, the number of required calculations tends to "blow up" rapidly when additional state variables are introduced (Hillier and Lieberman, 2001), this phenomenon is known as "curse of dimensionality".

2. Methodology

2.1 Model Objectives

A production plan required which stated the quantities of each product *j* produced per period *i* so as to meet the demand for the period at a minimal total cost. The cost function was made up of two components (production and inventory costs).

The Production cost for product *j* in period *i* given by;

$$\text{Production cost for product j} = c_{ij} z_{ij} + k_{ij} \tag{1}$$

The cost of carrying *x* units of product *j* from period *i* to period *i + 1* given by;

$$\text{Inventory cost of product j} = h_{ij} x_{j,i+1} \tag{2}$$

The cost function for product *j* for period *i* given by;

$$(c_{ij} z_{ij} + k_{ij}) + h_{ij} x_{j,i+1} \tag{3}$$

The total cost for all the products over the planning horizon given by;

$$\sum_{j=1}^{J} \sum_{i=1}^{I} \left[\left(c_{ij} z_{ij} + k_{ij} \right) + h_{ij} x_{j,i+1} \right] \tag{4}$$

2.2 Model Assumption

The following assumptions were set to construct the mathematical model of the production planning problem.

1. The average periodic demand varies appreciably among the different quarters.
2. Raw materials are available, but there is periodic change in their prices.
3. The model will handle both the production rate and inventory level simultaneously.
4. Multiple products are produced.
5. Only conservative of material constrained will be considered.
6. Single objective i.e minimizing total cost.
7. The model is deterministic.
8. Shortages are not allowed.
9. Unit production cost vary from period to period
10. Unit holding cost is unchanged for all period.

2.3 Mathematical Model of the Problem

The planning problem may be stated as:

$$\text{Minimize (G)} = \sum_{j=1}^{J} \sum_{i=1}^{I} \left[\left(c_{ij} z_{ij} + k_{ij} \right) + h_{ij} x_{j,i+1} \right]$$

$$S.t,$$

$$x_{j,i+1} = x_{ij} + z_{ij} - d_{ij} \tag{5}$$

for i = 1, 2,..........., I.
j = 1, 2,..........., J.

$$x_{ij}, z_{ij} \geq 0, \forall i, j$$

2.4 Solution Procedure

Wagner and Whitin solution procedure was used and under the given conditions it can be proved that:

1. Given the initial inventory $x_i = 0$, then at any period *i* of the I periods model, it is optimal to have a positive production quantity z_i^* or positive entering inventory x_i^* but not both; that is $z_i^* x_i^* = 0$.
2. The amount produced z_i at any period *i* is optimal only if it is zero or if it satisfies the exact demand of one or more succeeding periods. ($z_i = 0$, d_i, $d_i + d_{i+1}$, $d_i + d_{i+1} + d_{i+2}$ e.t.c). These succeeding periods are such that

if the demand in period $i + m$ (< I) is satisfied by z_i* then the demands of period i, i+1, i+2,....i+m-1, must also be satisfied.

The solution procedure begins by finding the optimal policy for the first stage. The optimal policy for the first stage prescribes the optimal policy decision for each of the possible states at that stage. There was a recursive relationship that identifies the optimal policy for period i, given the optimal policy for period $i+1$ is available. This recursive relationship is;

$$G_i(x_{j,i+1}) = \text{Minimize } ((c_{ij}z_{ij} + k_{ij}) + h_{ij}x_{j,i+1}) \qquad (6)$$

Therefore, finding the optimal policy decision at period i require finding the minimizing value of x_i and the corresponding minimum cost is achieved by using this value of x_i and then following the optimal policy when you start at period i + 1. The precise form of the recursive relationship differs somewhat among dynamic programming problems.

The recursive relationship keeps recurring as we move from period to period. When the current period i is increased by 1, the new function is derived by using the $G_{i+1}(x_{j,i+1})$ function that was just derived during the preceding iteration, and then this process keeps repeating, until it finds the optimal policy starting at the final period. This optimal policy immediately yields an optimal solution for the entire problem.

2.5 Data Collection and Analysis

Data for the period of the year under consideration were collected and analyzed for easy model application. Aggregate demand for four (4) quarters of the year and the setup cost corresponding to each quarter were shown in Fig 1. and Fig 2. respectively.

Likewise, data on unit production cost (which is made up of labour cost, machine cost and cost of raw materials), inventory holding cost as obtained from the company under study is presented in Fig 3.

