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Finite element analysis of radial stress distribution on axisymmetric variable thickness Dual Mass Flywheel using ANSYS.

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Abstract
Flywheels are applied in storing inertial energy in rotating machine engines and to limit speed fluctuations. In Dual Mass flywheel (DMF) the rotating mass is split into two and is joined by a damping mechanism. It is commonly in hardest use during engine start up and shut down. In flywheel design, important aspects to consider include geometry (cross-section), rotational speed and material strength. Also, to consider is the mass moment of inertia which when too much the system will be sluggish and unresponsive whereas when too little the system would lose momentum over time. The material strength directly determines the energy level that can be produced safely when coupled with rotor speed. This together with rotational speed result to the flywheel being very highly stressed hence necessary to determine stresses accurately using a discrete method as provided for by ANSYS software. During shaft rotation, centrifugal forces generate stresses in the circumferential as well as radial directions. This paper describes studies on the analysis of axisymmetric solid DMF geometry under radial stress distribution at high revolution using ANSYS. Finally, a discussion of the generated results which would be applied in modifications of existing structures for improved new operating service conditions and academic instruction.

Keywords: DMF, radial stress, FEA analysis, axisymmetric load.

1.0 Introduction
Flywheels are used in storage and release of energy in rotating machine engines known as inertial energy. They are basically meant to limit speed fluctuations through the amount of inertia contained i.e. the mass moment of inertia. In addition, it smoothes out torsional excitation of crankshaft and avoids vibrations. This is majorly accomplished by a flywheel mass. In DMF, the mass is split into two that are torsional linked by elastic springs. Installation is by mounting a flywheel onto one of the axes of the machine, integral with one of the rotating shafts as shown in Figure 1. Applications include; automobile engines, industrial punch presses and other ICE's. In all cases their design is intended to be most economical. Examples are solid disk and composite flywheels.

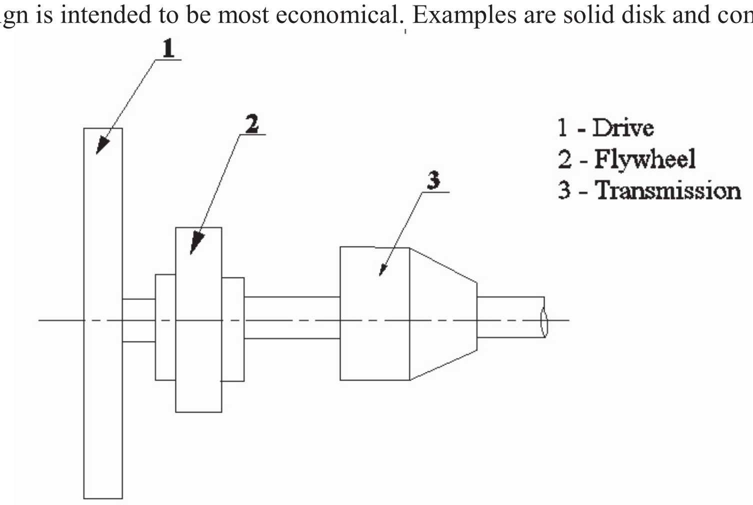


Figure 1:Schematic transmission configuration (2D- CAD drawing).

2.0 Literature
The first Dual Mass flywheel configurations in automobile power train systems appeared over three decades ago

(Theodossiades S. et al, 2006). However the technology for mechanical energy storage began hundreds of years ago and gained development throughout the industrial revolution. They were designed to specifically reduce torsional impulses in the gear box to significant levels which arise due to engine power output fluctuations. Usually in the confirmative designs, torsional resonance range between 700 – 2000RPM hence DMF introduces a secondary inertia near the transmission shaft as shown in Figure 1 which then shifts the resonance below the engine idle speed.

