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Modeling and Trajectory Tracking Control of Robot Manipulators for Laser Cutting Industrial Application.

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Abstract:

Robot manipulators are electromechanical devices whose functionality is similar to that of the arms of human beings. The problem with robot manipulators is that they are open loop unstable systems so it is important to stabilize the response of the robot manipulator. In this research project the robot manipulator is used for an industrial application of Laser Cutting. For this purpose trajectory tracking is important. The user will define any desired trajectory and the robot manipulator's end effector is supposed to track the desired generated trajectory. For this purpose a PID controller is used which is tuned using iterative method. In this research work, simulations are carried out on a 2R robot manipulator with end griper using Simulink / Matlab Software. The end gripper is supposed to hold the laser that is meant to cut an object in a shape defined by user. In this paper, kinematic & dynamic models of robot manipulator, with and without uncertainties, will be presented and a PID controller will be designed to track the desired trajectory of the end effectorfor laser cutting applications
Keywords: Robot manipulator, Kinematics, trajectory tracking, PID controller, Joint Space, Joint variables, Work Space, End effector.

1. Introduction and Problem Statement

Different control laws have been designed by taking into account system model and parameters. For this purpose a correct and detailed model of robot manipulator is very important [1,2]. In literature, e.g., [3,4] one can find the modeling of 2R robotic manipulator with a supposition that whole of the mass of the manipulator's links are concentrated at the respective joints. But practically the mass of the robot manipulator's links is distributed along the arms that can be modeled with different functions. This has been discussed in [5]. In this research work, a new consideration is presented where joint masses are considered along with any kind of mass distribution along the links of a robot manipulator. Different control mechanisms like robust control, adaptive control have been discussed in e.g., [1,6,7,8] but in this paper a PID controller has been used for the trajectory tracking of the robot manipulator.

2. Mathematical Model of the System

Robot manipulators are concerned with two different domains or spaces. One is the Joint Space which gives information of orientation of each joint of a robot manipulator and the second one is the Cartesian Space which gives the coordinates of the end effector of a robot manipulator. Controllers are designed using joint variables that are basically the angles at which the actuators (DC motors) should be rotated so that the robot manipulator's end effector can track the desired trajectory. But in practice for a robot manipulator to track a certain trajectory, the trajectory is not in joint space but is in Cartesian space. So a transformation model is required that can map the Cartesian variables into joint space variables and vice versa. This is done by the kinematic model of a robot manipulator.

A 2R manipulator consists of two links with link lengths l_1 and l_2 and link masses m_1 and m_2, "q_1" and "q_2" are the joint angles. The manipulator is shown in the Figure 1.

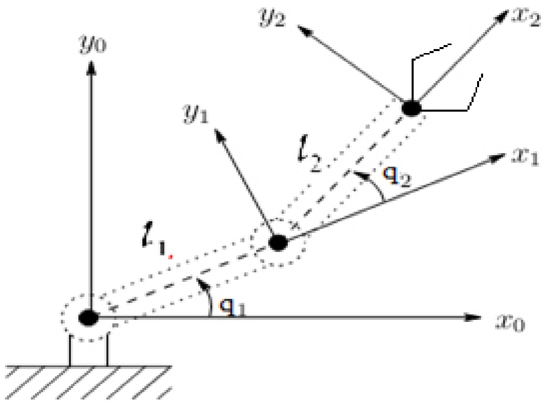

Figure 1 A 2R Planner Robot Manipulator

The coordinates (x, y) of the end effector in terms of the joint angles q_1 and q_2 can be expressed as,

$x = l_1 c_1 + l_2 c_{12}$ (1)

$y = l_1 s_1 + l_2 s_{12}$ (2)

Where

$c_1 = \cos(q_1)$

$c_{12} = \cos(q_1 + q_2)$

$s_1 = \sin(q_1)$

$s_{12} = \sin(q_1 + q_2)$

In matrix notation the kinematic equation can also be written as

$$X = \begin{bmatrix} l1c1 + l2c12 \\ l1s1 + l2s12 \end{bmatrix} \quad (3)$$

The instantaneous motion of the position vector (x, y) of the end effector is

$\delta x = -(l_1 s_1 + l_2 s_{12})\,\delta\Theta_1 - l_2 s_{12}\,\delta\Theta_2$ (4)

