

## Mathematical Relationship Between Percentile & Grade Point Numbers

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### Abstract

Since the advent of *course system* of education in tertiary institutions world-over, Cumulative Point Average (CGPA) has been in use as an Assessment Instrument instead of Cumulative Weighted Average Mark (CWAM). Consequently, mapping of percentage marks into an n-grade points system which is required to generate the much needed CGPA has become necessary. Countless methods of mapping have been witnessed across different tertiary institutions. In addition, the number of grade point, 'n' varies from institutions to institutions. While it is a universal fact that 'n' can take any value less than 100, it is nevertheless important to know that the value of 'n' has never been greater than 12. In Nigerian tertiary institutions, the value of 'n' varies between 4 and 7 while 5 is the most common. However, simply equating 'n' to 100 is not sufficient to convert the percentile system to an n-grade points system in order to generate the required CGPA. It is discovered that there is no sound mathematical method employed to relate the CGPA ranges normally used to classify degree to the percentage scores earned by individual students. This paper (paper 2) is primarily written to establish the required parameters and the most suitable format of an n-grade points system which is referred to as a Non-Graded Fail Grading System with a fail grade, 'F' assigned a zero value ( $NGF/GS_{F=0}$ ) while another paper (paper 3) presents the development of the required mathematical relationship between CGPA and percentage scores ranges.

### 1. Introduction

There are many opinions about grading systems. As a matter of facts, there are as many as there are users of grading systems. Every training institution that is required to assess its trainees has its own format of grading system since a grading system is a platform for the application of Assessment Instruments. There are also many different Assessment Instruments that are also used by different training institutions. All these grading systems do not address the same objectives and purposes. Because of these different shades of opinions and freedom to use whichever is considered suitable for a given situation, much study has not been done on the subject. It is discovered that people copy one format or the other without knowing fully the original purpose for which what is copied is intended. This consequently leads to many assumptions, one of which is to think that there is nothing to teach anybody about grading systems. The study carried out on grading system is presented in four different papers, namely, Paper 1: Assessment of Student Performance: Grading Systems, Paper 2: Parameters of Grading Systems, ***Paper 3: Mathematical Relationship Between Percentile & Grade Point Numbers***, Paper 4: Evaluation of Grading Systems of Some Tertiary Institutions in Nigeria. The Assessment Instrument considered in this paper is the Cumulative Grade Point Average, CGPA which is the one adopted in most tertiary institutions around the world because of its unique features. Therefore, the grading system suitable for such an instrument is the subject of this paper. There are still many types of this kind depending on such factors as objectives of assessment, understanding of the CGPA and other demands for graduates being assessed. However, there are basically two types of Grading Systems being considered for CGPA. These are **Non-Graded Fail** (where only one class is allowed in a Failure Zone) and **Graded Fail** (where there are more than one class in a Fail Zone).

### 2. Grouping Pass Score/Mark Range ( $R_H, R_{H-1}, R_{H-2}, \dots R_F$ )

It is good to be reminded that the expression for a valid and unique grading system is given by equation 2.1 (from paper 2).

$$[NGF/GS]_{F=a} = \{[(m - point)a, x], [P(a, d, n)], [(M_H + 1)_x, (M_L)_n] \dots \dots \dots (2.1)$$

Equation 2.1 generates the CGPA ranges that match the Score/Mark ranges,  $[(R_H, R_{H-1}, R_{H-2}, \dots R_F)]$  to be used to assess the students.

It is, therefore, required to establish the score/mark ranges as specified. There are numerous ways of grouping the remaining range of score/mark,  $R_{PR} = [(M_H+1) \text{ to } M_L]$  to suit a given policy of the user institution. In general, this can be achieved from two points of considerations, namely;

- a) **Equal Distribution:** That is, equal intervals between the high and low limits of the mark ranges as follows:  $R_{PR}/(m-1) = [M_L - (M_H+1)]/(m-1)$ .
- b) **Unequal Distribution:** The intervals between the high ( $M_L$ ) and low ( $M_H+1$ ) limits of  $R_{PR}$  are chosen at random. This is governed by the probability of any learner's score/mark falling into any of the score/mark ranges beginning from the highest ( $R_H$ ) to the fail range ( $R_F$ ). This is mathematically expressed in equation (2.2)

$$y_H\% + y_{H-1}\% + y_{H-2}\% + \dots + y_L\% + y_F\% = 100\% \dots\dots\dots (2.2)$$

Where,

$y_H\%$  = probability that a learner's score/mark will fall into the highest pass mark range ( $R_H$ )

$y_{H-1}\%$  = probability that a learner's score/mark will fall into the next range lower than ( $R_H$ ) by 1

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$y_L\%$  = probability that a learner's score/mark will fall into the least pass mark range ( $R_L$ )

$y_F\%$  = probability that a learner's score/mark will fall into the fail mark range ( $R_H$ )

These probabilities are equivalent to the percentages of students expected to have their scores falling into the different pass mark ranges and they are fundamental variables that are governed by normal (Gaussian) probability distribution.

After the pass score/mark ranges have been determined, letter grades and grade points (n) are assigned to each group/range as follows:

$R_H$  = the highest score range = [ $M_L$  to 100] is assigned letter grade 'H or A' and grade point (n-0)

$R_{H-1}$  = the next highest score range = [ $M_{L-1}$  to ( $M_L - 1$ )] to  $R_H$  is assigned letter grade 'H-1, or B' and grade point (n-x)

$R_{H-2}$  = the next highest score range = [ $M_{L-2}$  to ( $M_{L-1} - 1$ )] to  $R_{H-1}$  is assigned letter grade 'H-2, or C' and grade point (n-2x)

$R_{H-3}$  = the next highest score range = [ $M_{L-3}$  to ( $M_{L-2} - 1$ )] to  $R_{H-2}$  is assigned letter grade 'H-3, or D' and grade point (n-3x)

$R_{H-4}$  = the next highest score range = [ $M_{L-4}$  to ( $M_{L-3} - 1$ )] to  $R_{H-3}$  is assigned letter grade 'H-4, or E' and grade point (n-4x)

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$R_L$  = the least pass score range = [ $(M_H+1)$  to  $M_{HL}$ ] is assigned letter grade 'L' and grade point 'x'

$R_F$  = the fail score range next to  $R_L$  = [0 to  $M_H$ ] is assigned letter grade 'F' and grade point (n-n) = 0

**NOTE**

For  $m = 8$ ,  $R_L = R_{H-7}$ . For  $m = 5$ ,  $R_L = R_{H-4}$ . For  $m = 4$ ,  $R_L = R_{H-3}$ . For  $m = 3$ ,  $R_L = R_{H-2}$ . For  $m = 2$ ,  $R_L = R_{H-1}$ . That is, for  $m$ ,  $R_L = R_{H-(m-1)} = R_{H-m+1}$ .

In order to drive this concept home, examples are given as illustrations in Table 1 & Table 2.

Example 1: Obtain the parameters of an appropriate grading system to suit the given variables shown in Table 1.

Example 2: Obtain the parameters of an appropriate grading system to suit the given variables shown in Table 2.