2.6 Model Application

The production and inventory model as stated in equation (7) can thus be expressed as;

$$\begin{aligned}\text{Minimize } (G) = {} & ((c_{11}z_{11} + k_{11}) + h_{11}x_{21}) + ((c_{21}z_{21} + k_{21}) + h_{12}x_{31}) + ((c_{31}z_{31} + k_{31}) + h_{13}x_{41}) \\ & + ((c_{41}z_{41} + k_{41}) + h_{14}x_{51}) + ((c_{12}z_{12} + k_{12}) + h_{21}x_{22}) + ((c_{22}z_{22} + k_{22}) + h_{22}x_{32}) \\ & + ((c_{32}z_{32} + k_{32}) + h_{23}x_{42}) + ((c_{42}z_{42} + k_{42}) + h_{24}x_{52}) \qquad (7)\end{aligned}$$

Non negativity constraint:

$z_{11}, z_{21}, z_{31}, z_{41}, z_{12}, z_{22}, z_{32}, z_{42}, x_{21}, x_{31}, x_{41}, x_{51}, x_{22}, x_{32}, x_{42}, x_{52} \geq 0$

3. Discussion of Results.

The model was solved and the results showing quantity of each product to produce, and inventory to be carried from one period to the other is presented in Fig 4 and 5.

From the result it is required that in the first quarter, only 1,112,750Kilograms of Layers feed and 325,500Kilograms of Growers feed should be produced to meet the demand for period 1 leaving no inventory. A total production cost of ₦ 44,359,250 will be incurred.

For the 2nd quarter 2,878,755Kilograms of Layers feed should be produced to meet the demand of 1,478,500Kilograms, leaving 1,400,255Kilograms of Layers feed in inventory. The total production cost for layers feed in this quarter is ₦ 95,310,415, which is ₦ 46,207,915 more than the cost of producing to meet up the demand for 2nd quarter only.

Also, 728,750Kilograms of Growers feed should be produced to meet the demand of 371,750Kilograms, leaving 357,000Kilograms of Growers feed in inventory. The total production cost for growers feed in 2nd quarter is ₦ 21,445,250, which is also ₦ 10,353,000 more than the cost of producing to meet the demand for 3rd quarter only.

For the 3rd quarter no production should take place, and the inventory of 1,400,255Kilograms of layers feed and 357,000Kilograms of growers feed, carried over from period 2 will satisfy the demands in period 3. The reason for this is because prices of the major raw materials go up most time around this period and this is justifiable because the storage cost incurred here is not to be compared with the cost of producing in this period.

The total cost of producing layer feeds in 2nd quarter to meet the demand of 3rd quarter and the inventory holding cost from 2nd quarter to 3rd quarter is ₦ 59,122,210, which is ₦ 4,152,765 lesser than the cost of producing layers feed in the 3rd quarter.

While the total cost of producing growers feed in the 2nd quarter to meet the demand of 3rd quarter and the inventory holding cost from 2nd quarter to 3rd quarter is ₦ 13,877,500, which is ₦ 1,380,000 lesser than the cost of producing growers feed in the 3rd quarter.

In the 4th quarter, 1,004,000Kilograms of layers feed should be produced to exactly meet the demand leaving no inventory. While 327,000Kilograms of growers feed should be produced to exactly meet the demand leaving no inventory. And the total production cost of this plan is ₦ 48,774,000.

Finally, the advantage of this production plan, as exhibited by this study is that it reduces cost. From ₦231,859,975 which is the total cost of production in previous year in which there was production in every quarter; to ₦ 225,704,210, which is ₦ 6,155,765 lesser.

4. Conclusion

Production planning problem as it relates to an animal feed producing company was observed and tackled, using a dynamic programming approach to make production and inventory level decisions, the objective is to minimize the total cost of production and the annual inventory cost, at the same time meets the customer's demand.

Wagner and whitin inventory model was used, stipulating the minimum quantities of the product to produce per quarter and the corresponding inventory levels such that total production cost is minimized over the planning period. The model considered the four quarters planning horizon from June 2011 to May 2012 and the corresponding production forecast data was used.

The results show that a total minimum cost will be achieved with production in period 1, 2, and 4. While demand for period 3 are satisfied with inventory from period 2. The total cost of this plan is ₦225, 704,210.00 for the two products, which is ₦6, 155, 765.00 less than the existing plan.