$\delta y = (l_1 c_1 + l_2 c_{12})\,\delta\Theta_1 + l_2 c_{12}\,\delta\Theta_2$ (5)

Group the coefficients of δx&δy, we obtain a matrix equation which can be written as

$$\delta x = \begin{bmatrix} \delta x \\ \delta y \end{bmatrix} = \begin{bmatrix} -y & -l2s12 \\ x & l2c12 \end{bmatrix} \begin{pmatrix} \delta\Theta 1 \\ \delta\Theta 2 \end{pmatrix} \quad (6)$$

or

$\delta x = J(q)\,\delta\Theta$

or normally we use q instead of Θ, so the above equation can also be written as

$\delta x = J(q)\,\delta q$ (7)

As it is clear from the above equation that J matrix is a function of the vector q = (q_1, q_2) so

$$J = \begin{pmatrix} \frac{\partial x}{\partial \theta 1} & \frac{\partial x}{\partial \theta 2} \\ \frac{\partial y}{\partial \theta 1} & \frac{\partial y}{\partial \theta 2} \end{pmatrix} \quad (8)$$

Now we could obtain relationship between $x^{\cdot}$ and q', we differentiate equation (7) w.r.t time,

$$\dot{x} = J(q)\dot{q} \quad (9)$$

Equation (9) relates the Cartesian velocity of the end effector with the angular velocities of the joints of the robot manipulators.

Similarly, if we are to find q_1 and q_2 for given coordinates (x,y) of end effector i.e inverse problem is to dealt with. By the inverse kinematic model, the joint angles can be found by the given formulas

$$\cos(q2) = (x^2+y^2-l_1^2-l_2^2)/(2l_1l_2) = D \quad (10)$$

Or

$$q_2 = \cos^{-1}(D) \quad (11)$$

In a similar fashion, $q_1$is given by,

$$q_1 = \tan -1\left(\frac{y}{x}\right) - \tan - 1\left(\frac{l2\,\sin q2}{l1+l2\cos q2}\right) \quad (12)$$

From equation(12), it is clear that q_1 depends on q_2, that is if end effector has to move by 45 degrees and q_2 has rotated by 25 degrees, then remaining 20 degrees rotations will be made by q_1 but if q_2 has rotated by 20 degrees then q_1 will have to move by 25. Hence, q_1 value is dependent on q_2.

Dynamic Model

The mathematical dynamics model of a generalized n-links planner robotic manipulator is represented by the following non-linear differential equation.

$$F_c = M(q)\ddot{q} + V(q, \dot{q})\dot{q} + N(q, \dot{q}) \quad (13)$$

where

$$N(q, \dot{q}) = G(q) + F_d\dot{q} + F_s(\dot{q}) \quad (14)$$

Where M(q) is an $n \times n$ matrix known as inertial matrix, , F_d is an identity matrix of size nxn that contains dynamic friction coefficients, G(q) is a vector of size nx1 and contains the gravity terms while q(t) is a vector of size nx1 and contains the joint angles of all the joint, Fc is a vector of size nx1 and contains control inputs that is torques, generalized forces, V(q,$\dot{q}$) is also an nxn matrix and contains the centrifugal and coriolis terms , and $F_s(\dot{q})$ is a vector of size nx1 that is known as Nixon static friction vector.

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The robot in this research is a two link planner robot, so the above dynamic model shown in equation (13) and (14) is implemented for the two link planner manipulator. As shown in the Figure 2, m_1 and m_2 are the joint masses of joint1 and joint2, respectively. While, m_{1r} and m_{2r} represent the masses that are distributed along the arm1 and arm2, respectively. Similarly, f_1 and f_2 represent the control torques and in vector form they are represented as $Fc = [f_1\ f_2]^T$. And notations for link lengths and joint angles have been mentioned earlier.

