

Parameters of Grading Systems

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Abstract

Since the advent of *course system* of education in tertiary institutions world-over, Cumulative Point Average (CGPA) has been in use as an Assessment Instrument instead of Cumulative Weighted Average Mark (CWAM). Consequently, mapping of percentage marks into an n-grade points system which is required to generate the much needed CGPA has become necessary. Countless methods of mapping have been witnessed across different tertiary institutions. In addition, the number of grade point, 'n' varies from institutions to institutions. While it is a universal fact that 'n' can take any value less than 100, it is nevertheless important to know that the value of 'n' has never been greater than 12. In Nigerian tertiary institutions, the value of 'n' varies between 4 and 7 while 5 is the most common. However, simply equating 'n' to 100 is not sufficient to convert the percentile system to an n-grade points system in order to generate the required CGPA. It is discovered that there is no sound mathematical method employed to relate the CGPA ranges normally used to classify degree to the percentage scores earned by individual students. This paper (paper 2) is primarily written to establish the required parameters and the most suitable format of an n-grade points system which is referred to as a Non-Graded Fail Grading System with a fail grade, 'F' assigned a zero value ($NGF/GS_{F=0}$) while another paper (paper 3) presents the development of the required mathematical relationship between CGPA and percentage scores ranges.

1. Introduction

There are many opinions about grading systems. As a matter of facts, there are as many as there are users of grading systems. Every training institution that is required to assess its trainees has its own format of grading system since a grading system is a platform for the application of Assessment Instruments. There are also many different Assessment Instruments that are also used by different training institutions. All these grading systems do not address the same objectives and purposes. Because of these different shades of opinions and freedom to use whichever is considered suitable for a given situation, much study has not been done on the subject. It is discovered that people copy one format or the other without knowing fully the original purpose for which what is copied is intended. This consequently leads to many assumptions, one of which is to think that there is nothing to teach anybody about grading systems. The study carried out on grading system is presented in four different papers, namely, Paper 1: Assessment of Student Performance: Grading Systems, ***Paper 2: Parameters of Grading Systems***, Paper 3: Mathematical Relationship Between Percentile & Grade Point Numbers, Paper 4: Evaluation of Grading Systems of Some Tertiary Institutions in Nigeria. The Assessment Instrument considered in this paper is the Cumulative Grade Point Average, CGPA which is the one adopted in most tertiary institutions around the world because of its unique features. Therefore, the grading system suitable for such an instrument is the subject of this paper. There are still many types of this kind depending on such factors as objectives of assessment, understanding of the CGPA and other demands for graduates being assessed. However, there are basically two types of Grading Systems being considered for CGPA. These are **Non-Graded Fail** (where only one class is allowed in a Failure Zone) and **Graded Fail** (where there are more than one class in a Fail Zone).

2. Non-Graded Fail (F) Type of Grading System

The Non-Graded Fail Grading System (NGF/GS) is expressed by equation (2.1) as follows:

$$[NGF/GS]_{F=a} = \{[(m - point)a, x], [P(a, d, n)], [(M_H + 1)_x, (M_L)_n] \dots \dots \dots\} \quad (2.1)$$

$P(a, d, n)$ = Arithmetical Progression (AP) with first term = 'a' and a Common Difference of 'd'.

That is, AP \equiv a, [a+d], [a+2d], [a+3d],[a+(r-1)d].

The (r+1)th term = a + rd (2.2)

x = The grade point (n_x) assigned to the **least pass** score range.

- n = The highest grade point assigned to the **highest pass** score range with a letter grade of 'A'.
 a = The first term of Arithmetical Progression assigned to the fail score range with a letter grade of 'F'
 m = The number of degree classification which is expected to be equal to 'n' for a non-complex grading system.
 $(M_H + 1)_x$ = The **least pass** score and the lower limit of the **least pass** score range.
 $(M_L)_n$ = The lower limit of the **highest pass** score range.

The reason for choosing Arithmetical Progression is presented in Table 1.

The following relationships among the defined parameters above are valid and germane for designing a simple grading system that takes into consideration the properties of Cumulative Grade Point Average (CGPA) which is expected to be mapped into it in order to classify students' academic performance:

- $d = 2^{\text{nd}} \text{ term} - 1^{\text{st}} \text{ term} = 3^{\text{rd}} \text{ term} - 2^{\text{nd}} \text{ term} \dots\dots\dots = r^{\text{th}} \text{ term} - (r - 1)^{\text{th}} \text{ term} \dots\dots (2.3)$
- a = First term of AP, value of which is the n_F assigned to the failure grade, (F)/mark/score range (0 to M_H) = (0 to 39) or (0 to 49) or (0 to 59), etc. In order to take full advantage of the Separation Property of CGPA, the value of the n_F assigned MUST be zero.
 That is, $n_F = F = a = 0 \equiv (0 \text{ to } M_H) = (0 \text{ to } 39) \text{ or } (0 \text{ to } 49) \text{ or } (0 \text{ to } 59)$, etc.
a = 0 (2.4)
- x = Second term of AP to which the least of grade point (n_x) other than zero is assigned which corresponds to the least pass grade/mark/score, $(M_H + 1)$. That is, $n_x - n_F = x - a = d$
x - a = d (2.5)
- m = number of degree classification given by
m = $n_0 - a$ (2.6)

n_0 = The highest grade point assigned to the highest score range when the first term of AP is zero, ($F = a = 0$).

Using the established relationship above, two options are considered as follows:

2.1 Option 1: a = 0: Table 3 Through Table 6

Substituting equation (2.4) into (2.5) & (2.6), we have

x = d, m = n_0

$$[NGF/GS]_{F=a} = \{[(n_0 - \text{point})a, d], [P(a, d, n_0)], [(M_H + 1)_d, (M_L)_{n_0}] \dots\dots\dots (2.1.1)$$

or

$$[NGF/GS]_{F=0} = \{[(n_0 - \text{point})0, d], [P(0, d, n_0)], (M_H + 1)_d, (M_L)_{n_0}\} \dots\dots\dots (2.1.2)$$

For $d = 1$,

$$[NGF/GS]_{F=0} = \{[(n_0 - \text{point})0, 1], [P(0, 1, n_0)], (M_H + 1)_d, (M_L)_{n_0}\} \dots\dots\dots (2.1.3)$$

Therefore,

CGPA $\geq x = d = 1$ is in a pass zone.

CGPA $< x = d = 1$ is in a Failure zone.

2.2 Option 2: a = 1

Substituting a = 1 into equations (1.5) & (1.6), we have

x = d + 1,

m = $n_1 = n_0 + 1$ (see Table 2). Where,

n_1 = The highest grade point assigned to the highest score range when the first term of AP is one, ($F = a = 1$).

$$[NGF/GS]_{F=a} = \{[(n_1 - \text{point})a, (1 + d)], [P(a, d, n_1)], [(M_H + 1)_x, (M_L)_{n_1}] \dots\dots\dots (2.2.1)$$

or

$$[NGF/GS]_{F=1} = \{[(n_1 - \text{point})1, (1 + d)], [P(1, d, n_1)], [(M_H + 1)_x, (M_L)_{n_1}] \dots\dots\dots (2.2.2)$$

or

$$[NGF/GS]_{F=0} = \{[(n_0 - \text{point})0, (1 + d)], [P(0, d, n_0)], (M_H + 1)_x, (M_L)_{n_0}\} \dots\dots\dots (2.2.3)$$

If $d = 1$,

$$[NGF/GS]_{F=1} = \{[(n_1 - \text{point})1, 2], [P(1, 2, n_1)], [(M_H + 1)_2, (M_L)_{n_1}] \dots\dots\dots (2.2.4)$$

Therefore,

CGPA $\geq x = 2$ is in a pass zone.
 CGPA $< x = 2$ is in a Failure zone

Comparing equation (2.1.3) with (2.2.4), they are of the same form except that the values of 'x', 'a' and 'n' are shifted up by a unit (1) as shown on the graph in Figure 1.

However, the disadvantage of equation (2.2.4) is the loss of the Separation Property of CGPA that this grading system is supposed to serve. That is, the CGPA of academically weak students are increased, thereby creating the tendency to reduce the gap between the weak and the strong students and the advantage of CGPA as against CPMA (Cumulative Percentage Mark Average) is totally lost. In addition, the highest grade point 'n' is equally increased to (n + 1) for F = a = 1 as demonstrated in Table 2.