From this study, the following recommendations can be drawn:

i. Operations research or management science techniques as used in this study are very useful in providing mathematically feasible solution to the problem of production planning. However, the management must still play a major role of reconciling the scientific solution with the environmental conditions and other intangible effects to arrive at wise decisions.
ii. It is advisable that this type of model is adopted when dealing with making decisions on production and inventory levels for varying period. It helps, without exhaustive enumeration to determine the minimum quantities of product to produce to meet demand at the same time not incurring excessive storage cost by way of inventory in an attempt to meet all demand.
iii. This project as an attempt to improve on the use of Wagner and Whitin model to solve more than one product problem can be improved upon in the nearest future to solve more than two products problem through the proper exploitation of the Microsoft excel solver or other applicable software.
iv. The only limitation of this solution approach is known as "curse of dimensionality", a situation in which the number of required calculations tends to "blow up" rapidly when additional state variables are introduced.

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Figures

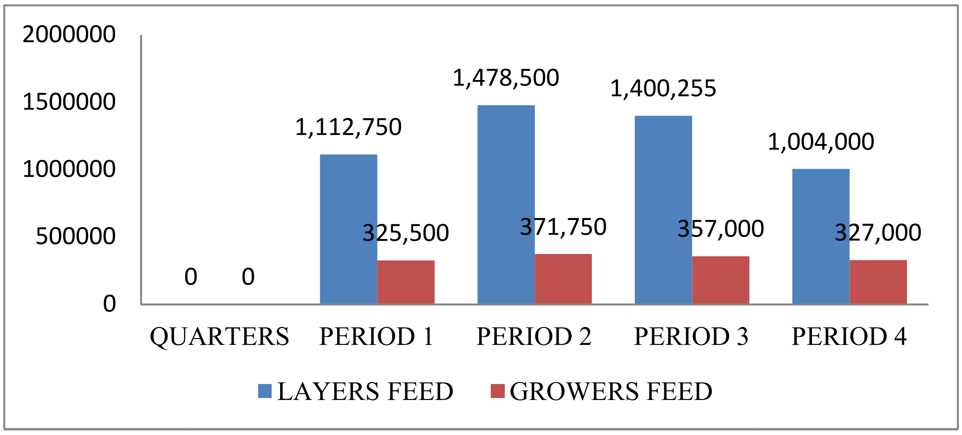

Fig 1: Demand for Layers and Growers Feed (In Kilogram).

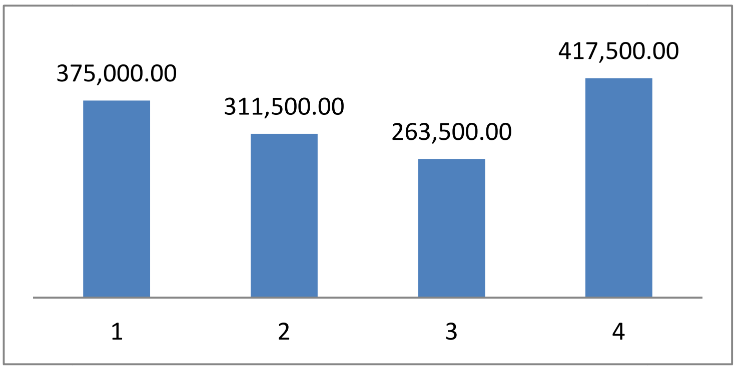

Fig 2: Total Setup Cost per Quarter (In Nigerian Naira).

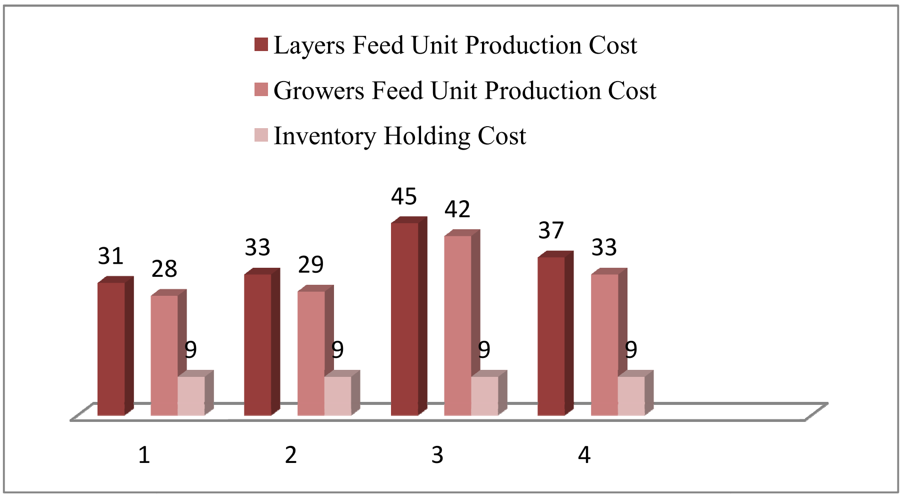

Fig 3: Production and Inventory Holding Cost (All costs are in Nigerian Naira).