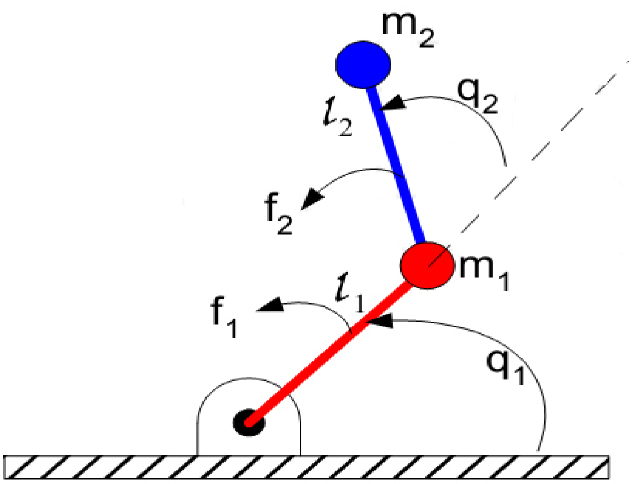

Figure2 Dynamics of a 2R planner robot manipulator

As 2R planner robot has two arms hence M(q) will be a 2x2 matrix, $V(q,\dot{q})$ will also be a 2x2 matrix, G(q) will be a 2x1 vector, Fc(q) and $Fs(\dot{q})$ both will be 2x1 vectors. Thus the matrices and vectors are given by

$$M(q) = \begin{bmatrix} M11 & M12 \\ M21 & M22 \end{bmatrix} \quad (15)$$

Where

$$M_{11} = (m_1 + \xi_1 m_{r1} + m_2 + m_{r2})l_1^2 + (m_2 + \xi_2 m_{r2})l_2^2 + 2(m_2 + \zeta_2 m_{r2})l_1 l_2 \cos q_2$$

$$M_{12} = M_{21} = (m_2 + \xi_2\, m_{r2})l_2^2 + (m_2 + \zeta_2\, m_{r2})l_1 l_2 \cos q_2$$

$$M_{22} = (m_2 + \xi_2\, m_{r2})l_2^2$$

$$V(q,\dot{q}) = -(m_2 + \zeta_2\, m_{r2})l_1 l_2 \dot{q}_2 \sin q_2 \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \quad (16)$$

$$G(q) = g\begin{bmatrix} (m1 + \zeta 1\, mr1 + m2 + mr2)l1\cos q1 + \\ (m2 + \zeta 2\, mr2)l2\cos(q1 + q2) \\ (m2 + \zeta 2\, mr2)l2\cos(q1 + q2) \end{bmatrix} (17)$$

Where g is the gravitational acceleration, m_{r1} and m_{r2} are the total masses of joint1 and joint 2, respectively. ξ_1, ξ_2, ζ_1 and ζ_2 are scaling coefficients and are defined by

$$\xi_1 = \int_0^{li} \rho i(l) Si(l) l^2 dl / mri\, l_i^2 \quad (18)$$

$$\zeta_1 = \int_0^{li} \rho i(s) Si(l) l\ \ dl / mri\, l_i \quad (19)$$

$$m_{ri} = \int_0^{li} \rho i(l) Si(l) dl,\ \ for i = 1,2 \quad (20)$$

Where $\rho_1(l)$ and $\rho_2(l)$ are the arms mass density functions along their length l, $S_1(l)$and $S_2(l)$ are the arm cross sectional area functions along the length l.

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Dynamic Model of 2R Planner Manipulator (joint masses only)

For simplicity, assume that there exist only joint masses, so equations (15-20) of the dynamic model will change by using
$m_{ri}= 0,\ \xi_i= 0,\ \zeta_i= 0$ for i = 1, 2 (21)

So the inertial matrix M (q) will become

$M_{11} = (m1 + m2)l_1^2+m_2l_2^2+2m_2l_1l_2\cos(q_2)$
$M_{12} = M_{21} = m_2 (l_2^2 + l_1l_2\cos(q_2))$
$M_{22} = m_2l_2^2$
Also from [4] , we find an interesting result that is

$$\dot{M}(q) = V(q, \dot{q})$$

In a similar fashion equations of $V(q, \dot{q})$ and G(q) are given by

$$V(q,\dot{q}) = -m_2l_1l_2\dot{q}2\sin q_2\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}(22)$$

$$G(q)= g\begin{bmatrix} (m1 + m2)l1\cos q1 + m2l2\cos(q1 + q2) \\ m2l2\cos(q1 + q2) \end{bmatrix}$$

Using above equations, the centrifugal and coriolis matrix can also be written equivalently

$$V(q,\dot{q}) = m_2l_1l_2\sin q_2\begin{bmatrix} -\dot{q}2 & -\dot{q}1 - \dot{q}2 \\ -\dot{q}2 & 0 \end{bmatrix}(23)$$