The equal class intervals concept (Table 1 & 2) is recommended for the following reasons:

- It is more logical and it suggests equal opportunity for every learner to have his/her scores fall into any of the score/mark ranges or groups/classes of score/marks so developed.
- It is easier to adopt it as a uniform standard for national use if it is required for all assessing institutions in a country to standardize their grading systems.
- It justifies the use of Arithmetic Progression (AP) in assigning grade-points to each group/class of score/marks or range. Though the common difference of any value can be used like the case of (8-point) scale in the example of Table 2, the common practice is to use a common difference of a unit (1).
- When the class intervals are unequal, it introduces complication in computation. Specifically the relationship between the number of assessment divisions, 'm' and the maximum value of grade point, 'n' is no longer governed by the minimum value of grade point, 'x' normally assigned to the least pass letter grade earned by the learner, below which the learner cannot graduate. That is, the validity test of a grading system is violated.

**3. Matching Pass Score/Mark Ranges with CGPA Ranges – Forward Integration**

The CGPA ranges ( $R_{CG}$ ) needs to be correlated with the pass score/mark ranges ( $R_{MR} \equiv R_H, R_{H-1}, \dots, R_L, R_F$ ) such that the intervals between each class of  $R_{MR}$  bear the same ratio with the intervals of  $R_{CG}$ .

That is, if  $R_{MR} \equiv (R_H, R_{H-1}, \dots, R_L)$ ;  $R_{CG} \equiv (R_{CGH}, R_{CGH-1}, \dots, x)$  and  $R_{MR} \equiv R_{CG}$ , then

$(R_H, R_{H-1}, \dots, R_L) \equiv (R_{CGH}, R_{CGH-1}, \dots, x)$  where,

$R_H = [M_L \text{ to } 100]$  with an interval =  $100 - M_L = I_H$ .

$R_{H-1} = [M_{L-1} \text{ to } (M_L - 1)]$  with an interval =  $(M_L - 1) - M_{L-1} = I_{H-1}$ .

$R_{H-2} = [M_{L-2} \text{ to } (M_{L-1} - 1)]$  with an interval =  $(M_{L-1} - 1) - M_{L-2} = I_{H-2}$ .

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$R_{H-(m-1)} = R_L = [(M_H+1) \text{ to } (M_{L-m} - 1)]$  with an interval =  $(M_{L-m} - 1) - (M_H+1) = I_L$ .

Similarly,

$R_{CGH} = [R_{CL} \text{ to } 5]$  with an interval =  $5 - R_{CL} = I_{CGH}$ .

$$R_{CGH-1} = [R_{CL-1} \text{ to } (R_{CL}-0.01)] \text{ with an interval} = (R_{CL} - 0.01) - R_{CL-1} = I_{CGH-1}.$$

$$R_{CGH-2} = [R_{CL-2} \text{ to } (R_{CL-1}-0.01)] \text{ with an interval} = (R_{CL-1} - 0.01) - R_{CL-2} = I_{CGH-2}.$$

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$$R_{CGH-(m-1)} = R_{CGL} = [(x \text{ to } (R_{CL-m}-0.01)] \text{ with an interval} = (R_{CL-m} - 0.01) - x = I_{CGL}.$$

The relationship between  $R_{MR}$  and  $R_{GP}$  is shown in Figure 1 where 100% is taken as being assigned grade point, 5 while in Figure 2 the lower limit of maximum obtainable pass score/mark range ( $R_H$ ) is assigned grade point, 5 for a (5-point) grading system. That is,  $M_L \equiv 5$  instead of  $100 \equiv 5$ . These two possibilities result in different CGPA ranges as shown and they are analysed as follows:

Figure 1: This configuration gives rise to five valid sections ( $I_{CGH}$ ,  $I_{CGH-1}$ ,  $I_{CGH-2}$ ,  $I_{CGH-3}$ , &  $I_{CGL}$ ) as required but it generates a CGPA value that is too low to be used as the lower limit for the First Class category. That is, a lower limit of  $R_{CGH}$  resulting from this configuration is too low to be acceptable (see Figure 1).

Figure 2 on the other hand produces only four sections ( $I_{CGH}$ ,  $I_{CGH-1}$ ,  $I_{CGH-2}$ , &  $I_{CGL}$ ) instead of the required five. As a result, it produces a CGPA of 5.00 at both the lower and upper limits of  $R_{CGH}$  (see Figure 2). Consequently, this configuration is considered invalid.

It is therefore, obvious that a score greater than  $M_L$  and less than 100% is required to be assigned the CGPA of 5. It is for this reason that a point between  $M_L$  and 100 (say point P with a score/mark  $M_p$  in Figure 3) is considered appropriate for equal chances for all students. That is,  $M_p \equiv 5$ .

By graphical construction of Figure 3 and linearization at the highest range,  $R_H = (M_L \text{ to } 100)$  a relationship between the CGPA and  $M_L$  is derived as follows:

If  $M_p \equiv 5$  (instead of  $M_L$  or 100% as shown in Figure 1 & Figure 2); then, for  $M_L\%$  marks, we have

$$CGPA \text{ for } M_L\% = CGPA (M_L\%) \equiv \frac{5M_L}{M_p} \dots \dots \dots (3.1)$$

Equation (3.1) is an equation of a straight line starting from the origin (that is, having y-intercept of zero) with CGPA on the y-axis, score/mark ( $M_L\%$ ) on the x-axis and a slope of  $5/M_p$ .

Alternatively the principle of similar triangles is employed to relate the class intervals of  $R_{MR}$  to those of  $R_{CG}$  as follows: In Figure 3,  $\Delta ALP$  is similar to  $\Delta AJK$ . That is,  $\frac{\Delta ALP}{\Delta AJK}$ .

$$\text{Hence, } \frac{AL}{AJ} = \frac{AP}{AK} = \frac{LP}{JK} \dots \dots \dots (3.2)$$

But  $AL = 5$ ,  $AP = M_p$  and  $AK = M_L$ . Therefore, equation (3.2) becomes

$$AJ = \frac{5M_L}{M_p} = R_{CL} \dots \dots \dots (3.3)$$

Therefore, for (n-point) grading system, equation (3.3) becomes

$$AJ = \frac{n * M_L}{M_P} = R_{CL} \dots \dots \dots (3.4)$$

Equation (3.3) or (3.4) is in the same form as equation (3.1) and it is the lower limit of  $I_{CGH}$ .

Hence,  $R_{CGH} = [AJ \text{ to } 5.00] = [R_{CL} \text{ to } 5.00]$  (see Figure 3)

The remaining range (RR) to be shared among the rest of  $R_{CG}$  is given by equation (3.5) (see Figure 4)

$$RR = \frac{5M_L}{M_P} - 1 \dots \dots \dots (3.5)$$

Equation (3.5) is shared in the ratio of  $I_L:I_{H-3}:I_{H-2}:I_{H-1}$  as follows:

Sum of ratio ( $S_R$ ) is given by

$$S_R = I_L + I_{H-3} + I_{H-2} + I_{H-1} \dots \dots \dots (3.6)$$

The proportional share (PS) of  $I_L$  is given by

$$PS(I_L) = I_{CGL} = \frac{I_L}{S_R} RR \dots \dots \dots (3.7)$$

The proportional share (PS) of  $I_{H-3}$  is given by

$$PS(I_{H-3}) = I_{CGH-3} = \frac{I_{H-3}}{S_R} RR \dots \dots \dots (3.8)$$

The proportional share (PS) of  $I_{H-2}$  is given by

$$PS(I_{H-2}) = I_{CGH-2} = \frac{I_{H-2}}{S_R} RR \dots \dots \dots (3.9)$$

The proportional share (PS) of  $I_{H-1}$  is given by

$$PS(I_{H-1}) = I_{CGH-1} = \frac{I_{H-1}}{S_R} RR \dots \dots \dots (3.10)$$

Equations (3.7) through (3.10) is plotted on the CGPA Axis in Figure 5 and the lower limits of each CGPA range are identified.