Consequent upon the above analysis, equation (2.1.3) is the highly recommended format to be adopted as a Grading System to be used to classify students' performances when a CGPA Assessment Instrument is employed in tertiary institutions to avoid complexity and confusion.

$$[NGF/GS]_{F=0} = \{[(n - \text{point})0, 1], [P(0, 1, n)]\}, [(M_H + 1)_1, (M_L)_n] \dots \dots (2.1.3)$$

From the above equation (2.1.3), the only parameters that can constitute different variations in grading system from one university to another are those in RED. That is, the maximum pass score $(M_L)_n$ to which the highest grade point (n) is assigned and the least pass score $(M_H + 1)_1$ to which the least grade point (1) is assigned.

For examples, most Nigerian universities, especially the public (Federal & State Government universities) have

$$(M_L)_n = 70$$

$$(M_H + 1)_1 = 40$$

Some private universities have

$$(M_L)_n = 80 \text{ or } 90 \text{ or } 95$$

$$(M_H + 1)_1 = 40 \text{ or } 50 \text{ or } 60$$

The variations in the values/choice of $(M_L)_n$ and $(M_H + 1)_1$ must generate different values of CGPA ranges for the degree classifications. It has been found that this is not the case in many private universities. Despite the variations observed in mark/score ranges, the same CGPA ranges for $(M_L)_n = 70$ and $(M_H + 1)_1 = 40$ are still being used in many cases. In addition, the score ranges between $(M_L)_n$ and $(M_H + 1)_1$ are biased in favour of a particular degree class as against chances of equal opportunities to /probabilities of all students in the allocation of the score ranges as demanded by the analysis presented here.

Consequent upon the above scenario amongst other factors beyond the scope of this paper, the degree classes of one university differ from another by a wide margin, even when they appear to be carrying the same CGPA values. THIS IS A SERIOUS ANOMALY WHICH THIS PAPER INTENDS TO CORRECT!

Below are examples of what are the correct relationships between CGPA and Score ranges displayed on Tables 3, 4, 5 & 6. These tables are produced in excel. The figures in WHITE on BLACK background are parameters required to be agreed upon as a policy. Every other figures are calculated by an excel calculator which is so programmed. The results (mark/score ranges and GP ranges) to be used are in BLACK on YELLOW. All other figures are required to adequately calculate the values needed. The theoretical derivation of the required formula is presented in a separate paper (paper 3).

3. Graded Fail ($F_1 = a, F_2 = a+d, F_3 = a+2d\dots\dots$) Type of Grading System

Similarly, the Graded Fail Grading System (GF/GS) is given by equation (3.1) as follows:

$$[GF/GS]_{F=a,(a+d)\dots} = \{[(m - \text{point})a, (a + d), \dots, x], [P(a, d, n)]\}, [(M_H + 1)_x, (M_L)_n] \dots \dots (3.1)$$

The parameters have the same meanings as for NGF/GS except that the failure zone is divided into more than one class (say, $F_1 = a$, $F_2 = a+d$, $F_3 = a+2d$,) to which grading points are assigned in an Arithmetical Progression manner beginning from the least fail range. That is,

1. $a =$ First term of AP, value of which is the n_{F_1} assigned to the failure grade, (F_1)/mark/score range The advantage of the Separation Property of CGPA (Cumulative Grade Point Average) is lost in this type of grading system. Hence, the value of the n_{F_1} assigned to F_1 can be any integer or fraction or combination of integer and fraction as desired. That is, **$n_{F_1} = F_1$ (3.2)**
2. $d =$ Common difference of Arithmetical Progression, AP. That is, **Second term of AP – $a + d$ (3.3)** (see Tables 6 through 9)
3. For a two-class fail zone, 'x' serves as the third term of AP to which grade point (n_x) is assigned which corresponds to the least pass grade/mark/score, ($M_H + 1$). That is, $GP_x - GP_{F_1} = x -$ Second term of AP = d . That is, **$x -$ Second term of AP = d (3.4)**
4. $m =$ number of degree classifications

It is necessary to examine the score ranges within Pass and Fail Zones and the effect of assigning grade points to these ranges according to the principles of Arithmetical Progression in order to adequately establish parameters for Grading System with Graded Fail Zone. The tables below (Tables 7, 8 & 9) are used to illustrate the required principles.

In general, equation (3.5) is derived.

$$m = 1 + \frac{n - n_x}{d} \dots \dots \dots (3.5)$$

Where,

$n =$ The highest GP assigned to the highest score range, (say, 80 to 100).

$n_x =$ The GP assigned to the least pass score range, (say, 50 to 59) = 'x' in equation (2.1)

3.1 Option 1: Common Difference, $d = 1$, $a = 0$: Table 10 Through Table 13

$$[GF/GS]_{F=F_1, F_2, F_3} = \{[(m - point)F_1, F_2, F_3, x], [P(F_1, d, n)], (M_H + 1)_x, (M_L)_n\}$$

$$[GF/GS]_{F=0,1,2} = [(4 - point)0, 1, 2, 3], [P(0, 1, 6), (50, 80)] \dots \dots \dots (3.1.1)$$

$$[GF/GS]_{F=0,1} = [(4 - point)0, 1, 2], [P(0, 1, 5), (50, 80)] \dots \dots \dots (3.1.2)$$

$$[GF/GS]_{F=0} = [(4 - point)0, 1], [P(0, 1, 4), (50, 80)] \dots \dots \dots (3.1.3)$$

$$[NGF/GS]_{F=0} = [(4 - point)0, 1], [P(0, 1, 4), (50, 80)] \dots \dots \dots (3.1.4)$$

Note,

From equation (3.1.1), we have, $m = 1 + (6 - 3)/1 = 4$.

From equation (3.1.2), we have, $m = 1 + (5 - 2)/1 = 4$

From equation (3.1.3), we have, $m = 1 + (4 - 1)/1 = 4$.

Hence, equation (3.5) is still valid here.

Note the closeness of equations (3.1.3) and (3.1.4). The expressions are the same but the calculation is obtained differently. In Equation (3.1.3), the fail class includes the fail class score intervals in the calculation of the corresponding CGPA range while that of equation (3.1.4) is not. Both equations (3.1.3) and (3.1.4) have their classification numbers, 'm' equal to the highest grade points, 'n' assigned to the highest pass score range. Usually no CGPA is assigned to fail score ranges except to say that $CGPA < x$ is not utilized to graduate students that fall into this category. Hence, the most appropriate grading system format is therefore, equation (3.1.4). From equation (3.5), for 'm' to be equal to 'n', $d = n_x = 1$. These are the necessary conditions for a valid grading system. Equation (3.1.4) satisfies these conditions.

The corresponding CGPA ranges are calculated by an Excel Calculator shown in Tables 10 through 13.

3.2 Option 2: Common Difference, d = 1, a = 1: Table 14 Through Table 17

$$[GF/GS]_{F=F_1, F_2, F_3} = \{[(m - \text{point})F_1, F_2, F_3, x], [P(F_1, d, n)], (M_H + 1)_x, (M_L)_n\}$$

$$[GF/GS]_{F=1,2,3} = [(4 - \text{point})1, 2, 3, 4], [P(1, 1, 7), (50, 80)] \dots \dots \dots (3.2.1)$$

$$[GF/GS]_{F=1,2} = [(4 - \text{point})1, 2, 3], [P(1, 1, 6), (50, 80)] \dots \dots \dots (3.2.2)$$

$$[GF/GS]_{F=1} = [(4 - \text{point})1, 2], [P(1, 1, 5), (50, 80)] \dots \dots \dots (3.2.3)$$

$$[NGF/GS]_{F=1} = [(4 - \text{point})1, 2], [P(1, 1, 5), (50, 80)] \dots \dots \dots (3.2.4)$$

Note,

From equation (3.2.1), we have, $m = 1 + (7 - 4)/1 = 4$.

From equation (3.2.2), we have, $m = 1 + (6 - 3)/1 = 4$

From equation (3.2.3), we have, $m = 1 + (5 - 2)/1 = 4$.

Similarly note the closeness of equations (3.2.3) and (3.2.4). The expressions are the same but the calculation is obtained differently for the same reason given above. None of these equations have their classification numbers equal to the highest grade point assigned to the highest pass score range due to the shift in the origin by '1'. Thus, it can be misinterpreted. Though the conditions for validity is met by equation (2.5), but the format is misleading because a (4-point) grading system is expected to have a maximum of 4.00 CGPA and not 5.00 as shown in equations (3.2.3) & (3.2.4).

The corresponding CGPA ranges are calculated by an Excel Calculator shown in Tables 14 through 17.

3.3 Option 3: Common Difference, d = 2, a = 0: Table 18 Through Table 21

$$[GF/GS]_{F=F_1, F_2, F_3} = \{[(m - \text{point})F_1, F_2, F_3, x], [P(F_1, d, n)], (M_H + 1)_x, (M_L)_n\}$$

$$[GF/GS]_{F=0,2,4} = [(8 - \text{point})0, 2, 4, 6], [P(0, 2, 12)], [(50, 80)] \dots \dots \dots (3.3.1)$$

or

$$[GF/GS]_{F=0,2,4} = 2\{[(4 - \text{point})0, 1, 2, 3], [P(0, 1, 6)], [(50, 80)] \dots \dots \dots (3.3.1a)$$

Note that equation (3.3.1a) is the same as equation (3.3.1) when multiplied by a factor of $d = 2$ as shown.

