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Bridging the Divide Between The Modulus Of Elasticity Obtained From Direct Test And Flexural Test Methods: Paper 1 Central Point Load

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Abstract

The modulus of elasticity E is a fundamental material constant which is usually determined from experimentation and is an index of the stiffness of the material. The Direct Tensile or compressive test method is usually used for the determination of this constant, while the flexural test method is an indirect test method. Three samples of each of Glass, Mahogany timber, Massonia timber, Cotton timber, Iroko timber, Y16 steel, Y12 steel, Y10 steel and R12 steel were tested in flexure with a central point load in a simply supported arrangement. The respective deflections were recorded via dial gauges for each incremental load and repeated when unloading. The average load and deflection are back substituted into the deflection equation of a point loaded simply supported beam and the circular beam theory equation from where E_1 and E_2 are obtained respectively. For all the materials tested (Glass, Timber and Steel), It was discovered that the ratio E_2/E_1 was 1.5. This raises the question of which of the two equations is correct. However the surprising constant of $E_2/E_1 = 1.5$ informed the need to get the average of E_1 and E_2 as the required E. This way the average E so obtained compares with quoted E from the direct Tensile or compressive test methods for the various Engineering materials tested. Therefore within the limits of experimental errors, $E_{Direct} = 1.25\ E_{Flexure.}$

Keywords: Young's Modulus, Tensile Test, Flexural Test, Deflection, Moment, Circular Bending Theory

1.0 Introduction

Young's Modulus, is a measure of how a material or structure will deform and strain when placed under stress. Materials deform differently when loads and stresses are applied, and the relationship between stress and strain typically varies. The ability of materials to resist or transmit stress is important, and this property is often used to determine if a particular material is suitable for a specific purpose. The young's modulus is often determined in a laboratory, using an experimental technique known as *tensile testing*, which is usually conducted on a sample of material with a specific shape and dimensions. This will require machining of the sample to the required shape and size. There are a variety of testing devices available that apply very precise loads and stresses to the sample, and accurately measure and record any resulting strain in the material. Modulus of elasticity is based on Hooke's Law of elasticity and can be calculated by dividing the stress by the strain. For many materials at low levels of stress and under tension, the stress and strain are proportional — meaning they increase and decrease in a constant way, relative to each other. Deformation of a material that occurs when the stress and strain behave proportionally is known as *elastic deformation* or *elastic strain*. Modulus of elasticity describes the relationship between stress and strain when under these conditions. The modulus of elasticity is known for a wide variety of structural materials, including metals, wood, glass, rubber, ceramics, concrete, and plastics.

Driven by the incessant failure and collapses of various civil infrastructure in the country (Nigeria) and its direct adverse effect on the safety of lives and properties and on the economy of the country, there arose the need to compliment various laboratory testing of materials with field (insitu.) testing of materials, to ensure that design standards and material quality are not compromised. To achieve this simple field experimental procedure needs to be developed and a correlation between field and laboratory results established. The Young's modulus is one of such material constant and is given attention in this paper. Several researchers have shown that quoted results which are within 20% error are too general to be used if accuracy is desired. This means and it is true that every sample of a given material will produce a different result of young modulus and to this end, field test with samples tested in their original shapes and sizes need to be tested in the form, shape and size they will be used. To achieve this, a flexural test for simply supported beam of the material loaded and their central displacement recorded will suffix. The displacement so obtained when back substituted in the deflection equation for the load type the value of the young's modulus E_1 will be obtained. This same value of displacement will be back substituted in the equation of circular bending theory and another value of young's modulus E_2 will be obtained. The value of E_2 is usually greater than that of E_1.(1)(6)(8) & (9). The result patterns of this proposed method are in agreement with those of these researchers i.e. they are usually smaller than those obtained in direct tensile testing. The reason always put forward is that the effect of shear deformation is not considered in the deflection equation and if the correction due to shear deformation is made, then the E_{direct} will be obtained. The correction factor of 27% has been obtained and quoted in (10) for a central point loaded beam i.e $E_{direct}=1.27E_1$. They argued that if the simply supported beam is uniformly loaded using under four 4 points, the influence of shear deflection is not very significant. The novelty in the method now presented is the determination of E_2. Here the method suggests that the values of E_1 and E_2 need to be the same for one to conveniently say the value of E_{direct} has been obtained. Where they are not of the same value, a simple average of E_1 and E_2 gives the value of E_{direct}, this way correcting the effect of shear deformation without further expansion of the deflection equation or experimentation. This work therefore sets to re-validate the value of the correction factor of 27% or otherwise for a centrally point loaded simply supported beam.