The lower limit of  $I_{CGH-1} = AJ - PS(I_{H-1}) = R_{CL} - PS(I_{H-1}) = R_{CL-1}$ ; (see Figure 5).

Hence,  $R_{CGH-1} = [R_{CL-1}] \text{ to } [R_{CL} - 0.1]$

The lower limit of  $I_{CGH-2} = [R_{CL-1} - PS(I_{H-2})] = R_{CL-2}$ .

Hence,  $R_{CGH-2} = [R_{CL-2}] \text{ to } [R_{CL-1} - 0.1]$

The lower limit of  $I_{CGH-3} = [R_{CL-2} - PS(I_{H-3})] = R_{CL-3}$ .

Hence,  $R_{CGH-3} = [R_{CL-3}] \text{ to } [R_{CL-2} - 0.1]$

The lower limit of  $I_{CGL} = [R_{CL-3} - PS(I_L)] \cong 1$ .

Hence,  $R_{CGL} = [1] \text{ to } [R_{CL-3} - 0.1]$

If the value of  $R_{CL}$  given by equation (3.4) or (3.5) is small, thereby specifying high percentage of  $R_{CG}$ , point P in Figure 3 can be made a Median ( $M_{Ref70}$ ), a quartile ( $M_q$ ) between points K & M. When this happens,  $M_P$  of equation (3.4) or (3.5) is replaced by  $M_{dh}$  or  $M_q$ . For different highest obtainable pass score/mark ranges,  $R_H$ , we have different values for the lower limits of the highest CGPA range as shown in Table 3 including a point described as Ref 70 ( $M_{Ref70}$ )

which is a point made reference to the popular CGPA range, (5.00 to 4.50) for First Class classification with a score/mark range of (70 to 100) in Nigerian universities.

Changing the value of  $M_L$  without the corresponding value of  $R_{CL}$  invalidates the evaluation exercise. This practice is very common as a result of lack of in-depth knowledge of the derivation of the assessment tool known as CGPA.

Hence, one of the very important requirements that must be decided upon aside the grading systems parameters or variables death with in Paper 2 is whether or not to use the median ( $M_{dH}$ ) or the quartile ( $M_q$ ) or the Ref 70 ( $M_{Ref70}$ ) in equation (3.3) or (3.4). That is, at what point in Figure 3 should point 'P' be located?

This principle is best illustrated by an example as follows:

Given the parameters of  $R_{MR}$  as follows and making point 'P' in Figure 3 the Ref 70-point between points 'K' and 'M' (ie  $M_p = M_{Ref70}$ ), for a (5-point) grading system, we have,

$R_H = (70 \text{ to } 100)$  with its median,  $M_{Ref70} = 77.78$ , letter grade of 'A', grade point of 5 and class interval of 30

$R_{H-1} = (65 \text{ to } 69)$  with its median, letter grade of 'B', grade point of 4 and class interval of 4

$R_{H-2} = (60 \text{ to } 64)$  with its median, letter grade of 'C', grade point of 3 and class interval of 4

$R_{H-3} = (55 \text{ to } 59)$  with its median, letter grade of 'D', grade point of 2 and class interval of 4

$R_L = (50 \text{ to } 54)$  with its median, letter grade of 'E', grade point of 1 and class interval of 4

Substituting the value of  $M_{Ref70} = 77.78$  in equation (3.4) or (3.5), we have

$$CGPA = \frac{5}{77.78} M_L \dots \dots \dots (3.11)$$

Therefore, the lower limit of  $R_{CGH}$  is calculated from equation (3.11) using  $M_L = 70$ .

That is,  $CGPA (70) = 350/77.78 = 4.50$

Thus, the lower limit of  $R_{CGH}$  is 4.50. Hence,  **$R_{CGH} = [4.50 \text{ to } 5.00]$** .

The remaining interval =  $4.50 - 1 = 3.50$  which is divided into either (i) 4 equal parts or (ii) at the same ratios of  $R_{MR}$ . That is, the ratio of 4:4:4:4 is used to share 3.50 CGPA.

Thus,

**(i) Equal proportions**

$3.50/4 = 0.88$  (to two decimal places).

Hence, the lower limit of  $R_{CGH-1}$  is  $[4.50 - 0.88] = 3.62$ . Therefore,  **$R_{CGH-1} = [3.62 \text{ to } 4.49]$** .

Similarly,

the lower limit of  $R_{CGH-2}$  is  $[3.62 - 0.88] = 2.74$ . Therefore,  **$R_{CGH-2} = [2.74 \text{ to } 3.61]$** .

the lower limit of  $R_{CGH-3}$  is  $[2.74 - 0.88] = 1.86$ . Therefore,  **$R_{CGH-3} = [1.86 \text{ to } 2.73]$** .

But, the lower limit of  $R_{CGH-4}$  is 1.00. Therefore,  **$R_{CGH-4} = [1.00-1.75]$** .

**(ii) Ratio of  $R_{MR}$**

Sum of the ratio = 4+4+4+4 = 16. Therefore, the shared CGPA is calculated as follows:

$$\text{For } R_{H-1}, 4 * 3.50 / 16 = 0.88.$$

Hence, the lower limit of  $R_{CGH-1}$  is  $[4.50 - 0.88] = 3.62$ . Therefore,  $R_{CGH-1} = [3.62 \text{ to } 4.49]$ .

$$\text{For } R_{H-2}, 4 * 3.50 / 16 = 0.88$$

Hence, the lower limit of  $R_{CGH-2}$  is  $[3.62 - 0.88] = 2.74$ . Therefore,  $R_{CGH-2} = [2.74 \text{ to } 3.61]$ .

$$\text{For } R_{H-3}, 4 * 3.50 / 16 = 0.88$$

Hence, the lower limit of  $R_{CGH-3}$  is  $[2.74 - 0.88] = 1.86$ . Therefore,  $R_{CGH-3} = [1.86-2.73]$ .

$$\text{For } R_{H-4}, 4 * 3.50 / 16 = 0.88$$

But, the lower limit of  $R_{CGH-4}$  is 1.00. Therefore,  $R_{CGH-4} = [1.00-1.75]$ .

The result of the above example is presented in summary in Table 4. The two methods agree and produce the same results because the score/mark range intervals form perfect equal values. An **Excel Calculator** is developed and used to calculate CGPA ranges from given score/mark ranges in the same way the above example is carried out. This is shown on Tables 5, 6 & 7.