Similarly,

$$[GF/GS]_{F=0,2} = [(8 - \text{point})0, 2, 4], [P(0, 2, 10), (50, 80)] \dots \dots \dots (3.3.2)$$

or

$$[GF/GS]_{F=0,2} = 2\{[(4 - \text{point})0, 1, 2], [P(0, 1, 5)], [(50, 80)] \dots \dots \dots (3.3.2a)$$

and

$$[GF/GS]_{F=0} = [(8 - \text{point})0, 2], [P(0, 2, 8), (50, 80)] \dots \dots \dots (3.3.3)$$

or

$$[GF/GS]_{F=0} = 2\{[(4 - \text{point})0, 1], [P(0, 1, 4)], [(50, 80)] \dots \dots \dots (3.3.3a)$$

and

$$[NGF/GS]_{F=0} = [(8 - \text{point})0, 2], [P(0, 2, 8), (50, 80)] \dots \dots \dots (3.3.4)$$

or

$$[NGF/GS]_{F=0} = 2\{[(4 - \text{point})0, 1], [P(0, 2, 4)], [(50, 80)] \dots \dots \dots (3.3.4a)$$

Note the principle of factorization that is applicable to all grading systems (Graded and Non-Graded Fail) formats. That is, increasing the value of 'd' merely expands the grading system by the same factor when the first fail class is assigned grade point of zero ($a = 0$). This is what is referred to as **Extended Grading System (ExGS)**.

The corresponding CGPA ranges are calculated by an Excel Calculator shown in Tables 18 through 21.

3.4 Option 4: Common Difference, d = 2, a = 1: Table 22 Through Table 25

$$[GF/GS]_{F=F_1, F_2, F_3} = \{[(m - \text{point})F_1, F_2, F_3, x], [P(d, F_1, n)], (M_H + 1)_x, (M_L)_n\}$$

$$[GF/GS]_{F=1,3,5} = [(4 - \text{point})1, 3, 5, 7], [P(1, 2, 13), (50, 80)] \dots \dots \dots (3.4.1)$$

$$[GF/GS]_{F=1,3} = [(4 - \text{point})1, 3, 5], [P(1, 2, 11), (50, 80)] \dots \dots \dots (3.4.2)$$

$$[GF/GS]_{F=1} = [(4 - \text{point})1, 3], [P(1, 2, 9), (50, 80)] \dots \dots \dots (3.4.3)$$

$$[NGF/GS]_{F=1} = [(4 - \text{point})1, 3], [P(1, 2, 9), (50, 80)] \dots \dots \dots (3.4.4)$$

Also note the closeness of equations (3.4.3) and (3.4.4). The expressions are the same but the calculation is obtained differently for the same reason given earlier. For the same reason deduced to reject equations (3.2.3) & (3.3.4), equations (3.4.3) & (3.4.4) are also rejected, not adequate to be used to classify CGPA data.

The corresponding CGPA ranges are calculated by an Excel Calculator shown in Tables 22 through 25.

3.5 Another Type of Graded Fail Grading System

The Graded Fail Grading System (GF/GS) is given by equations (3.5.1) & (3.5.2) as follows:

$$[GF/GS]_{F=a\dots} = \{[(m - \text{point})a, \dots, x], [P(a, d, n)], [(M_H + 1)_x, (M_L)_n] \dots \dots \dots (3.5.1)$$

Equation (3.5.1) is a Grading System where the Pass Zone is graded separately as in Equation (2.1)

$$[GF/GS]_{F=a,-d,-2d\dots} = \{[(m_F - \text{point})a, -d, -2d \dots, x], [P(a, -d, n_F)], [(M_H)_a, (0)_n] \dots \dots \dots (3.5.2)$$

Equation (3.5.2) is like a Grading System where the Fail Zone is graded separately like Equation (2.1)

It is necessary to examine the score ranges within Pass and Fail Zones and the effect of assigning grade points to these ranges according to the principles of Arithmetical Progression in order to adequately establish parameters for Grading System with Graded Fail Zone. Tables 26, 27 & 28 are used to illustrate the required principles.

From Table 26 through Table 28, it is obvious that there are two separate classification which are independent of each other, the Pass and Fail Zone classifications. The Pass Zone classification is precisely the same as the Non-Grade Fail Grading System dealt with in section 2 of this paper. In a similar manner, Fail Zone can be classified using the same set of formula with a negative value of 'd'. However, for what purpose will this classification be done when no award will be given for graduates that failed?

4. General Observations & Deductions

From the analysis presented here and the summary presented in Tables 29 & 30, grading systems of different types (Non-Graded & Graded Fail) can be developed with appropriate parameters chosen according to the policies of the institutions desired to use CGPA as their Assessment Instrument.

OBSERVATIONS FROM TABLES 29 & 30:

1. Graded Fail increases the CGPA of all classes except the first one whether or not the first fail (F₁) class is assigned 0 or 1.
2. By advancing the value of the first fail (F₁) grade by '1', all classes are advanced by the same amount in both graded and non-graded fail format.
3. The classification number, m defers from the highest grade point, n assigned to the highest score range for Graded Fail format.

4. Doubling the Common Difference, d of Arithmetical Progression doubles all the parameters of the grading system in all cases as expected.

For $F = 0$, $2[\text{TABLE 10, } (d = 1)] = \text{TABLE 18 } (d = 2) \dots\dots\dots (1)$

For any of the grading system here developed to be adopted, the following conditions MUST be adhered to:

- Arithmetical Progression MUST be used and no other, if equal chances are to be given to all students whether or not they are academically weak or strong. **That is, the design MUST NOT be biased in favour of the weak or strong.**
- The Common Difference of the Arithmetical Progression must not a unit (1) in order to avoid misinterpretation of the grading system as either an ExGS or EIGS.
- In other to fully take the advantage of the Separation Property of CGPA amongst other features, the following conditions MUST be obeyed:
 - a) A zero grade point must be assigned to all failure grades/scores. That is, $F = a = 0$. Assigning a finite value to failure grades potent a bias in favour of the weak students as discussed and presented in Tables 29 & 30.
 - b) The grading system to be employed MUST be a Non-Graded Fail type since this is the only one type that ensures that all failure grades/scores are assigned a zero grade point. In addition, Graded Fail type alters the distribution/classification of the CGPA upward in favour of lower classes thereby closing the gap between the strong and the weak students. This defeats the essence of using CGPA as an Assessment Instrument which is designed to distinctly separate the weak from the strong without any bias.

4.1 Graded Fail Grading System

The following reasons are valid to reject Graded Fail Grading System (GF/GS):

- a) Degree classifications normally refer to levels of certification of a passed examination and NOT a failed examination. Hence, only the pass zone is graded and NOT the fail zone. This is diagrammatically shown in Figure 2.
- b) If no certificate is issued for a failed examination why then is it necessary to classify fail zone?
- c) Consequent upon the (a) above, the expressions for a Grading System [Equations (2.1) & (3.1)] are expected to produce a solution where the parameters 'm' and 'n' are equal. That is, the number of degree classification, 'm' must have the same value as the maximum number of grade point, 'n' assigned to the highest score range. That is, $(m = n)$.
- d) GF/GS does not support the Separation Property of the Cumulative Grade Point Average, CGPA for which a grading system is expected to serve. This is because fail grades are assigned finite grade points thereby increasing the overall CGPA of academically weak students in a greater proportion than the strong students (see Tables 29 & 30). This is a serious misrepresentation of student evaluation. For the above reasons, the GF/GS format is rejected.

4.2 Non-Graded Fail Grading System

This is more appropriate than the Graded Fail format because it eliminates all the drawbacks stated against the Graded Fail Grading System in addition to its simplicity in expression [equation (2.1)] and calculation [Tables 3 through Table 6 and Tables 13, 17, 21 & 26].

However, certain conditions may be imposed in order to make its application still much simpler. These conditions are derived from equation (3.5) as follows:

$$m = 1 + \frac{n - n_x}{d} \dots\dots\dots (3.5)$$

Equation (3.5) can be re-arranged as follows:

$$m = 1 + \frac{n}{d} - \frac{n_x}{d} \dots\dots\dots (4.2.1)$$

Condition 1: $d = n_x$

That is, the Common Difference, 'd' of the required Arithmetical Progression must be equal to the grade point, 'n_x' (x) assigned to the **least pass** grade/score range. Thus, equation (4.2.1) becomes

$$m = \frac{n}{d} \dots \dots \dots (4.2.2)$$

Equation (4.2.2) is the Validity Test equation for a Non-Graded Grading System.