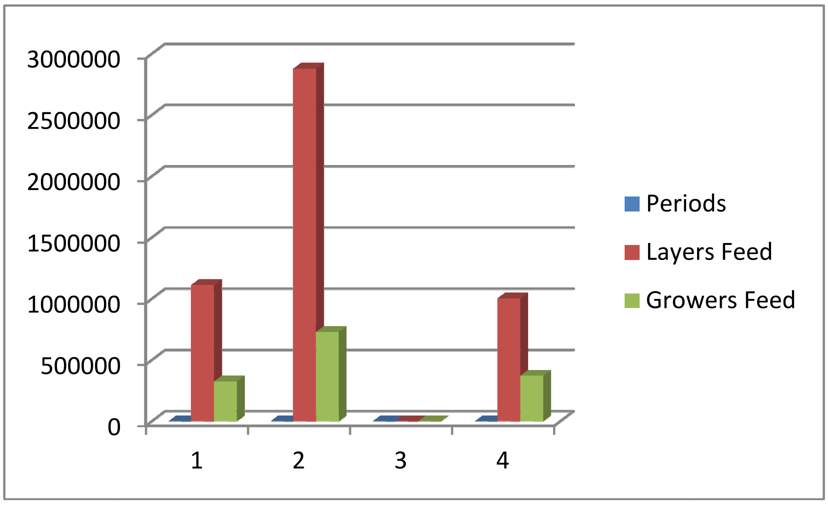

Fig 4: Quarterly Production Plan in Kilograms.

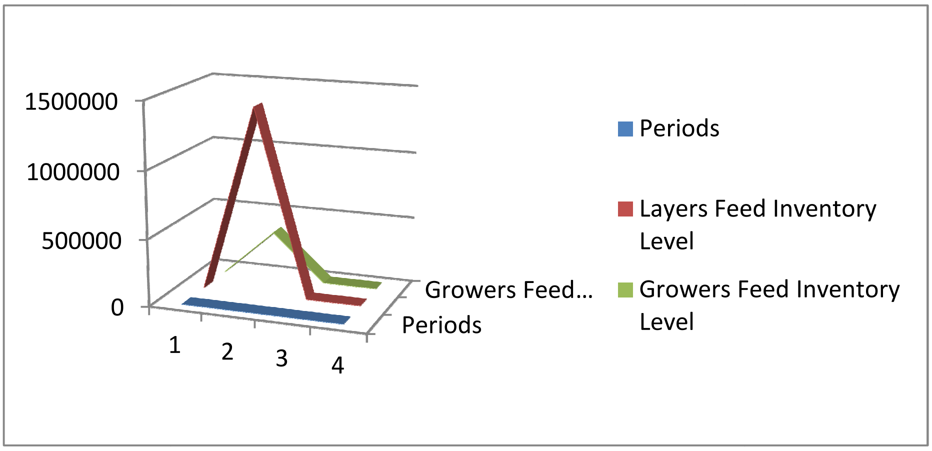

Fig 5: Inventory Levels in Kilograms.

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APPENDIX A: ITERATIONS TO THE SOLUTION PROCEDURE OF PRODUCT 1

LAYERS FEED

NUMBER OF PERIOD N = 4

CURRENT PERIOD =		**1**	**D_{11} =**	**1,112,750**		**Current optimum**	
PERIOD	**Z_{11}=**	1,112,750	2,591,250	3,991,505	4,995,505	**Period 1**	
F0	**$C_{11}Z_{11}$=**	34,870,250	80,703,750	124,111,655	155,235,655	**G1(₦)**	**Z_{11}**
$h_{11}X_{21}$	0	34,870,250				34,870,250	1,112,750
$h_{11}X_{21}$	13,306,500		94,010,250			94,010,250	2,591,250
$h_{11}X_{21}$	25,908,795			150,020,450		150,020,450	39,91,505
$h_{11}X_{21}$	34,944,795				190,180,450	190,180,450	4,995,505

CURRENT PERIOD =		**2**	**D_{21}=**	**1,478,500**		**Current optimum**	
PERIOD	**Z_{21}=**	0	1478500	2,878,755	3,882,755	**Period 2**	
F1	**$C_{21}Z_{21}$=**	0	49,102,000	95,310,415	128,442,415	**G2(₦)**	**Z_{21}**
$h_{21}X_{31}$	0	94,010,250	83,972,250			83,972,250	0
$h_{21}X_{31}$	12,602,295	162,622,745		142,782,960		142,782,960	2,878,755
$h_{21}X_{31}$	21,638,295	211,818,745			184,950,960	184,950,960	3,882,755