#### 4. Matching CGPA Ranges with Pass Score/Mark Ranges – Backward Integration

*Where fixed CGPA ranges are associated with classes of degree, the corresponding pass mark ranges are required to evaluate the students. The following example is worked out to illustrate how other values could be obtained from any given degree classification:*

##### **Example-1**

For a classification shown below, calculate the corresponding mark ranges :

Given CGPA ranges as

$$4.50 - 5.00 = 0.50$$

$$3.50 - 4.49 = 0.99 \approx 1$$

$$2.50 - 3.49 = 0.99 \approx 1$$

$$1.50 - 2.49 = 0.99 \approx 1$$

$$1.00 - 1.49 = 0.49 \approx 0.5$$

##### **Solution**

$$GP_L = \frac{5M_L}{M_p} \dots \dots \dots (4.1)$$

For  $M_L = 70$ , &  $GP_L = 4.5$ , from equation (1),  $M_p = 77.78$

$$S_R = I_L + I_{H-3} + I_{H-2} + I_{H-1} \dots \dots \dots (4.2)$$

$$RR = GP_L - 1 = 4.5 - 1 = 3.5$$

$$P_{H-1} = \frac{I_{H-1}}{S_R} RR \dots \dots \dots (4.3a)$$

From equation (4.3a),

$$\frac{1}{3.5} = 0.286 = \frac{I_{H-1}}{S_R}$$

Hence,  $I_{H-1} = 0.286S_R$  ..... (4.4a)

$$P_{H-2} = \frac{I_{H-2}}{S_R} RR \dots \dots \dots (4.3b)$$

From equation (4.3b),

$$\frac{1}{3.5} = 0.286 = \frac{I_{H-2}}{S_R}$$

Hence,  $I_{H-2} = 0.286S_R$  ..... (4.4b)

$$P_{H-3} = \frac{I_{H-3}}{S_R} RR \dots \dots \dots (4.3c)$$

From equation (4.3c),

$$\frac{1}{3.5} = 0.286 = \frac{I_{H-3}}{S_R}$$

Hence,  $I_{H-3} = 0.286S_R$  ..... (4.4c)

$$P_L = \frac{I_L}{S_R} RR \dots \dots \dots (4.3d)$$

From equation (4.3d),

$$\frac{0.5}{3.5} = 0.143 = \frac{I_{H-1}}{S_R}$$

Hence,  $I_L = 0.143S_R$  ..... (4.4d)

From equations (4.4a, 4.4b & 4.4c),  $I_{H-1} = I_{H-2} = I_{H-3} = 0.286S_R = x$  ..... (4.5)

Putting equations (4.2) & (4.5) into equation (4.4d), we have

$$I_L = (I_L + I_{H-3} + I_{H-2} + I_{H-1})0.143 = (I_L + 3x)0.143$$

$0.857I_L = 0.429x$ . Therefore,

$$\frac{0.429}{0.857} = 0.5 = \frac{I_L}{x}$$

That is,

$$2I_L = x = I_{H-3} \dots \dots \dots (4.6)$$

But from the diagram,  $I_{H-3} = [(M_{L-2}-1) - (M_{L-3})]$  ..... (4.6a)

But from the diagram,  $I_{H-2} = [(M_{L-1}-1) - (M_{L-2})]$  ..... (4.6b)

But from the diagram,  $I_{H-1} = [(M_L-1) - (M_{L-1})] = [69 - (M_{L-1})]$  ..... (4.6c) for  $M_L = 70$

But from the diagram,  $I_L = [(M_{L-3}-1) - (M_H + 1)] = [(M_{L-3} - 1) - 40]$  ..... (4.6d) for  $M_H + 1 = 40$

Substituting equations (4.6a) & (4.6d) into equation (4.6), we have

$$\text{For } (M_H + 1) = 40: 2[(M_{L-3} - 1) - 40] = [(M_{L-2}-1) - (M_{L-3})]. \text{ For } (M_H + 1) = 50: 2[(M_{L-3} - 1) - 50] = [(M_{L-2}-1) - (M_{L-3})].$$



$$-M_{L-2} + 3M_{L-3} = 81 \dots\dots\dots (4.7a) \text{ and } -M_{L-2} + 3M_{L-3} = 101 \dots\dots\dots (4.7b)$$

From equation (4.5),  $I_{H-1} = I_{H-2} \dots\dots\dots (4.8)$

Substituting equations (4.6b) & (4.6c) into equation (4.8), we have

$$[69 - (M_{L-1})] = [(M_{L-1}-1) - (M_{L-2})]$$

$$2M_{L-1} - M_{L-2} = 70 \dots\dots\dots (4.9)$$

From equation (4.5),  $I_{H-2} = I_{H-3} \dots\dots\dots (4.10)$

Substituting equations (4.6a) & (4.6b) into equation (4.10), we have

$$[(M_{L-1}-1) - (M_{L-2})] = [(M_{L-2}-1) - (M_{L-3})]$$

$$M_{L-1} - 2M_{L-2} + M_{L-3} = 0 \dots\dots\dots (4.11)$$

Solving equations (4.7a), (4.7b), (4.9) & (4.11) simultaneously, we have

$$\begin{bmatrix} M_{L-1} \\ M_{L-2} \\ M_{L-3} \end{bmatrix} \begin{bmatrix} 0 & -1 & 3 \\ 2 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 81 \\ 70 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} M_{L-1} \\ M_{L-2} \\ M_{L-3} \end{bmatrix} \begin{bmatrix} 0 & -1 & 3 \\ 2 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 101 \\ 70 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0 & -1 & 3 \\ 2 & -1 & 0 \\ 1 & -2 & 1 \end{vmatrix} = -(-1)[2] + (3)[-4 + 1] = 2 - 9 = -7$$

**For  $M_H + 1 = 40$**

$$\Delta 1 = \begin{vmatrix} 81 & -1 & 3 \\ 70 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 81[-1] - (-1)[70] + (3)[-140] = -81 + 70 - 420 = -431$$

**$M_{L-1} = 431/7 = 61.57$**

**For  $M_H + 1 = 50$**

$$\Delta 1 = \begin{vmatrix} 101 & -1 & 3 \\ 70 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 101[-1] - (-1)[70] + (3)[-140] = -101 + 70 - 420 = -451$$

**$M_{L-1} = 451/7 = 64.43$**

**For  $M_H + 1 = 40$**

$$\Delta 2 = \begin{vmatrix} 0 & 81 & 3 \\ 2 & 70 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -81[2] + 3[-70] = -162 - 210 = -372$$

**$M_{L-2} = 372/7 = 53.14$**

**For  $M_H + 1 = 50$**

$$\Delta 2 = \begin{vmatrix} 0 & 101 & 3 \\ 2 & 70 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -101[2] + 3[-70] = -202 - 210 = -412$$

**$M_{L-2} = 412/7 = 58.86$**

**For  $M_H + 1 = 40$**

$$\Delta 3 = \begin{vmatrix} 0 & -1 & 81 \\ 2 & -1 & 70 \\ 1 & -2 & 0 \end{vmatrix} = -(-1)[-70] + 81[-4 + 1] = -70 - 243 = -313$$

$$\underline{M_{L,3} = 313/7 = 44.71}$$

For  $M_H + 1 = 50$

$$\Delta 3 = \begin{vmatrix} 0 & -1 & 101 \\ 2 & -1 & 70 \\ 1 & -2 & 0 \end{vmatrix} = -(-1)[-70] + 101[-4 + 1] = -70 - 303 = -373$$

$$\underline{M_{L,3} = 373/7 = 53.29}$$

The summary of this analysis is given in Table 8 and a similar Excel Calculator is developed to automatically calculate the values of the score/mark ranges (see Tables 9, & 10).