2.0 Material

The materials tested are Glass (obtained from Louvre blades), Timber (Mahogany, Massonia, Cotton & Iroko), steel (Y16, Y12, Y10 & R12). They were tested in their finished state. However the timber samples were cut out into flat shapes and sizes. These are commonly used materials in construction and are commercially available. Timber is used in roofs, partioning, trestles & wooden bridges, columns, beams, boats etc. The use of glass and steel are all encompassing. The measured parameters are indicated in the tables 1 to 10 below.

3.0 Experimental procedure and theoretical formulation.

A wide variety of experimental techniques have been used to determine the deflections of beams. The beams could be cantilevered, Clamped or simply supported. In this work the direct testing results are quoted from existing works. In this direct test method

$$E = \tan \Theta = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{Stress}{Strain} .$$

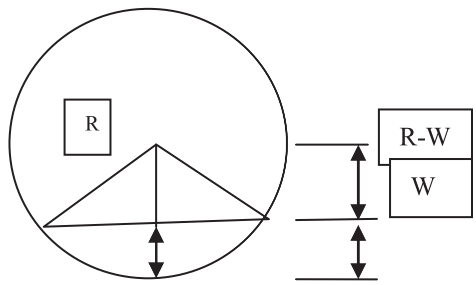
Here a lot of load is needed to put the sample in tension or compression. For many common structural materials, the strain is an essentially linear function of the stress over the range of stresses normally used in load-carrying members. In this experiment, the flexural load- displacement diagram for each of the material will be obtained by loading a simply supported beam. With the dimensions of the beam known, the young's modulus can be calculated quite accurately for a given central point load (p) from;

$$W = \frac{pl^3}{48EI} \rightarrow E_1 = \frac{pl^3}{48WI} .$$

$$1/R = M/EI \rightarrow E_2 = \frac{(PLR)}{4I}$$

where

$$R = \frac{L^2 + 4w^2}{8W} \quad \text{and} \quad M = \frac{PL}{4}$$



The deflection w at the midspan was measured by a dial guage fixed at the point and calibrated masses are used as loads p at the same point. Here knife edges (Triangular prisms) were used as simple supports. The acceleration due to gravity g is taken as $10m/s^2$.

4.0 RESULTS

The following tables 1 to 3 below show the results from the laboratory investigation for the samples test

TABLE 1: GLASS LOUVRES (Length L=800mm, Breadth b=77mm Thickness t=4mm, I= $\frac{bt3}{12}$)

s/no	Mass	Load	SAMPLE A		SAMPLE B		SAMPLE C		Avg	Deflection
	(kg)	(N)	Loading	unloading	Loading	unloading	Loading	unloading	(mm)	(mm)
0	0	0	25.00	24.90	31.50	31.40	40.00	39.90	32.1166	0
1	0.2	2	24.06	24.01	30.56	30.51	39.06	39.01	31.2016	0.9149
2	0.4	4	23.12	23.04	29.62	29.54	36.12	38.04	30.2466	1.870
3	0.6	6	22.16	21.99	28.66	28.49	37.07	36.90	29.2100	2.905
4	0.8	8	21.20	21.12	27.70	27.62	36.11	36.03	28.296	3.790
5	1.0	10	20.14	20.13	26.64	26.63	35.03	35.02	27.265	4.8516.
6	1.2	12	19.06	19.03	25.56	25.53	33.95	33.92	26.175	5.905
7	1.4	14	18.18	18.15	24.68	24.65	33.07	33.05	25.296	6.820
8	1.6	16	17.27	17.17	23.77	23.67	32.16	32.06	24.386	7.730
9	1.8	18	16.27	16.18	22.77	22.68	21.16	31.07	23.396	8.720
10	2.0	20	15.27	15.27	21.77	21.77	30.16	30.16	22.436	9.680
Average		11								5.3186

TABLE 2: Timber (Mahogany, Massonia, Cotton, Iroko)