Condition 2: m = n

That is, the number, 'm' of degree classification must be equal to the maximum grade point, 'n' assigned to the **highest pass** grade/score range. This requires **d = n_x = 1**, which is the condition already dealt with above (Condition1). Thus, equation (4.2.2) becomes

$$m = n \dots \dots \dots (4.2.3)$$

Since, d = n_x = 1 and by definition of n_x (grade point assigned to the least pass grade/score range), the score range below the least pass grade/score range is certainly a fail grade/score range which automatically is assigned a zero grade point in order to maintain mathematical equilibrium. Hence, equation (4.2.3) compels fail grade/score range to be assigned zero grade point, thereby supporting the major characteristic of CGPA. Thus, the recommended expression for a Grading System (NGF/GS) is derived from equation (2.1) with the above derived parameters. Hence,

$$[NGF/GS]_{F=0} = \{[(n - point)0, 1], [P(0, 1, n)], [(M_H + 1)_x, (M_L)_n] \dots \dots \dots (4.2.4)$$

However, if 'd' is assigned a greater integer number (say, 2), from equation (4.2.2) a double grade point value (2m) is assigned across the Arithmetical Progression and all calculated values are equally doubled. This is what gives rise to an **Expanded/Extended Grading System (ExGS)** such as presented in Table 18 through 26. That is, from equation (4.2.2), we have

$$md = n \dots \dots \dots (4.2.5)$$

Hence, equation (2.1) becomes

$$[ExGS]_{F=0} = \{[(m - point)0, d], [P(0, d, md)], [(M_H + 1)_x, (M_L)_n] \dots \dots \dots (4.2.6)$$

From equation (4.2.6), it is obvious that ExGS has the same characteristics as Non-Graded Fail Grading System (NGF/GS) when d = 1. **Note that the degree classification, 'm' is not extended.** Though 'm' is not extended but 'n' is extended (n = md). For this reason, this format is not recommended.

It is also possible to assign a fraction to the Common Difference, 'd'. When this is done, the fraction MUST be a factor of 'n' such that the total sum of AP will be equal to 'n'. That is,

$$[n] = \sum_0^n d_i \dots \dots \dots (4.2.7)$$

And from equations (4.2.2) and (2.1), we have

$$m = nd_i \dots \dots \dots (4.2.8)$$

$$[ElGS]_{F=0} = \{[(nd_i - point)0, d_i], [P(0, d_i, n)], [(M_H + 1)_x, (M_L)_n] \dots \dots \dots (4.2.9)$$

Where,
 d_i = fractional Common Difference of AP.

Equation (4.2.9) is what is referred to as an **Elongated Grading System (ELGS)** which is developed with its own characteristics. The corresponding CGPA ranges calculated by an Excel Calculator using equation (4.2.9), [ELGS] is shown in Table 31. Comparing the classifications with those obtained from a Non-Grade Fail Grading System, [NGF/GS] there exist a great discrepancies as follows:

- The CGPA distribution is different since the score distribution is different as expected.
- Since the Common Difference is lowered, it is expected that the corresponding scores will be lowered and consequently the CGPA.

- Therefore, more candidates are admitted into higher classes than lower classes.

However, if the original first class (3.76 to 4.00) is to be maintained, each class created by the Arithmetical Progression must be upheld. Hence, this gives an (8-point) grading system with the maximum grade point of 4 instead of a normal (4-point) grading system with a maximum grade point of 4. Note that in order to group into original 4 classes, two divisions must be recombined to obtain 4 classes as shown in Table 31. Otherwise, 8 nomenclatures (1st Upper, 1st Lower, 2nd Upper, 2nd Lower, 3rd Upper, 3rd Lower, 4th Upper, 4th Lower,) will have to be maintained to cover all the 8 classes produced by the grading system, EIGS. **The resulting degree classification, 'm' is elongated from 4 to 8 for a 'd' value of 0.5.** Hence, this format is rejected.

The comparison between the Non-Graded Fail Grading System (NGF/GS) and the Elongated Grading Systems are shown in Table 32 below. This is also diagrammatically presented in Figure 3.

Finally, the recommended expression for a suitable grading system is given by equation (4.2.4) which produces the following variables that are used to generate the appropriate CGPA ranges that are suitable for degree classification:

- Pass Score Ranges [$R_{MA} = (M_L \text{ to } 100)$, $R_{MB} = (M_B \text{ to } \{M_L - 1\})$, $R_{MC} = (M_C \text{ to } \{M_B - 1\})$, etc]. The total number of pass score ranges = n. The equal intervals are obtained using equation (4.2.10).

$$Equal\ Int = \frac{M_L - (M_H + 1)}{m - 1} \dots \dots \dots (4.2.10)$$

When equal score intervals do not give perfect integers, the nearest to perfect integers (whole numbers) could be used allowing the higher numbers at the lower classes in order to strengthen the Separation Property of CGPA.

- Fail Score Range [$R_{MF} = 0 \text{ to } M_H$]
- The grade points from zero to 'n' with a common difference of 1. That is, (0 to n)
- Letter grades with assigned grade points [A (n), B (n-1), C (n-2) F (n-n)].

The Excel Calculator that is necessary to calculate the CGPA ranges using the above variables requires appropriate mathematical relationships between the score ranges and the grade points (0 to n). The theory required to develop this mathematical relationship is a subject matter of another paper (Paper 3).

5. A Recommended Grading System as An Example

The author recommends a (5-point) Non-Grade Fail (Normal) Grading System for the following reasons:

- God used a (5-point) assessment scale to evaluate His three servants in the Parable of the Talents in Matthew 25:15-28.
- Figure '5' is a Biblical number for completeness. Elisha asked the king of Israel to smite upon the ground the arrows of the LORD's deliverance (the arrow of deliverance from Syria), signifying how Israel would defeat the Syrians in a battle but the king smote thrice and stayed instead of five (5) or six (6) times in order that Israel may defeat the Syrians completely. Because the arrows were smitten only three times, Israel defeated the Syrians in only three occasions. – 2 King 13: 14-19. (5-point) grading system is required to completely evaluate and classify students' performance optimally.
- Figure '5' is a Biblical victory/miracle number. David picked five (5) stones to satisfy the required number for complete victory but one of them gave him victory over his enemies, Goliath – 1 Samuel 17:40-50. The relevance of this to a (5-point) grading system is that First Class of a degree classification which has an upper limit of 5 stands for the winning stone out of the five while any of the five stones could have been the winning stone, suggesting equal chances for the five stones (degree classes).
- There are five (5) porches in Bethesda pool which was a miracle pool (John 5:1-9). The relevance of this to a (5-point) grading system is that First Class of a degree classification which has an upper limit of 5 stands for the first person whosoever stepped in, after the pool is troubled, was made whole of whatsoever diseases he had while the five porches represent the degree classification. Any of the

five porches could have been troubled by the angel at any given time, suggesting equal chances for the five porches (degree classes).

- There are five (5) senses created in MAN [Eyes (seeing), Nose (smell), Ears (hearing), Mouth (taste) & Body (touch or feelings)].
- A human palm/hand consists of five (5) fingers which are organized/set/arranged, one relative to the other, in such a way that they represent a Gaussian form of probability distribution (Bell Curve). When both hands are considered representing ten (10) fingers, they both represent the Extended (5-point) Grading System (ExGS). The toes of both legs are similarly set to produce a Bell Curve shape (Normal Probability Distribution) when brought together.
- (5-point) grading system has been the most common among Nigerian universities and most English-speaking countries or British former Colonies, simply because after the descriptive and qualitative expression of levels of achievement, no further vocabularies is available without ambiguity (see Table 33).

5.1 **Minimum Academic Standard (MAS)**

The following parameters are to be fixed as minimum academic standard:

- $n = m = 5$
- $(M_H + 1) = 40\%$
- $M_L = 70\%$

Using the above parameters, the score & CGPA ranges are generated by the Excel Calculator as presented in Table 34 below.

5.2 **An Example of a University with Parameters higher than MAS**

The following parameters are to be fixed by a University:

- $n = m = 5$
- $(M_H + 1) = 45\%$
- $M_L = 80\%$

Using the above parameters, the score & CGPA ranges are generated by the Excel Calculator as presented in Table 35 below.