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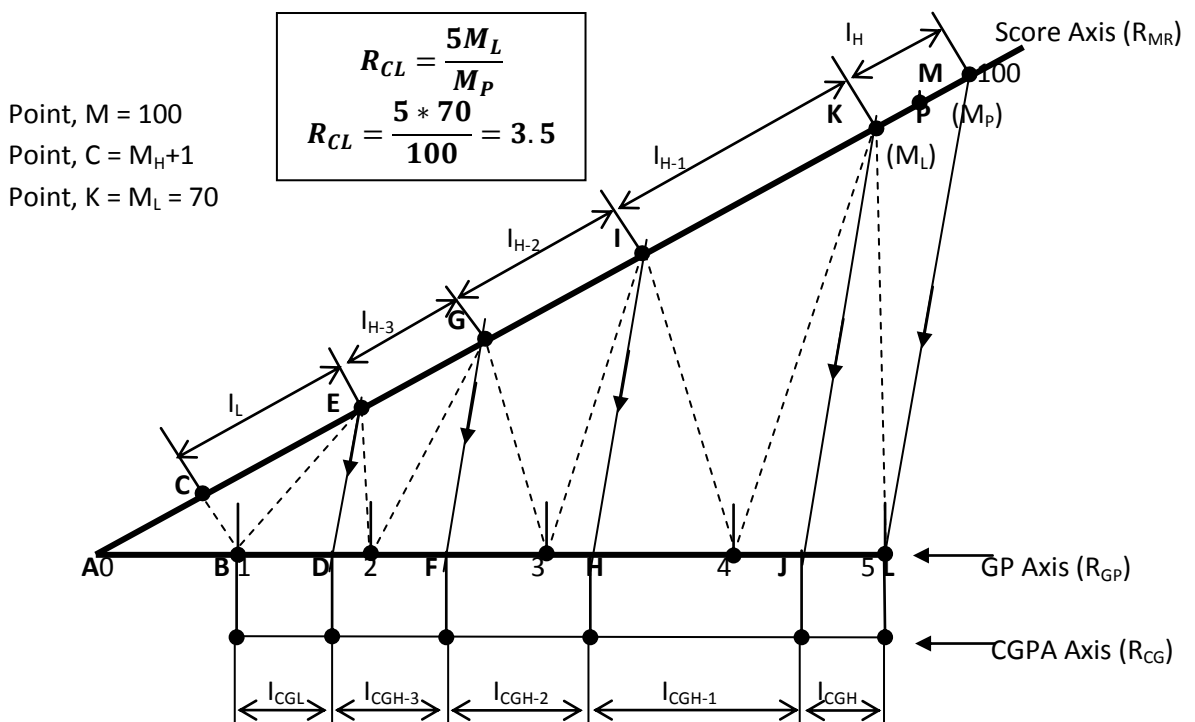


Figure 1: Triangular (Model) Relationship amongst  $R_{MR}$ ,  $R_{GP}$  &  $R_{CG}$  ( $100 \equiv 5$ )

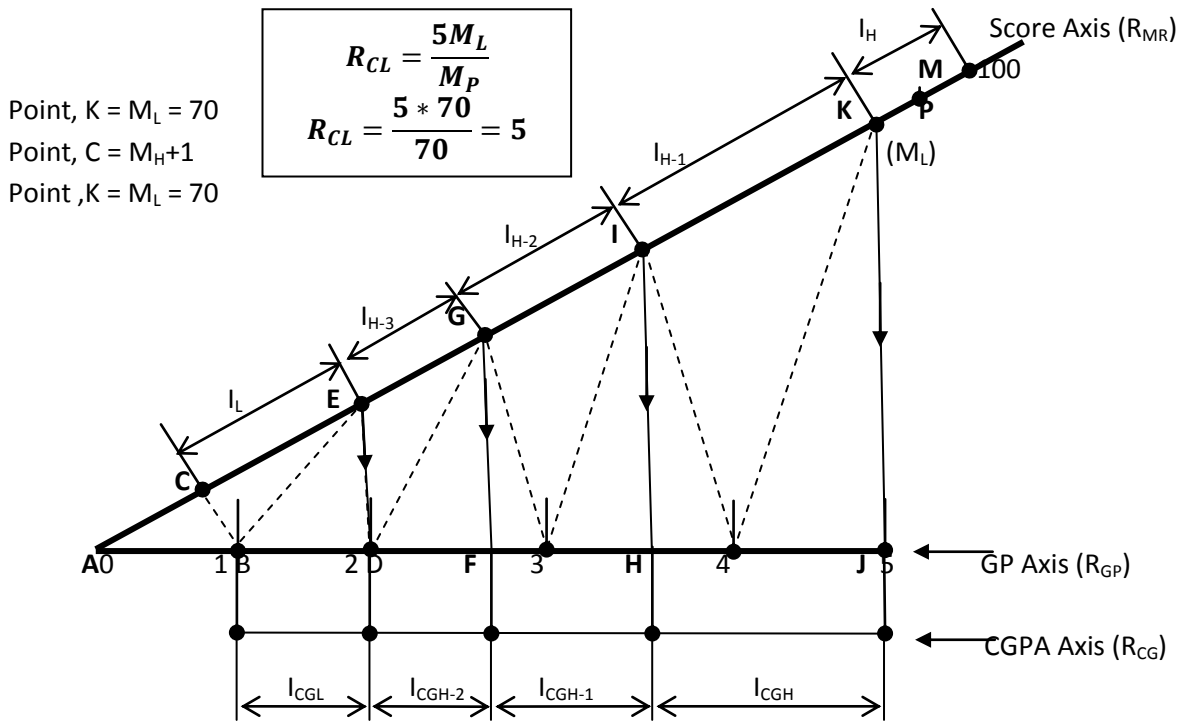


Figure 2: Triangular (Model) Relationship amongst  $R_{MR}$ ,  $R_{GP}$  &  $R_{CG}$  ( $M_L \equiv 5$ ).