Load (2) (N)	Deflection (2) (mm)	Load (3) (N)	Deflection (3) (mm)	Load (4) (N)	Deflection (4) (mm)	Load (5) (N)	Deflection (5) (mm)
0	0	0	0	0	0	0	0
2	0.806	2	3.051	2	1.757	2	2.15
4	1.571	4	5.926	4	3.570	4	3.17
6	2.411	6	9.915	6	3.558	6	5.155
8	3.246	8	13.38	8	7.805	8	6.745
10	4.121	10	16.971	10	9.96	10	8.65
12	4.981	12	20.202	12	11.12	12	10.50
14	5.821	14	23.532	14	14.12	14	11.88
16	6.789	-	-	16	16.05	16	13.545
18	7.671	-	-	18	17.783	18	15.61
20	8.646	-	-	20	19.683	20	16.715
average							
11	4.6063	8	13.282429	11	10.5406	11	9.412

TABLE 3: STEEL (Y16,Y12,Y10, R12)

Load (6) (N)	Deflection (6) (mm)	Load (7) (N)	Deflection (7) (mm)	Load (8) (N)	Deflection (8) (mm)	Load (9) (N)	Deflection (9) (mm)
0	0	0		0	0	0	0
10	0.370	10	1.183	10	2.276	10	1.115
20	0.754	20	2.375	20	4.647	20	2.23
30	1.120	30	3.572	30	6.967	30	3.26
40	1.642	40	4.772	40	9.326	40	4.31
50	1.864	50	5.975	50	12.045	50	5.375
60	2.230	60	7.192	60	14.675	60	6.41
70	2.617	70	8.392	70	17.175	70	7.45
80	2.990	80	9.602	80	20.025	80	8.47
90	3.340	90	10.793	90	22.335	90	9.475
100	3.71	100	11.985	-	-	-	-
Average							
55	2.0637	55	6.5841	50	12.163444	50	5.3438889

Test Samples specification

[2] Mahogany: (Length L=1000mm, Breadth b=25.4mm Thickness t=12.7mm, I= $\frac{bt3}{12}$)

[3] Massonia: (Length L=900mm, Breadth b=23mm Thickness t=8mm, I= $\frac{bt3}{12}$)

[4] Cotton: (Length L=1000mm, Breadth b=25.4mm Thickness t=12.7mm, I= $\frac{bt3}{12}$)

[5] Iroko: (Length L=1000mm, Breadth b=25.4mm Thickness t=12.7mm, I= $\frac{bt3}{12}$)

[6]: STEEL Y16: (Length L=1000mm, Diameter d=16mm, I= $\frac{\pi d4}{64}$)

[7] STEEL Y12: (Length L=1000mm, Diameter d=12mm, I= $\frac{\pi d4}{64}$)

[8] STEEL Y10: (Length L=1000mm, Diameter d=10mm, I= $\frac{\pi d4}{64}$)

[9] STEEL R12: (Length L=1000mm, Diameter d=12mm, I= $\frac{\pi d4}{64}$)