6. References

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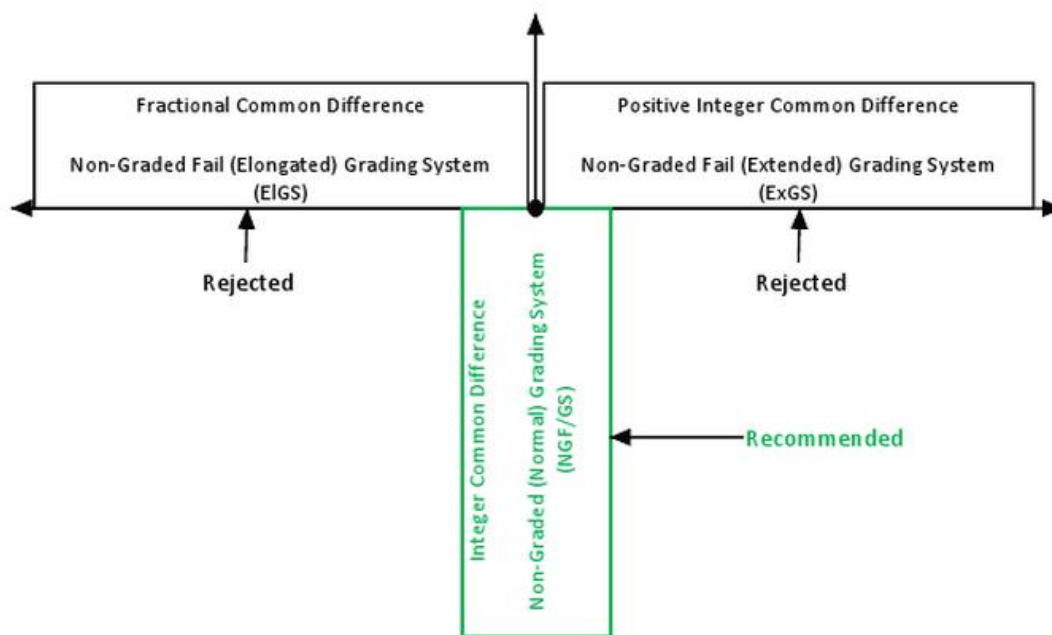
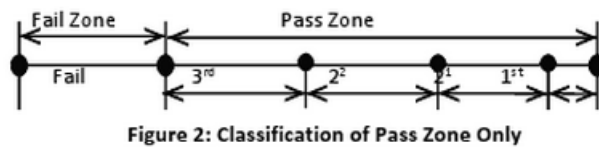
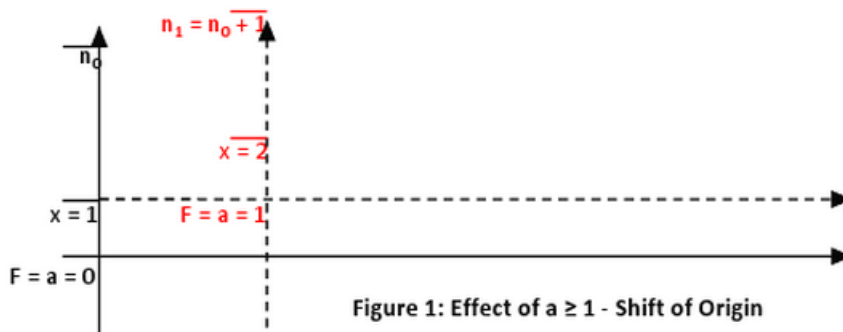


Table 1: Reason For Choosing Arithmetical Progression And Not Any Other

ARITHMETICAL PROGRESSION FOR GRADE POINTS		GEOMETRICAL PROGRESSION FOR GRADE POINTS	
COMMON DIFFERENCE = 1 $2 - 1 = 3 - 2 = 1$	DIFFERENCE BETWEEN SUCCESSIVE GP	COMMON RATIO = 2 $4/2 = 8/4 = 16/8 = 32/16 = 2$	DIFFERENCE BETWEEN SUCCESSIVE GP
5		32	
4	$5 - 4 = 1$	16	$32 - 16 = 16$
3	$4 - 3 = 1$	8	$16 - 8 = 8$
2	$3 - 2 = 1$	4	$8 - 4 = 4$
1	$2 - 1 = 1$	2	$4 - 2 = 2$
0	$1 - 0 = 1$	0	$2 - 0 = 2$
The difference between successive GP is the same throughout the mark/score ranges and it is equal to the common difference. Hence, it offers equal chances to all students and in addition, it agrees with mark/score ranges that have equal class intervals		The difference between successive GP is not the same throughout the mark/score ranges. Hence, it does not offer equal chances to all students and in addition, it does not agree with mark/score ranges that have equal class intervals	

Table 2: Effect Of 'F = A = 1'

ARITHMETICAL PROGRESSION FOR GRADE POINTS	
n_0	$n_1 = n_0 + 1$
5	6
4	5
3	4
2	3
1	2
0	1
This is preferred because: - Number of class ($m = 5$) is the same as highest grade point ($n_0 = 5$) as expected. - This is the correct counting in Base Number Systems. That is, it includes a null symbol.	This is not preferred because: - Number of class ($m = 5$) is not the same as highest grade point ($n_1 = 6$). - This is an incorrect counting in Base Number Systems. That is, it does not include a null symbol.

Table 3: Non-Graded Fail Grading System (70,40,5,1)

$[NGF/GS]_{F=0} = (5 - \text{point})0, 1, [P(0, 1, 5), 40, 70]$ (POLICY PARAMETERS):- $M_L = 70, (M_H + 1) = 40, \text{Max Grade Point} = 5$ and the Least Pass Grade Point = 1											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(5)	1	70	100	77.78	30	4.50	5.00		3.50	1 st	5
B(4)	2	63	69		6.5	3.62	4.49	0.874968	$d = x$	2 nd U	4
C(3)	3	55	62		6.5	2.75	3.61	0.874968	1	2 nd L	3
D(2)	4	48	54		6.5	1.87	2.74	0.874968		3 rd	2
E(1)	5	40	47		6.5	1.00	1.86	0.874968		Pass	1
F(0)		0	39							Fail	0
		ΣCI (Pass only)			26						

Table 4: Non-Graded Fail Grading System (70,50,5,1)

$[NGF/GS]_{F=0} = (5 - \text{point})0, 1, [P(0, 1, 5), 50, 70]$ (POLICY PARAMETERS):- $M_L = 70, (M_H + 1) = 50$, Max Grade Point = 5 and the Least Pass Grade Point = 1											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(5)	1	70	100	77.78	30	4.50	5.00		3.50	1 st	5
B(4)	2	65	69		4	3.62	4.49	0.874968	$d = x$	2 nd U	4
C(3)	3	60	64		4	2.75	3.61	0.874968	1	2 nd L	3
D(2)	4	55	59		4	1.87	2.74	0.874968		3 rd	2
E(1)	5	50	54		4	1.00	1.86	0.874968		Pass	1
F(0)		0	49							Fail	0
		ΣCI (Pass only)			16						

Table 5: Non-Graded Fail Grading System (70,40,4,1)

$[NGF/GS]_{F=0} = (4 - \text{point})0, 1, [P(0, 1, 4), 40, 70]$ (POLICY PARAMETERS):- $M_L = 70, (M_H + 1) = 40$, Max Grade Point = 4 and the Least Pass Grade Point = 1											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(4)	1	70	100	77.78	30	3.60	4.00		2.60	1 st	4
B(3)	2	60	69		9	2.73	3.59	0.866632	$d = x$	2 nd U	3
C(2)	3	50	59		9	1.87	2.72	0.866632	1	2 nd L	2
D(1)	4	40	49		9	1.00	1.86	0.866632		3 rd	1
F(0)		0	39							Fail	0
		ΣCI (Pass only)			27						

Table 6: Non-Graded Fail Grading System (70,50,4,1)

$[NGF/GS]_{F=0} = (4 - \text{point})0, 1, [P(0, 1, 4), 50, 70]$ (POLICY PARAMETERS):- $M_L = 70, (M_H + 1) = 40$, Max Grade Point = 4 and the Least Pass Grade Point = 1											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(4)	1	70	100	77.78	30	3.60	4.00		2.60	1 st	4
B(3)	2	63	69		5.6667	2.73	3.59	0.866632	$d = x$	2 nd U	3
C(2)	3	57	62		5.6667	1.87	2.72	0.866632	1	2 nd L	2
D(1)	4	50	56		5.6667	1.00	1.86	0.866632		3 rd	1
F(0)		0	49							Fail	0
		ΣCI (Pass only)			17						

Table 7: 1 Class of Failure

L/G	ML	MH	INT	GP (n)	TABLE 7: 1 Class of Failure	L/G	ML	MH	INT	GP (n)	
A(4)	80	100	20	4	5 th term	A(5)	80	100	20	5	
B(3)	70	79	9	3		B(4)	70	79	9	4	
C(2)	60	69	9	2		C(3)	60	69	9	3	
D(1)	50	59	9	1		$x = a + d$ 2 nd term	D(2)	50	59	9	2
F(0)	0	49	49	0	1 st term = a	F(1)	0	49	49	1	1 st term = a