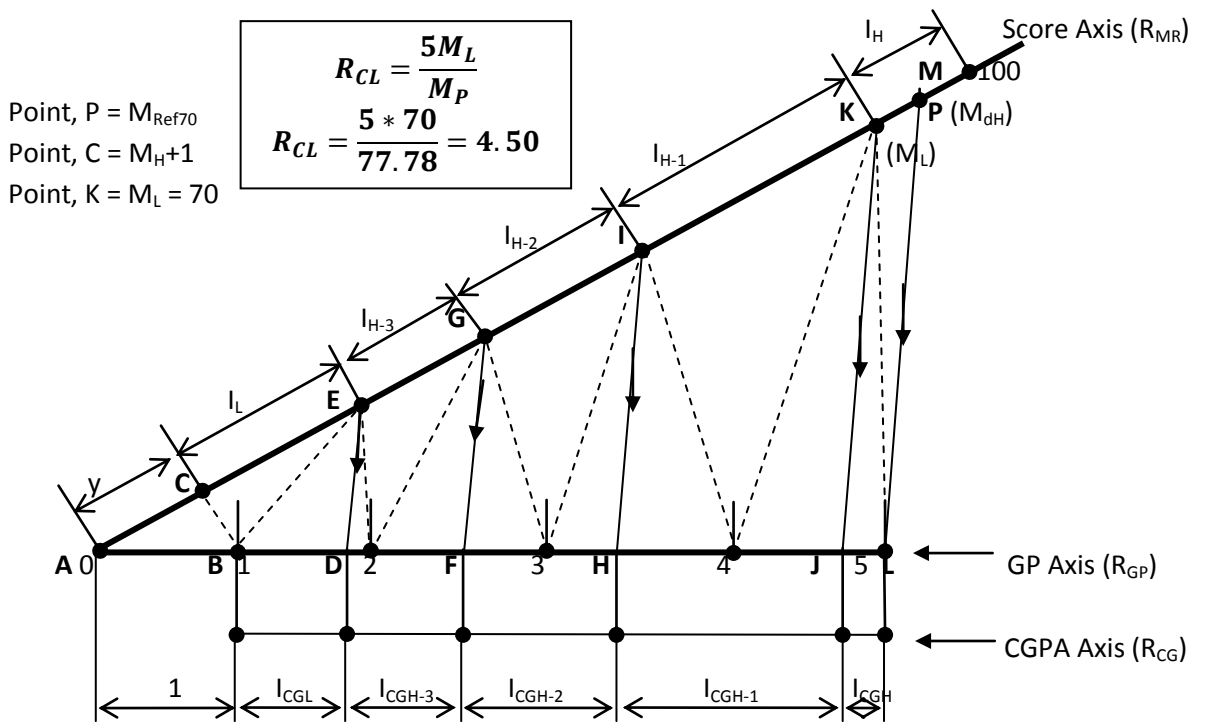


Figure 3: Triangular (Model) Relationship amongst  $R_{MR}$ ,  $R_{GP}$  &  $R_{CG}$  ( $M_P \equiv 5$ ).

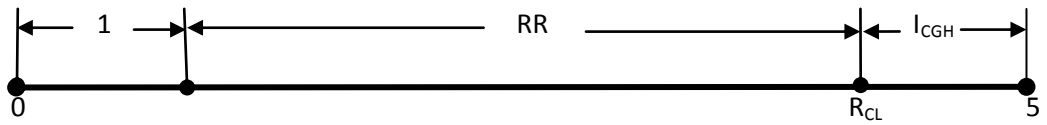
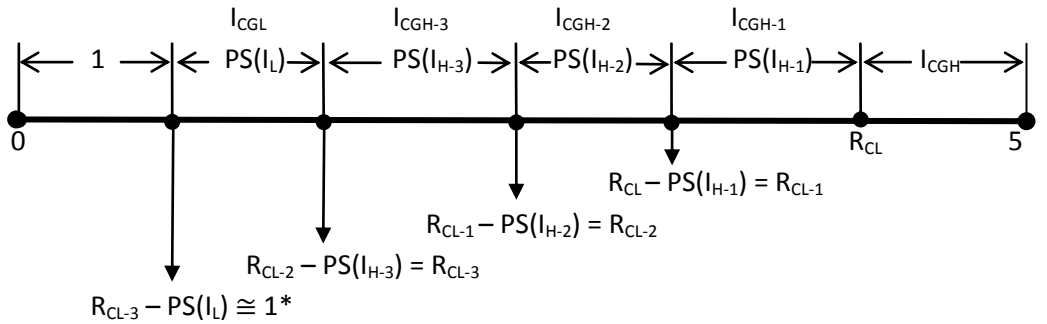


Figure 4: CGPA Axis Showing equation (3.4) & (3.5).



\*due to rounding errors/approximations, this equality may not be exact.

Figure 5: CGPA Axis Showing equation (3.7) through (3.10).

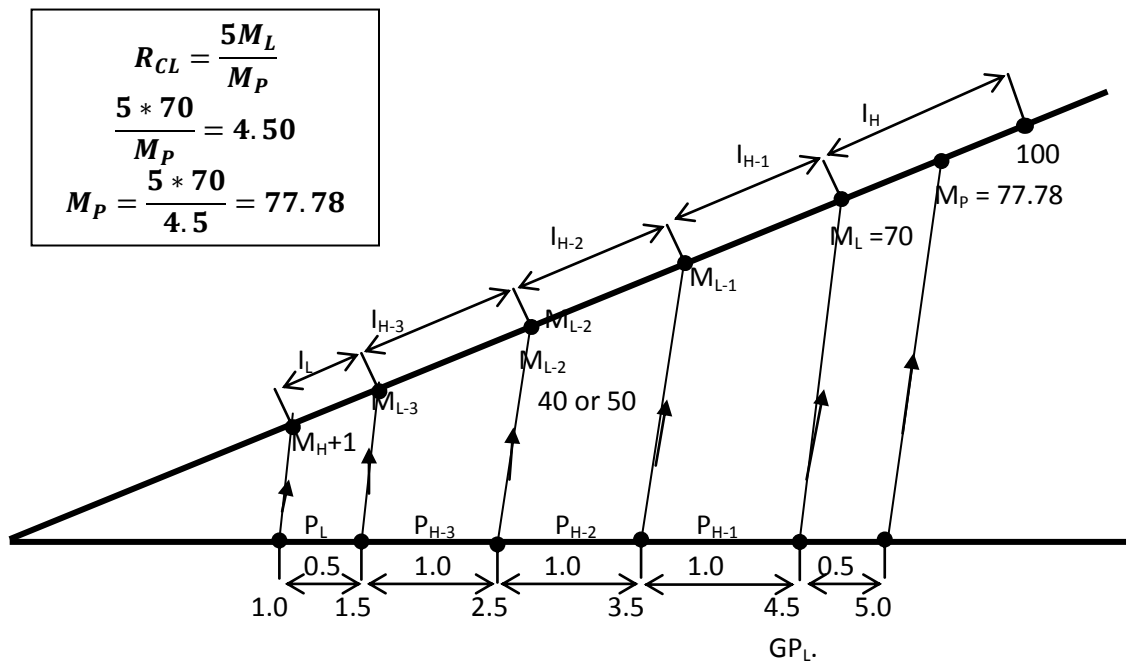


Figure 6: Relationship Between CGPAs and Score/Mark Ranges.