Dynamic Model of 2R Planner Manipulator (Disturbances and Uncertainties)

The dynamic model represented by equation (13) and equation (14) does not take into account the possible uncertainties**.** If possible disturbances and uncertainties are considered, in the new model, each term of the above mentioned equations can be decomposed into two parts, that is one known part and another one will be unknown perturbed part as shown in the equation.
$M = M_o + \Delta M,\ N = N_o + \Delta N$ and $V = V_o + \Delta V$(24)
Where M_o, N_o and V_o model the known parts and ΔM, ΔN and ΔN model the unknown parts. With these changes, the dynamic model can be used not only for total uncertain robot systems with uncertain parameters, but also for a known part with their nominal parameters of the system.

3. Proposed Control Mechanism

A trajectory tracking controller is designed for a 2R planner manipulator with uncertainties so the modified dynamic model considers disturbance and uncertainties. For such a model, the M, V and N matrices change according to the following equation
$M = M_o + \Delta M,\ N = N_o + \Delta N$ and $V = V_o + \Delta V$
With these changes, now the dynamic model can be used not only for total uncertain robot systems with uncertain parameters, but also for a known part with their nominal parameters.
The controlled input is given by
$Fc = Mo(q)\ddot{q}d + Vo(q,\dot{q})\dot{q} + No(q,\dot{q}) - Mo(q)u$ (25)
$e = q_d - q$ (26)
From the above equations (13, 25,26), we get

$\ddot{e} = M^{-1}(q)[\Delta M(q)\ddot{q}+\Delta V(q,\dot{q})\dot{q}+\Delta N(q,\dot{q})+Mo(q)u]$
or $= w+E\dot{e}+Fu+u$ (27)
where
$E = -M^{-1}(q)\Delta V(q,\dot{q})$,
$F = -M^{-1}(q)\Delta M(q)$ and
$w = -F\ddot{q}d-E\dot{q}d+M^{-1}\Delta N$
The norms of w, E and F are bounded i.e
$||E||<\sigma_e, ||w||<\sigma_w ||F||<\sigma_f$ (28)
Hence the state space representation of the system will be
$\dot{x} = Ax + Bu + B[0\ E]x + BFu + Bw$ (29)

The uncertainties in the system is modeled by the last three terms of the equation (29)
Hence,
$x = [e\ \dot{e}]^T = [e_1\ e_2\ \dot{e}_1 \dot{e}_2]^T, A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$

The desired trajectory qd, the velocity $\dot{q}d$ and the acceleration $\ddot{q}d$ can be generated by the model of the type
$\ddot{q}d(t) + Kv\dot{q}d(t) + Kpq_d(t) = r(t)$ (30)

4. Simulation Results

The performance of designed optimal controller for the 2R robot manipulator is evaluated in the simulations. The stability of the system and controller implemented for industrial applications that is laser cutter application for a periodic changing pattern is implemented in Simulink/Matlab.The whole mechanism is described in the block diagram given in Figure 3 & this block diagram is implemented in Simulink.

In this model the Robot block implements the above mentioned 2R robot manipulator's model, the trajectory generator block generated the desired trajectory, the controller (Control System) is a PID controller what generates the control input that the torques for joint 1 and joint 2. The value of the controller output (that is the torqued exerted on each joint) depends upon the error signal at the input of the controller that is the difference of the desired trajectory and the actual trajectory tracked by the end effector of the robot manipulator.
The numerical simulation parameters of the 2R robot manipulator are shown in the Table 1.

Table 1

Parameter	Symbolic Notation	Value
Length of Link 1	*l*1	2m
Length of Link 2	*l*2	1m
Mass of Link 1	m_1	10kg
Mass of Link 2	m_2	10kg
Inertia of Link 1	*I*1	13.33
Inertia of Link 2	*I*2	3.33
Distance of Centre of Arm from Joint 1	*l*c1	1m
Distance of Centre of Arm from Joint 2	*l*c2	0.5m
Gravitational Acceleration	*G*	9.8 m/s

The general block diagram of the whole system, which is implemented above in Simulink, is given in Figure 3. The 2R robotic manipulator system is a multi-input multi-output system with two inputs that is desired joint position and desired joint velocity and two outputs that is actual joint position and actual joint velocity.