Note,

$$m = 1 + (4 - 1)/1 = 4.$$

$$m = 1 + (5 - 2)/1 = 4$$

Table 8: 2 Classes of Failure

L/G	ML	MH	INT	GP (n)	TABLE 8: 2 Classes of Failure	L/G	ML	MH	INT	GP (n)	
A(5)	80	100	20	5	5 th term	A(6)	80	100	20	6	
B(4)	70	79	9	4		B(5)	70	79	9	5	
C(3)	60	69	9	3		C(4)	60	69	9	4	
D(2)	50	59	9	2		$x = a + 2d$ 3 rd term	D(3)	50	59	9	3
F ₂ (1)	25	49	24	1	2 nd term	F ₂ (2)	25	49	24	2	2 nd term
F ₁ (0)	0	24	24	0	1 st term = a	F ₁ (1)	0	24	24	1	1 st term = a

$$m = 1 + (5 - 2)/1 = 4.$$

$$m = 1 + (6 - 3)/1 = 4$$

Table 9: 3 Classes of Failure

L/G	ML	MH	INT	GP (n)	TABLE 9: 3 Classes of Failure	L/G	ML	MH	INT	GP (n)	
A(6)	80	100	20	6	7 th term	A(7)	80	100	20	7	
B(5)	70	79	9	5		B(6)	70	79	9	6	
C(4)	60	69	9	4		C(5)	60	69	9	5	
D(3)	50	59	9	3		$x = a_3 + 3d$ 4 th term	D(4)	50	59	9	4
F ₃ (2)	33	49	16	2	3 rd term	F ₃ (3)	33	49	16	3	3 rd term
F ₂ (1)	16	32	16	1	1 st term = a	F ₂ (2)	16	32	16	2	1 st term = a
F ₁ (0)	0	15	15	0		F ₁ (1)	0	15	15	1	

$$m = 1 + (6 - 3)/1 = 4.$$

$$m = 1 + (7 - 4)/1 = 4$$

Table 10: Graded Fail Grading System (80,50,6,3)

$[GF/GS]_{F=1,2,3} = [(4 - point)0, 1, 2, 3], [P(0, 1, 6), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 6 and the Least Pass Grade Point = 3											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(6)	1	80	100	85.19	20	5.63	6.00		5.63	1 st	6
B(5)	2	70	79		9	4.95	5.62	0.6852995	(d)	2 nd U	5
C(4)	3	60	69		9	4.26	4.94	0.6852995	1	2 nd L	4
D(3)	4	50	59		9	3.58	4.25	0.6852995		3 rd	3
F ₃ (2)	5	33	49		16.000	2.36	3.57	1.218310		Fail	2
F ₂ (1)	6	16	32		16.000	1.14	2.35	1.218310		Fail	1
F ₁ (0)	7	0	15		15.000	0.00	1.13	1.142166		Fail	0
		ΣCI (Pass/Fail)			74.000						

Table 11: Graded Fail Grading System (80,50,5,2)

$[GF/GS]_{F=0,1} = [(4 - point)0, 1, 2], [P(0, 1, 5), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 5 and the Least Pass Grade Point = 2											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(5)	1	80	100	85.19	20	4.70	5.00		4.70	1 st	5
B(4)	2	70	79		9	4.13	4.69	0.5634685	(d)	2 nd U	4
C(3)	3	60	69		9	3.57	4.12	0.5634685	1	2 nd L	3
D(2)	4	50	59		9	3.01	3.56	0.5634685		3 rd	2
F ₂ (1)	5	25	49		24.000	1.50	3.00	1.502583		Fail	1
F ₁ (0)	6	0	24		24.000	0.00	1.49	1.502583		Fail	0
		ΣCI (Pass/Fail)			75.000						

Table 12: Graded Fail Grading System (80,50,4,1)

$[GF/GS]_{F=0} = [(4 - point)0, 1], [P(0, 1, 4), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 4 and the Least Pass Grade Point = 1											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(4)	1	80	100	85.19	20	3.76	4.00		3.76	1 st	4
B(3)	2	70	79		9	3.31	3.75	0.4448435	(d)	2 nd U	3
C(2)	3	60	69		9	2.87	3.30	0.4448435	1	2 nd L	2
D(1)	4	50	59		9	2.42	2.86	0.4448435		3 rd	1
F(0)	5	0	49		49.000	0.00	2.41	2.421926		Fail	0
		ΣCI (Pass/Fail)			76.000						

Table 13: Non-Graded Fail Grading System (80,50,4,1)

$[NGF/GS]_{F=0} = [(4 - point)0, 1], [P(0, 1, 4), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 4 and the Least Pass Grade Point = 1											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(4)	1	80	100	85.19	20	3.76	4.00		2.76	1 st	4
B(3)	2	70	79		9	2.84	3.75	0.9188188	(d)	2 nd U	3
C(2)	3	60	69		9	1.92	2.83	0.9188188	1	2 nd L	2
D(1)	4	50	59		9	1.00	1.91	0.9188188		3 rd	1
F(0)	5	0	49							Fail	0
		ΣCI (Pass Only)			27						

Table 14: Graded Fail Grading System (80,50,7,4)

$[GF/GS]_{F=1,2,3} = [(4 - point)1, 2, 3, 4], [P(1, 1, 7), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 7 and the Least Pass Grade Point = 4											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(7)	1	80	100	85.19	20	6.57	7.00		5.57	1 st	7
B(6)	2	70	79		9	5.90	6.56	0.6778944	(d)	2 nd U	6
C(5)	3	60	69		9	5.22	5.89	0.6778944	1	2 nd L	5
D(4)	4	50	59		9	4.54	5.21	0.6778944		3 rd	4
F ₃ (3)	5	33	49		16.000	3.33	4.53	1.205146		Fail	3
F ₂ (2)	6	16	32		16.000	2.13	3.32	1.205146		Fail	2
F ₁ (1)	7	0	15		15.000	1.00	2.12	1.129824			1
		ΣCI (Pass/Fail)			74.000						

$[GF/GS]_{F=1,2} = [(4 - point)1, 2, 3], [P(1, 1, 6), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 6 and the Least Pass Grade Point = 3											
L/G	m	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(6)	1	80	100	85.19	20	5.63	6.00		4.63	1 st	6
B(5)	2	70	79		9	5.08	5.62	0.5561622	(d)	2 nd U	5
C(4)	3	60	69		9	4.52	5.07	0.5561622	1	2 nd L	4
D(3)	4	50	59		9	3.97	4.51	0.5561622		3 rd	3
F ₂ (2)	5	25	49		24.000	2.48	3.96	1.483099		Fail	2
F ₁ (1)	6	0	24		24.000	1.00	2.47	1.483099		Fail	1
		ΣCI (Pass/Fail)			75.000						

Table 15: Graded Fail Grading System (80,50,6,3)

Table 16: Graded Fail Grading System (80,50,5,2)

$[GF/GS]_{F=1} = [(4 - point)1, 2], [P(1, 1, 5), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 5 and the Least Pass Grade Point = 2											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(5)	1	80	100	85.19	20	4.70	5.00		3.70	1 st	5
B(4)	2	70	79		9	4.26	4.69	0.4376334	(d)	2 nd U	4
C(3)	3	60	69		9	3.82	4.25	0.4376334	1	2 nd L	3
D(2)	4	50	59		9	3.38	3.81	0.4376334		3 rd	2
F ₁ (1)	5	0	49		49.000	1.00	3.37	2.382670		Fail	1
		ΣCI (Pass/Fail)			76.000						

Table 17: Non-Graded Fail Grading System (80,50,5,2)

$[NGF/GS]_{F=1} = [(4 - \text{point})1, 2], [P(1, 1, 5), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 5 and the Least Pass Grade Point = 2											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(5)	1	80	100	85.19	20	4.70	5.00		2.70	1 st	5
B(4)	2	70	79		9	3.80	4.69	0.8985235	(d)	2 nd U	4
C(3)	3	60	69		9	2.90	3.79	0.8985235	1	2 nd L	3
D(2)	4	50	59		9	2.00	2.89	0.8985235		3 rd	2
F(1)	5	0	49							Fail	1
		ΣCI (Pass Only)			27						

Table 18: Graded Fail Grading System (80,50,12,6)