**Table 1: Development of (5-Point) 1,5,70,40 for a Non-Graded Fail Grading System Using Equal Score/Mark Range Intervals**

Fundamental Variables (Arrived at by Policy)		Derived Variables	
Variables	Value	Variables	Values
'n'	5	$R_H = M_L$ to 100	70-100
'x'	1	$R_F = 0$ to $M_H$	0-39
$M_L$	70	$R_{PR} = M_L - (M_H+1)$	$70-40 = 30$
$M_H$	39	Equal Intervals = $R_{RR}/(m-1)$	$30/4 = 7$ (least integer) or 8 (largest integer)
'm'=5/1	5	$M_{L-1} = M_L - R_{RR}/(m-1)$	$70 - 7 = 63$ or $70 - 8 = 62$
		$R_{H-1} = [M_{L-1}$ to $(M_L-1)]$	[63 to 69] or [62 to 69]
		$M_{H-2} = M_{L-1} - R_{RR}/m-1$	$63 - 7 = 56$ or $62 - 8 = 54$
		$R_{H-2} = [M_{L-2}$ to $(M_{H-1}-1)]$	[56 to 62] or [54 to 61]
		$M_{H-3} = M_{L-2} - R_{RR}/m-1$	$56 - 7 = 49$ or $54 - 8 = 46$
		$R_{H-3} = [M_{L-3}$ to $(M_{H-2}-1)]$	[49 to 55] or [46 to 53]
		$M_{H-4} = M_{L-3} - R_{RR}/m-1$	$49 - 7 = 42$ or $46 - 8 = 36$
		$R_{H-4} = [M_{L-4}$ to $(M_{H-3}-1)] = R_L$	[42 to 48] or [36 to 45]
<b>THE RESULTS:</b>			
Because, the least pass score/mark range, $R_L$ is less than 40 when the largest integer, 8 is employed, this option is rejected. Hence,			
$R_H = (70 \text{ to } 100) = 30$ assigned letter grade 'H or A' and grade point 5			
$R_{H-1} = (63 \text{ to } 69) = 6$ assigned letter grade 'H-1 or B' and grade point 5-1 = 4			
$R_{H-2} = (56 \text{ to } 62) = 6$ assigned letter grade 'H-2 or C' and grade point 5-2 = 3			
$R_{H-3} = (49 \text{ to } 55) = 6$ assigned letter grade 'H-3 or D' and grade point 5-3 = 2			
$R_{H-4} = R_L = (40 \text{ to } 48) = 8$ assigned letter grade 'L or E' and grade point 5-4 = 1			
$R_{H-4} = R_F = (0-39) = 39$ assigned letter grade 'F' and grade point 5-5 = 0			

**Table 2: Development of (8-Point)2,4,80,50 for a Non-Graded Fail Grading System Using Equal Mark Range Intervals**

Fundamental Variables (Arrived at by Policy)		Derived Variables	
Variables	Value	Variables	Values
'n'	8	$R_H = M_L$ to 100	80-100
'x'	2	$R_F = 0$ to $M_H$	0-49
$M_L$	80	$R_{PR} = M_L - (M_H+1)$	$80-50 = 30$
$M_H$	49	Equal Intervals = $R_{RR}/(m-1)$	$30/3 = 10$
'm'=8/2	4	$M_{H-1} = M_L - R_{RR}/(m-1)$	$80 - 10 = 70$
		$R_{H-1} = [M_{L-1}$ to $(M_L-1)]$	[70 to 79]
		$M_{H-2} = M_{L-1} - R_{RR}/m-1$	$70 - 10 = 60$
		$R_{H-2} = [M_{L-2}$ to $(M_{H-1}-1)]$	[60 to 69]
		$M_{H-3} = M_{L-2} - R_{RR}/m-1$	$60 - 10 = 50$
		$R_{H-3} = [M_{L-3}$ to $(M_{H-2}-1)] = R_L$	[50 to 59]

**THE RESULTS:**  
 (8-point)2,4,80,50 Grading System with the following Mark ranges  
 $R_H = (80-100) = 20$  assigned letter grade 'H' and grade point 8-0 = 8  
 $R_{H-1} = (70-79) = 9$  assigned letter grade 'H-1' and grade point 8-2 = 6  
 $R_{H-2} = (60-69) = 9$  assigned letter grade 'H-2' and grade point 8-4 = 4  
 $R_{H-3} = R_L = (50-59) = 9$  assigned letter grade 'L' and grade point 8-6 = 2  
 $R_{H-4} = R_F = (0-49) = 49$  assigned letter grade 'F' and grade point 8-8 = 0

**Table 3: Variation of  $R_{CL}$  as  $M_L$  and  $M_p$  vary in a (5-point) Grading System– Graph on page 18c**

$R_H$	$R_{CL}$					
	$R_{CL} (M_{dH})$	% of $R_{CG}$ $(5-R_{CL})100/4$	$R_{CL} (M_q)$	% of $R_{CG}$ $(5-R_{CL})100/4$	$R_{CL} (Ref 70)$	% of $R_{CG}$ $(5-R_{CL})100/4$
70 to 100	4.12 (85.00)	22.0%	4.52 (77.50)	12.0%	4.50 (77.78)	12.5%
75 to 100	4.29 (87.50)	17.8%	4.62 (81.25)	9.5%	4.60 (81.48)	10.0%
80 to 100	4.44 (90.00)	14.0%	4.71 (85.00)	7.3%	4.70 (85.19)	7.5%
85 to 100	4.59 (92.50)	10.3%	4.79 (88.75)	5.3%	4.78 (88.89)	5.5%
90 to 100	4.74 (95.00)	6.5%	4.86 (92.50)	3.5%	4.86 (92.59)	3.5%
95 to 100	4.87 (97.50)	3.3%	4.94 (96.25)	1.5%	4.93 (96.30)	1.75

$M_p$  = Value of score/mark at point P in Figure 3  
 $M_{dH}$  = Median between points K & M in Figure 3  
 $M_q$  = Quartile between points K & M in Figure 3.

$R_{CL} (M_p) = R_{CL} (M_{dH})$  when  $M_p = M_{dH}$   
 $R_{CL} (M_p) = R_{CL} (M_q)$  when  $M_p = M_q$   
 $R_{CL} (M_p) = R_{CL} (M_{Ref70})$  when  $M_p = M_{Ref70}$

$$R_{CL} (M_p) = \frac{5M_L}{M_p}$$

**Table 4: CGPA Ranges calculated from Score/Mark Ranges in a (5-point) Grading System**

L/Grade GP (n)	R <sub>H</sub>	R <sub>CL</sub>			Remarks
		R <sub>CL</sub> (M <sub>dH</sub> )	% of R <sub>CG</sub> (5-R <sub>CL</sub> )100/4	R <sub>CL</sub> (M <sub>q</sub> )	
A (5)	70 to 100	30	4.50 to 5.00	0.50	
B (4)	65 to 69	4	3.62 to 4.49	0.87	Equal intervals in both CGPA and Score Ranges
C (3)	60 to 64	4	2.74 to 3.61	0.87	Equal intervals in both CGPA and Score Ranges
D (2)	55 to 59	4	1.86 to 2.73	0.87	Equal intervals in both CGPA and Score Ranges
E (1)	50 to 54	4	1.00 to 1.75	0.75	Equal intervals in both CGPA and Score Ranges
F (0)	0 to 49				