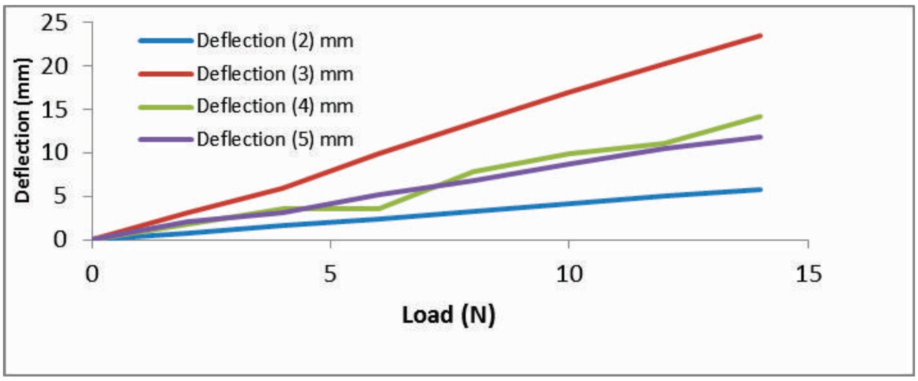


Fig 1: Showing plot of table 2

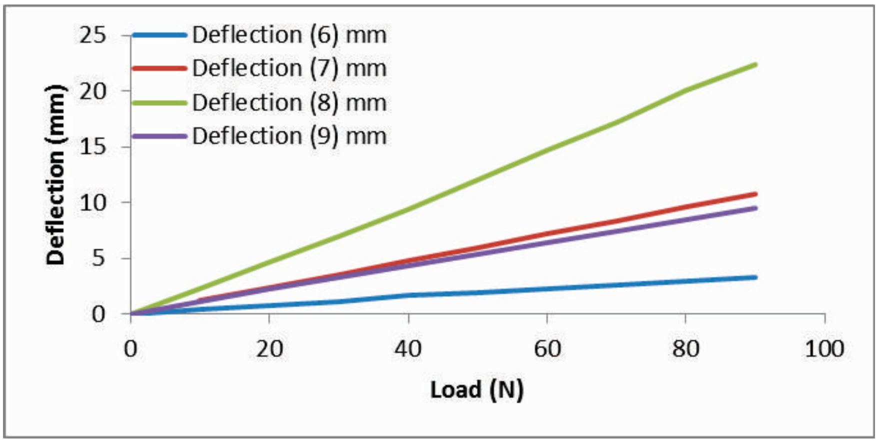


Fig 2: Showing plot of table 3

TABLE 4: YOUNG MODULUS FROM TABLES 1-3

Materials	$E_1 = \frac{(pL^3)}{48I\Delta}$ KN/mm^2 Deflection eqn Average of P & w	$E_2 = \frac{(PLR)}{4I}$ KN/mm^2 Circular bending theory	$\frac{E_2}{E_1}$	$\frac{(E_1 + E_2)}{2}$ KN/mm^2 Or $1.25E_1$	Reference
Glass	53.7207	80.595	1.50026	67.158	50-85
Mahogany	11.474	17.2136	1.500227	14.344	7-20
Massonia	9.394	14.106	1.501597	11.727	7-20
Cotton	4.889	7.336	1.500511	6.11	7-20
Iroko	5.6157	8.4266	1.500543	7.0212	7-20
High Yield Steel Y16mm Bar	172.65	258.895	1.499537	215.7	196-210
High Yield Steel Y12mm bar	170.974	256.505	1.500257	213.739	196-210
High Yield Steel Y10mm bar	174.55	261.986	1.500922	218.27	196-210
Mild Steel R12mm bar	191.685	287.529	1.500008	237.96	196-210

Where :
L = Length of Materials
W = Deflection
B = Breath of materials
H = Thickness of materials
M = Slope from graph
P =Point load
R = Radius of curvature of beam.

4.1 Discussion

The results of the average young's modulus from the central point loading of a simply supported beam from the deflection equation and circular bending theory compare with quoted results which were obtained from direct tensile load test in each case except for the mild steel samples which are 10.05%. The chemical properties of the samples were not investigated to see the composition of the steel. However the reason for it might be for the fact that the mild steel samples base metal thickness (bmt) or the total coated thickness (tct) was used in their strength calculations since they were not removed. (11) has shown that with bmt and tct not removed before specimens

are tested, E values are within the range of 190 to 230 KN/mm2.
The load used to achieve this result is nothing compared to the load required if it were to be tested in direct tension. The cost of preparing samples and the time required for such preparation is removed. Literature as shown that most anisotropic materials like timber and glass will not show a distinct linear stress-strain diagram. The E is therefore obtained from % proof stress, tangent modulus and secant modulus. The flexural test method will give that linear relationship. The hitherto calculated deflections must need to be corrected to bring theoretical and practical deflections to be the same. The practical deflection is much greater than the theoretical deflection and depends on the loading partner. The reason in literature that deformation due to shear deformation is the reason for the discrepancy is removed, since the same reason could be given for results from deflection equation and that of the circular beam theory. For a centrally loaded simply supported beam the correction factor is actually 1.25 which compares with 1.27 as has been experimentally obtained when shear deformation is considered

5.0 Conclusion and recommendation

It is therefore established that the Young's modulus of elasticity of a material can be tested directly jn flexure by loading a simply supported beam by central point load (three point loading) and taking the reading of the deflection at mid-span. The average load and displacement when back substituted in the deflection equation and factored by 1.25, gives you the needed Youngs Modulus measured in direct tension or compression (Edirect). It is also important to state here that though on the conservative side, the theoretical calculation of deflection as is presently practiced, does not give the true practical results. The theoretical value will need to be factored by 1.25 to obtain practical results for a centrally point loaded simply supported beam.
The test of brittle materials and anisotropic materials and composite materials are now feasible with some great level of accuracy using the central point loaded system as described in this work.

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