CURRENT PERIOD =		**3**	**D_{31}=**	**1,004,255**		**Current optimum**	
PERIOD	**Z_{31}=**	0	1,400,255	2,404,255		**Period 3**	
F2	**$C_{31}Z_{31}$=**	0	63,274,975	108,454,975		**G3(₦)**	**Z_{31}**
$h_{31}X_{41}$	0	142,782,960	147,247,225			142,782,960	0
$h_{31}X_{41}$	9,036,000	193,986,960		201,463,225		193,986,960	2,404,255

CURRENT PERIOD =		**4**	**D_{41}=**	**1,476,000**		**Current optimum**	
PERIOD	**Z_{41}=**	0	1,004,000			**Period 4**	
F3	**$C_{41}Z_{41}$=**	0	37,565,500			**G4(₦)**	**Z_{41}**
$h_{41}X_{51}$	0	193,986,960	180,348,460			180,348,460	1,004,000

Optimum policy is:

X_{51} =	**0**	**Z_{41}=**	**1,004,000**
X_{41} =	**0**	**Z_{31} =**	**0**
X_{31} =	**1,400,255**	**Z_{21} =**	**1,478,500**
X_{21} =	**0**	**Z_{11} =**	**1,112,750**

This gives:

Z_{11}* = 1,112,750 **Z_{21}* = 2,878,755** **Z_{31}* = 0** **Z_{41}* = 1,004,000**

At a total cost of ₦180, 348,460.

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APPENDIX B: ITERATIONS TO THE SOLUTION PROCEDURE OF PRODUCT 2

GROWERS FEED

NUMBER OF PERIOD N = 4

CURRENT PERIOD =		**1**	**D_{12} =**	**1,112,750**		**Current optimum**	
PERIOD	**Z_{12}=**	325,500	697,250	1,054,250	1,381,250	**Period 1**	
F0	**$C_{12}Z_{12}$=**	9,489,000	19,898,000	29,894,000	39,050,000	**G1(₦)**	**Z_{12}**
$h_{12}X_{22}$	0	9,489,000				9,489,000	325,500
$h_{12}X_{22}$	3,345,750		23,243,750			23,243,750	697,250
$h_{12}X_{22}$	6,558,750			36,452,750		36,452,750	1,054,250
$h_{12}X_{22}$	9,501,750				48,551,750	48,551,750	1,381,250

CURRENT PERIOD =		**2**	**D_{22}=**	**1,478,500**		**Current optimum**	
PERIOD	**Z_{22}=**	0	371,750	728,750	1,055,750	**Period 2**	
F1	**$C_{22}Z_{22}$=**	0	11,092,250	21,445,250	30,928,250	**G2(₦)**	**Z_{22}**
$h_{22}X_{32}$	0	23,243,750	20,581,250			20,581,250	0
$h_{22}X_{32}$	3,213,000	39,665,750		34,147,250		34,147,250	728,750
$h_{22}X_{32}$	6,156,000	54,707,750			46,573,250	46,573,250	1,055,750

CURRENT PERIOD =		**3**	**D_{32}=**	**1,004,255**		**Current optimum**	
PERIOD	**Z_{32}=**	0	357,000	684,000		**Period 3**	
F2	**$C_{32}Z_{32}$=**	0	15,257,500	28,991,500		**G3(₦)**	**Z_{32}**
$h_{32}X_{42}$	0	34,147,250	35,838,750			34,147,250	0
$h_{32}X_{42}$	2,943,000	49,516,250		52,515,750		49,516,250	684,000

CURRENT PERIOD =		**4**	**D_{42}=**	**1,476,000**		**Current optimum**	
PERIOD	**Z_{42}=**	0	327,000			**Period 4**	
F3	**$C_{42}Z_{42}$=**	0	11,208,500			**G4 (₦)**	**Z_{42}**
$h_{42}X_{52}$	0	49,516,250	45,355,750			45,355,750	327,000

Optimum policy is:

X_{52} =	**0**	**Z_{42}=**	**327,000**
X_{42} =	**0**	**Z_{32} =**	**0**
X_{32} =	**357,000**	**Z_{22} =**	**371,750**
X_{22} =	**0**	**Z_{12} =**	**325,500**

This gives:

Z_{12}* = 325,500 **Z_{22}* = 728,750** **Z_{32}* = 0** **Z_{42}* = 327,000**

At a total cost of ₦45, 355,750