**Table 5: Excel Calculator: CGPA Ranges calculated from Score/Mark Ranges Using Median Point, M<sub>dH</sub>**

$[NGF/GS]_{F=0} = \{[(5 - point)0, 1], [P(0, 1, 5)]\}, [40, 70]$ (POLICY PARAMETERS):- M <sub>L</sub> = 70, (M <sub>H</sub> +1) = 40, Max Grade Point = 5 and the Least Pass Grade Point = 1											
L/G	m	ML	MH	MdH	INT	GPL	GPH	SH	RR	Class	GP (n)
A	1	70	100	85	30	4.12	5.00		3.12	1st U	5
B	2	62.5	69.0		6.5	3.34	4.11	0.7794118	d	1st L	4
C	3	55.0	61.5		6.5	2.56	3.33	0.7794118	1	2nd U	3
D	4	47.5	54.0		6.5	1.78	2.55	0.7794118		2nd L	2
E	5	40	46.5		6.5	1.00	1.77	0.7794118		3 <sup>rd</sup>	1
F	5	0	39							Fail	0
		ΣCI (Pass Only)			26						

**Table 6: Excel Calculator: CGPA Ranges calculated from Score/Mark Ranges Using Ref &70-Point, M<sub>Ref70</sub>**

$[NGF/GS]_{F=0} = \{[(5 - point)0, 1], [P(0, 1, 5)]\}, [40, 70]$ (POLICY PARAMETERS):- M <sub>L</sub> = 70, (M <sub>H</sub> +1) = 40, Max Grade Point = 5 and the Least Pass Grade Point = 1											
L/G	m	ML	MH	Ref 70	INT	GPL	GPH	SH	RR	Class	GP (n)
A	1	70	100	77.78	30	4.50	5.00		3.50	1st U	5
B	2	62.5	69.0		6.5	3.62	4.49	0.8749679	d	1st L	4
C	3	55.0	61.5		6.5	2.75	3.61	0.8749679	1	2nd U	3
D	4	47.5	54.0		6.5	1.87	2.74	0.8749679		2nd L	2
E	5	40	46.5		6.5	1.00	1.86	0.8749679		3 <sup>rd</sup>	1
F	5	0	39							Fail	0
		ΣCI (Pass Only)			26						



**Table 7: Excel Calculator: CGPA Ranges calculated from Score/Mark Ranges Using Quartile-Point,  $M_q$**

$[NGF/GS]_{F=0} = \{[(5 - point)0, 1], [P(0, 1, 5)]\}, [40, 70]$ (POLICY PARAMETERS):- $M_L = 70, (M_H + 1) = 40$ , Max Grade Point = 5 and the Least Pass Grade Point = 1											
L/G	m	ML	MH	$M_q$	INT	GPL	GPH	SH	RR	Class	GP (n)
A	1	70	100	77.5	30	4.52	5.00		3.52	1st U	5
B	2	62.5	69.0		6.5	3.64	4.51	0.8790323	d	1st L	4
C	3	55.0	61.5		6.5	2.76	3.63	0.8790323	1	2nd U	3
D	4	47.5	54.0		6.5	1.88	2.75	0.8790323		2nd L	2
E	5	40	46.5		6.5	1.00	1.87	0.8790323		3 <sup>rd</sup>	1
F	5	0	39							Fail	0
		$\Sigma$ CI (Pass Only)		26							

**Table 8 : Calculation of Mark Ranges from Fixed CGPA Ranges (70/40 Or 50)**

FIXED CGPA RANGES		CALCULATED MARK RANGES			
RANGE (GIVEN)	INTERVAL	$M_p = 77.78, M_L = 70 \text{ \& } M_H + 1 = 40$		$M_p = 77.78, M_L = 70 \text{ \& } M_H + 1 = 50$	
		RANGE	INTERVAL	RANGE	INTERVAL
4.50 to 5.00	0.50	70.00 to 100	30.00	70.00 to 100	30.00
3.50 to 4.49	0.99	61.58 to 69.00	7.42	64.43 to 69.00	4.57
2.50 to 3.49	0.99	53.14 to 60.58	7.44	58.86 to 63.43	5.57
1.50 to 2.49	0.99	44.71 to 52.14	7.43	53.29 to 57.86	4.57
1.00 to 1.49	0.49	40.00 to 43.71	3.71	50.00 to 52.29	2.29

**Characteristic Equation:  $GP_L = \frac{5M_L}{M_P}$**

**Table 9 : Excel Calculator from Mark Ranges to CGPA Ranges and Vice-Visa**

CALCULATED DETAILS – BACKWARD INTEGRATION								
ML	MH	Ref 70	INT	GPL	GPH	SH	RR	
80	100	85.19	20	4.70	5.00		3.70	
70	79		9	3.77	4.69	0.9238926		
60	69		9	2.85	3.76	0.9238926	Figures on black background are given	
50	59		9	1.92	2.84	0.9238926		
40	49		9	1.00	1.91	0.9238926		
0	39					0		
$\sum$ CI			36					
<p>Note the zero difference between calculated values &amp; given value of Score Ranges. This is due to equal intervals which automatically eliminate any possible rounding error.</p>								
CALCULATED DETAILS – BACKWARD INTEGRATION								
RCL	RCH	Ref 70	CI	ML	MH	SH	RR	M <sub>p</sub>
4.70	5.00	4.70	0.30	80	100		40	85.19
3.77	4.69		0.92	70	79	10.019		
2.85	3.76		0.91	60	69	9.957306	Figures on black background are given	
1.92	2.84		0.92	50	59	10.066727		
1.00	1.91		0.91	40	49	9.957306		
$\sum$ CI			3.66					
FAIL RANGE				0	39			

**Table 10: Excel Calculator from CGPA Ranges to Mark Ranges and Vice-Visa**

CALCULATED DETAILS – BACKWARD INTEGRATION								
		Ref 70						
<b>70</b>	<b>100</b>	77.78	30	<b>4.50</b>	<b>5.00</b>			3.50
<b>60</b>	<b>69</b>		9	<b>3.29</b>	<b>4.49</b>	1.211494		
<b>50</b>	<b>59</b>		9	<b>2.08</b>	<b>3.28</b>	1.211494	Figures on black background are given	
<b>45</b>	<b>49</b>		4	<b>1.54</b>	<b>2.07</b>	0.5384418		
<b>40</b>	<b>44</b>		4	<b>1.00</b>	<b>1.53</b>	0.5384418		
<b>0</b>	<b>39</b>					0		
$\sum$ CI			26					
<p>Note the slight difference between calculated value, (49 to 59) &amp; given value, (50 to 59) of Score Ranges. This is due to rounding errors and unequal intervals.</p>								
CALCULATED DETAILS – BACKWARD INTEGRATION								
RCL	RCH	Ref 70	CI	ML	MH	SH	RR	M <sub>p</sub>
<b>4.50</b>	<b>5.00</b>	4.50	0.50	<b>70</b>	<b>100</b>		30	<b>77.78</b>
<b>3.29</b>	<b>4.49</b>		1.20	60	<b>69</b>	10.404		
<b>2.08</b>	<b>3.28</b>		1.2	49	<b>59</b>	10.404837	Figures on black background are given	
<b>1.54</b>	<b>2.07</b>		0.53	45	<b>48</b>	4.5954696		
<b>1.00</b>	<b>1.53</b>		0.53	40	<b>44</b>	4.5954696		
$\sum$ CI			3.46					
FAIL RANGE				0	<b>39</b>			