$[GF/GS]_{F=0,2,4} = 2\{[(4 - \text{point})0, 1, 2, 3], [P(0, 1, 6)], [(50, 80)]\}$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 12 and the Least Pass Grade Point = 6											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(12)	1	80	100	85.19	20	11.27	12.00		11.27	1 st	12
B(10)	2	70	79		9	9.90	11.26	1.370599	(d)	2 nd U	10
C(8)	3	60	69		9	8.53	9.89	1.370599	2	2 nd L	8
D(6)	4	50	59		9	7.16	8.52	1.370599		3 rd	6
F ₃ (4)	5	33	49		16.000	4.72	7.15	2.436620		Fail	4
F ₂ (2)	6	16	32		16.000	2.28	4.71	2.436620		Fail	2
F ₁ (0)	7	0	15		15.000	0.00	2.27	2.284332		Fail	0
		ΣCI (Pass/Fail)			74	$[GS]_F = [(4 - \text{point})0, 2, 4, 6], [P(2, 0, 12)], [(50, 80)]$					

Table 19: Extended Grading System (80,50,10,4)

$[GF/GS]_{F=0,2} = 2\{[(4 - \text{point})0, 1, 2], [P(0, 1, 5)], [(50, 80)] \dots \dots \dots (3.3.2a)\}$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 10 and the Least Pass Grade Point = 4											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(10)	1	80	100	85.19	20	9.39	10.00		9.39	1 st	10
B(8)	2	70	79		9	8.26	9.38	1.12693692	(d)	2 nd U	8
C(6)	3	60	69		9	7.14	8.25	1.12693692	2	2 nd L	6
D(4)	4	50	59		9	6.01	7.13	1.12693692		3 rd	4
F ₂ (2)	5	25	49		24.000	3.01	6.00	3.005165		Fail	2
F ₁ (0)	6	0	24		24.000	0.00	4.50	3.005165		Fail	0
		ΣCI (Pass/Fail)			75.000	$[GS]_F = [(4 - \text{point})0, 2, 4], [P(2, 0, 10), (50, 80)]$					

Table 20: Extended Graded Fail Grading System (80,50,8,2)

$[GF/GS]_{F=0} = 2\{[(4 - point)0, 1], [P(0, 1, 4)], [(50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 8 and the Least Pass Grade Point = 2											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(8)	1	80	100	85.19	20	7.51	8.00		7.51	1 st	8
B(6)	2	70	79		9	6.62	7.50	0.889687	(d)	2 nd U	6
C(4)	3	60	69		9	5.73	6.61	0.889687	2	2 nd L	4
D(2)	4	50	59		9	4.84	5.72	0.889687		3 rd	2
F ₁ (0)	5	0	49		49.000	0.00	4.83	4.843852		Fail	0
		ΣCI (Pass/Fail)			76.000	$[GS]_F = [(4 - point)0, 2], [P(2, 0, 8), (50, 80)]$					

Table 21: Extended Non-Graded Fail Grading System (80,50,8,2)

$[NGF/GS]_{F=0} = 2\{[(4 - point)0, 1], [P(0, 1, 4)], [(50, 80)] \dots \dots \dots (3.3.4a)$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 8 and the Least Pass Grade Point = 2											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(8)	1	80	100	85.19	20	7.51	8.00		5.51	1 st	8
B(6)	2	70	79		9	5.68	7.50	1.8376376	(d)	2 nd U	6
C(4)	3	60	69		9	3.84	5.67	1.8376376	2	2 nd L	4
D(2)	4	50	59		9	2.00	3.83	1.8376376		3 rd	2
F(0)	5	0	49							Fail	0
		ΣCI (Pass Only)			27	$[GS]_{F=y} = [(4 - point)0, 2], [P(2, 0, 8), (50, 80)]$					

Table 22: Graded Fail Grading System (80,50,13,7)

$[GF/GS]_{F=1,3,5} = [(4 - point)1, 3, 5, 7], [P(1, 2, 13), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 13 and the Least Pass Grade Point = 7											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(13)	1	80	100	85.19	20	12.21	13.00		11.21	1 st	13
B(11)	2	70	79		9	10.85	12.20	1.3631939	(d)	2 nd U	11
C(9)	3	60	69		9	9.48	10.84	1.3631939	2	2 nd L	9
D(7)	4	50	59		9	8.12	9.47	1.3631939		3 rd	7
F ₃ (5)	5	33	49		16.000	5.70	8.11	2.423456		Fail	5
F ₂ (3)	6	16	32		16.000	3.27	5.69	2.423456		Fail	3
F ₁ (1)	7	0	15		15.000	1.00	3.26	2.271990		Fail	1
		ΣCI (Pass/Fail)			74.000						

Table 23: Graded Fail Grading System (80,50,11,5)

$[GF/GS]_{F=1,3} = [(4 - \text{point})1, 3, 5], [P(1, 2, 11), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 11 and the Least Pass Grade Point = 5											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(11)	1	80	100	85.19	20	10.33	11.00		9.33	1 st	11
B(9)	2	70	79		9	9.21	10.32	1.1196306	(d)	2 nd U	9
C(7)	3	60	69		9	8.09	9.20	1.1196306	2	2 nd L	7
D(5)	4	50	59		9	6.97	8.08	1.1196306		3 rd	5
F ₂ (3)	5	25	49		24.000	3.99	6.96	2.985682		Fail	3
F ₁ (1)	6	0	24		24.000	1.00	3.98	2.985682		Fail	1
		ΣCI (Pass/Fail)			75.000						

Table 24: Graded Fail Grading System (80,50,9,3)

$[GF/GS]_{F=1} = [(4 - \text{point})1, 3], [P(1, 2, 9), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 9 and the Least Pass Grade Point = 3											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(9)	1	80	100	85.19	20	8.45	9.00		7.45	1 st	9
B(7)	2	70	79		9	7.57	8.44	0.8824769	(d)	2 nd U	7
C(5)	3	60	69		9	6.69	7.56	0.8824769	2	2 nd L	5
D(3)	4	50	59		9	5.80	6.68	0.8824769		3 rd	3
F(1)	5	0	49		49.000	1.00	5.79	4.804596		Fail	1
		ΣCI (Pass/Fail)			76.000						

Table 25: Non-Graded Fail Grading System (80,50,9,3)

GRADING SYSTEM (NON-GRADED FAIL WITH F1 = 1, d = 2) $[NGF/GS]_{F=1} = [(4 - \text{point})1, 3], [P(1, 2, 9), (50, 80)]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 9 and the Least Pass Grade Point = 3											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A(9)	1	80	100	85.19	20	8.45	9.00		5.45	1 st	9
B(7)	2	70	79		9	6.63	8.44	1.8173423	(d)	2 nd U	7
C(5)	3	60	69		9	4.82	6.62	1.8173423	2	2 nd L	5
D(3)	4	50	59		9	3.00	4.81	1.8173423		3 rd	3
F(1)	5	0	49							Fail	1
		ΣCI (Pass Only)			27						

Table 26: One Class of Failure

L/G	ML	MH	INT	GP (n)	Arithmetical Progression		REMARKS
A(4)	80	100	20	4		5 th term of AP with d = 1	This is exactly like a Normal Grading System. That is, (Non-Grade Fail)
B(3)	70	79	9	3		4 th term of AP with d = 1	
C(2)	60	69	9	2		3 rd term of AP with d = 1	
D(1)	50	59	9	1	$x = a + d$	2 nd term of AP with d = 1	
F(0)	0	49	49	0	a = 0	1 st term of AP with d = 1	

Table 27: Two Classes of Failure

					Arithmetical Progression		REMARKS
L/G	ML	MH	INT	GP (n)			
A(4)	80	100	20	4		5 th term of AP with $d = 1$	This is exactly like a Normal Grading System. That is, (Non-Grade Fail) AP in the opposite direction
B(3)	70	79	9	3		4 th term of AP with $d = 1$	
C(2)	60	69	9	2		3 rd term of AP with $d = 1$	
D(1)	50	59	9	1	$x = a + d$	2 nd term of AP with $d = 1$	
$F_1(0)$	25	49	24	0	$a = 0$	1 st term of AP with $d = 1$	
$F_2(-1)$	0	24	24	-1	$F_2 = a - d$	2 nd term of AP with $d = -1$	

Table 28: Three Classes of Failure

					Arithmetical Progression		REMARKS
L/G	ML	MH	INT	GP (n)			
A(4)	80	100	20	4		5 th term of AP with $d = 1$	This is exactly like a Normal Grading System. That is, (Non-Grade Fail) AP in the opposite direction AP in the opposite direction
B(3)	70	79	9	3		4 th term of AP with $d = 1$	
C(2)	60	69	9	2		3 rd term of AP with $d = 1$	
D(1)	50	59	9	1	$x = a + d$	2 nd term of AP with $d = 1$	
$F_1(0)$	33	49	16	0	$a = 0$	1 st term of AP with $d = 1$	
$F_2(-1)$	16	32	16	-1	$F_2 = a - d$	2 nd term of AP with $d = -1$	
$F_3(-2)$	0	15	15	-2	$F_3 = a - 2d$	3 rd term of AP with $d = -2$	