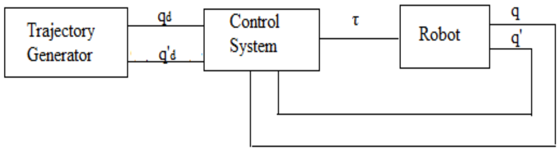

Figure 3 Block Diagram of the Close Loop 2R Manipulator

This block diagram has been implemented in Simulink as shown in Figure 3.

The desired trajectory of joint 1 is represented by q_{1d} and that of joint 2 is represented by q_{2d}. When the model is simulated, the actual trajectories of the joints are represented by q_1 and q_2. Similarly, the trajectory generator also generates the desired angular velocities of joint 1 and 2 and is represented by $\dot{q}1d$ and $\dot{q}2d$, respectively. When the model is simulated, the actual angular velocities of joint 1 and 2 are represented by $\dot{q}1$ and $\dot{q}2$, respectively. The simulation results are plotted in Figure 4, Figure 5, Figure 6 and Figure 7.

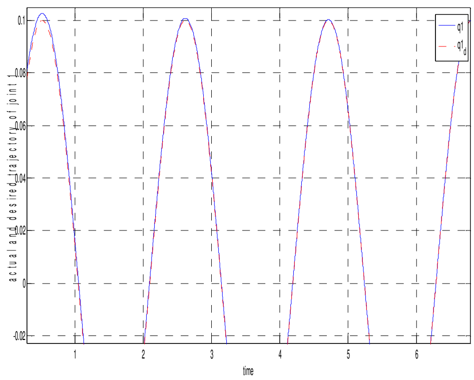

Figure 4 Simulation Results of Desired and Actual Trajectory of Joint 1

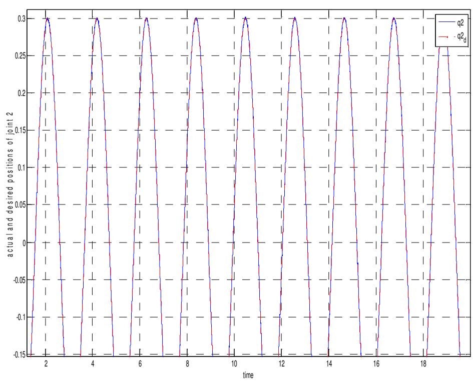

Figure 5 Simulation Results of Desired and Actual Trajectory of Joint 2

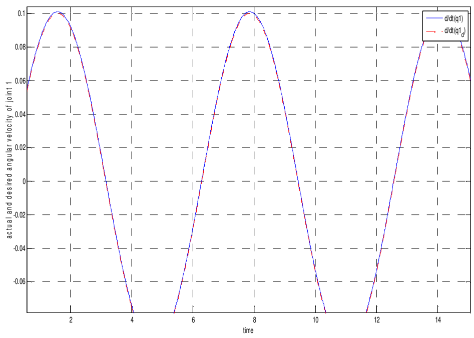

Figure 6 Simulation Results of Desired and Actual Velocities of Joint 1

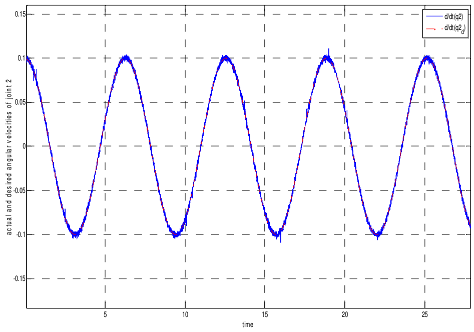

Figure 7 Simulation Results of Desired and Actual Velocities of Joint 2

5. Conclusion

A 2R robot manipulator was taken for the simulation of trajectory tracking of robot manipulators for industrial application of Laser Cutting. The mathematical model of the robot manipulator was presented while taking into account the disturbances & uncertainties. The model was implemented in Simulink. The user defined a desired trajectory. A PID controller was tuned using iterative method to track the desired trajectory of the end effector or gripper of the robot manipulator. Both the desired trajectory (In this case the sinusoids) defined by the user and the actual trajectory followed by the gripper of the robot manipulator were plotted in the Figures 4, 5, 6 and 7.

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