Table 29: Table of Comparison of 'GS' With Common Difference of '1' And First Fail Class of '0' And '1'

Score Ranges	NON-GRADED FAIL	GRADED FAIL			NON-GRADED FAIL	GRADED FAIL		
	F	F_1	F_1, F_2	F_1, F_2, F_3	F	F_1	F_1, F_2	F_1, F_2, F_3
	$F = 0, d = 1$	$F_1 = 0, d = 1$	$F_1 = 0, d = 1$	$F_1 = 0, d = 1$	$F = 1, d = 1$	$F_1 = 1, d = 1$	$F_1 = 1, d = 1$	$F_1 = 1, d = 1$
80 to 100	3.76 to 4.00	3.76 to 4.00	4.70 to 5.00	5.63 to 6.00	4.70 to 5.00	4.70 to 5.00	5.63 to 6.00	6.57 to 7.00
67 to 79	2.84 to 3.75	3.31 to 3.75	4.13 to 4.69	4.95 to 5.62	3.80 to 4.69	4.26 to 4.69	5.08 to 4.62	5.90 to 6.56
53 to 66	1.92 to 2.83	2.87 to 3.30	3.57 to 4.12	4.26 to 4.94	2.90 to 3.79	3.82 to 4.25	4.52 to 5.07	5.22 to 5.89
50 to 52	1.00 to 1.91	2.42 to 2.86	3.01 to 3.56	3.58 to 4.25	2.00 to 2.89	3.38 to 3.81	3.97 to 4.51	4.54 to 5.21
		0.00 to 2.41	1.50 to 3.00	2.36 to 3.57		1.00 to 3.37	2.48 to 3.96	3.33 to 4.53
			0.00 to 1.49	1.14 to 2.35			1.00 to 2.47	2.13 to 3.32
				0.00 to 1.13				1.00 to 2.12

Table 30: Table of Comparison of 'GS' With Common Difference of '2' And First Fail Class of '0' And '1'

Score Ranges	NON-GRADED FAIL	GRADED FAIL				NON-GRADED FAIL	GRADED FAIL			
	F	F ₁	F ₁ , F ₂	F ₁ , F ₂ , F ₃	F	F ₁	F ₁ , F ₂	F ₁ , F ₂ , F ₃		
	F = 0, d = 2	F ₁ = 0, d = 2	F ₁ = 0, d = 2	F ₁ = 0, d = 2	F = 1, d = 2	F ₁ = 1, d = 2	F ₁ = 1, d = 2	F ₁ = 1, d = 2		
80 to 100	7.51 to 8.00	7.51 to 8.00	9.39 to 10.00	11.27 to 12.00	8.45 to 9.00	8.45 to 9.00	10.33 to 11.00	12.21 to 13.00		
67 to 79	5.68 to 7.50	6.62 to 7.50	8.26 to 9.38	9.90 to 11.26	6.63 to 8.45	7.57 to 8.44	9.21 to 10.32	10.85 to 12.20		
53 to 66	3.84 to 5.67	5.73 to 6.61	7.14 to 8.25	8.53 to 9.89	4.82 to 6.62	6.69 to 7.56	8.09 to 9.20	9.48 to 10.84		
50 to 52	2.00 to 3.83	4.84 to 5.72	6.01 to 7.13	7.16 to 8.52	3.00 to 4.81	5.80 to 6.68	6.97 to 8.08	8.12 to 9.47		
		0.00 to 483	3.01 to 6.00	4.72 to 7.15		1.00 to 5.79	3.99 to 6.96	5.70 to 8.11		
			0.00 to 3.00	2.28 to 4.71			1.00 to 3.98	3.27 to 5.69		
				0.00 to 2.27				1.0 to 3.26		

Table 31: Elongated Grading System (80,50,4,0.5)

$[ELGS]_{F=0} = \{[(4 - \text{point})0, 0.5], [P(0, 0.5, n)], [50, 80]\}$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50, \text{Max Grade Point} = 4 \text{ and the Least Pass Grade Point} = 0.5$												
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)	Class
A(4.0)	1	80	100	85.19	20	3.76	4.00		3.26	1 st	4.0	1 st Upper
B(3.5)	2	76	79		3.2857	3.29	3.75	0.4652081	(d)		3.5	1 st Lower
C(3.0)	3	71	75		3.2857	2.83	3.28	0.4652081	0.5	2 nd U	3.0	2 nd Upper
D(2.5)	4	67	70		3.2857	2.36	2.82	0.4652081			2.5	2 nd Lower
E(2.0)	5	63	66		3.2857	1.90	2.35	0.4652081		2 nd L	2.0	3 rd Upper
G(1.5)	6	59	62		3.2857	1.43	1.89	0.4652081			1.5	3 rd Lower
H(2.0)	7	54	58		3.2857	0.97	1.42	0.4652081		3 rd	1.0	4 th Upper
I(0.5)	8	50	53		3.2857	0.50	0.96	0.4652081			0.5	4 th Lower
F(0.0)	5	0	49								0	Fail
		ΣCI (Pass Only)			23							

Table 32: Comparison of NGF/GS and EIGS

SCORE RANGES		L/Grade	Class	CGPA RANGES	
EIGS	NGF/GS			EIGS	NGF/GS
76 to 100	80 to 100	A	1st	3.29 to 4.00	3.76 to 4.00
67 to 75	70 to 79	B	2nd U	2.36 to 3.28	2.84 to 3.75
59 to 66	60 to 69	C	2nd L	1.43 to 2.35	1.92 to 2.83
50 to 58	50 to 59	D	3 rd	0.50 to 1.42	1.00 to 1.91

From the above analysis, the Non-Graded Grading System (NGF/GS) can be of three types; namely,

- Non- Graded Fail (Normal) Grading System (NGF/GS) (when d = 1)
- Non- Graded Fail (Extended) Grading System (ExGS) (when d > 1)
- Non- Graded Fail (Elongated) Grading System (EIGS) (when d < 1)

Table 33: Quantitative Assessment Parameters

Letter Grade	Description	General, Qualitative Of The Evaluation Criteria
A	Excellent	An excellent performance, clearly outstanding. The candidate demonstrates excellent judgement and a high degree of independent thinking.
B	Very Good	A very good performance. The candidate demonstrates sound judgement and a very good degree of independent thinking.
C	Good	A good performance in most areas. The candidate demonstrates a reasonable degree of judgement and independent thinking in the most important areas.
D	Satisfactory	A satisfactory performance, but with significant shortcomings. The candidate demonstrates a limited degree of judgement and independent thinking.
E	Sufficient	A performance that meets the minimum criteria, but no more. The candidate demonstrates a very limited degree of judgement and independent thinking.
F	Fail	A performance that does not meet the minimum academic criteria. The candidate demonstrates an absence of both judgement and independent thinking.

NOTE: Any other vocabulary will either be a synonym or a further expression of emphasis or duplication that may mean different things to different people, thereby causing confusion/ambiguity.

Table 34: Non-Graded Fail Grading System (80,40,5,1)

$[NGF/GS]_{F=0} = \{[(5 - point)0, 1], [P(0, 1, 5)]\}, [40, 70]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 50$, Max Grade Point = 5 and the Least Pass Grade Point = 1											
L/G	M	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A	1	70	100	77.78	30	4.50	5.00		3.50	1 st	5
B	2	62.5	69		6.5	3.62	4.49	0.8749679	(d)	2 nd U	4
C	3	55.0	61.5		6.5	2.75	3.61	0.8749679	1	2 nd L	3
D	4	47.5	54.0		6.5	1.87	2.74	0.8749679		3 rd	2
E	5	40	46.5		6.5	1.00	1.86	0.8749679		Pass	1
F	6	0	39.0							Fail	0
		ΣCI (Pass Only)			26						

Table 35: Non-Graded Fail Grading System (80,45,5,1)

$[NGF/GS]_{F=0} = \{[(5 - point)0, 1], [P(0, 1, 5)]\}, [45, 80]$ (POLICY PARAMETERS):- $M_L = 80, (M_H + 1) = 45$, Max Grade Point = 5 and the Least Pass Grade Point = 1											
L/G	m	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A	1	80	100	85.19	20	4.70	5.00		3.70	1 st U	5
B	2	71.3	79		7.75	3.77	4.69	0.9238926	(d)	1 st L	4
C	3	62.5	70.3		7.75	2.85	3.76	0.9238926	1	2nd U	3
D	4	53.8	61.5		7.75	1.92	2.84	0.9238926		2nd L	2
E	5	45	52.8		7.75	1.00	1.91	0.9238926		Pass	1
F	6	0	44.0							Fail	0
		ΣCI (Pass Only)			31						

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