Different papers have described the importance of flywheel geometry design, selection and contribution to energy storage hence performance. This In addition would have a reduction on operational loads exerted on the shafts, bearings due to reduced masses at high rotational speeds (Cui Deyu, Xu Yuanming, 2012; Akshay P.,Gattani G.K, 2013). Also stress deformation on materials varies with material type and concentration of lesser mass can be applied in flywheels to store energy at higher speed fluctuations (Akshay P.,Gattani G.K, 2013).In more recent technological advancements, critical programs focus on energy storage in such rotating masses at the expense of failure though gradual. Due to development of high strength materials with varied characteristics, the technology is getting much more focused unto. This allows designers to achieve higher operating speeds hence yielding more kinetic energy. Today's designed flywheels offer wide value of output magnitudes in both performance and service life which is critical to any operation application. This is in addition to large control torques and momentum storage capability on their application areas.

However it is necessary to meet full utilization and design challenges in order to maximize energy while satisfying stress constraints for performance. Tangential and radial stresses would give guide to designers in the choice of data. In all cases formulations has shown that design factors have the overall performance on flywheels. These include; material strength, rotational speed and geometry (Krack M et al 2011; Erasaslan A.N &Yusuf O, 2004; Ganesh B & Srithar K, 2010).

A further research effort is on energy storage improvement, delivery of high power rates at different transfer times and longer lasting. This would be much essential under failure consideration due to stresses resulting from centrifugal effects. Different designs existed all intended to be easy and most economical to produce. An efficient design became an art in the nineteenth century, progressed on to date. Better designs are achieved by moving the material from near the axis of rotation and placing it as far as practically from the axis (Ganesh B & Srithar K, 2010; Sharma et al 2012).

Ultimately analysis method on axisymmetric elements, with thick shells, solid bodies under revolution, a useful method proposed by E.L.Wilson ring element type having triangular cross-sections with radial and axial confined displacement directions (Cui & Yuanming, 2012).

J.E Shigley et al describes a way of determining stresses in a rotating element such as flywheels by simplying it to a rotating ring element as stated by the latter. By doing this it has been shown that the tangential and radial stresses exists as in the theory of thick-walled cylinders except that they are caused by inertial forces, a result of angular speed. The inertial forces act on all particles on the element. The described present tangential and radial stresses found are subject to; outer radius of the ring or disc is much larger compared with the thickness (t) i.e. $r_{outer} \geq 10t$ and secondly the thickness t is constant. For such rotating disc inner radius is zero as would be shown on analysis equations (Joseph E. et al 2004).

For complex cases where designs of thickness, t vary with radius, r it would not be straightforward to determine accurately stress values using general numerical proposed methods without more assumptions hence FEA analysis would be very useful.

3.0 FEM ANSYS

The finite element method is a versatile, powerful numerical procedure used to obtain solutions to different engineering problems by discrete analysis. ANSYS is a general purpose finite element analysis (FEA) modeling software capable of obtaining solutions either by graphical user interface (GUI) or by command files (Stolarski T, Nakasone Y, Yashimoto S, 2006).

Over the past, rotating structures have been modeled by the lumped mass approach which uses the centre of mass to calculate the effects of rotation. Amongst the major limitations was the inaccuracy in the calculation of internal forces and stresses in the structures.

The aim of this discussion is to apply the finite element code ANSYS for a FEA static analysis of axisymmetrical dual mass flywheel (DMF) loading of varying thickness due to its angular velocity. This would involve the major ANSYS procedures i.e.; creation of geometry model and meshing, applying loads and solving, viewing results and discussion.

3.1 Geometry and Material properties

The DMF is made of AISI steel 316 with Ø20mm shaft, thickness t_{min} = 6mm, and with the following structural properties (J Carvill, 2003);

- Young Modulus, **E** = 193Gpa

- Yield strength, σ_y = 205 Mpa
- Tensile strength, = 515 Mpa
- Compression strength, = 170Mpa
- Density, ρ = 8000kg/m^3
- Poisson's ratio, υ = 0.29

For this case of axisymmetric loading under revolution, the displacements would be confined to only two directions as stated by E.L Wilson on analysis of solids under revolution.

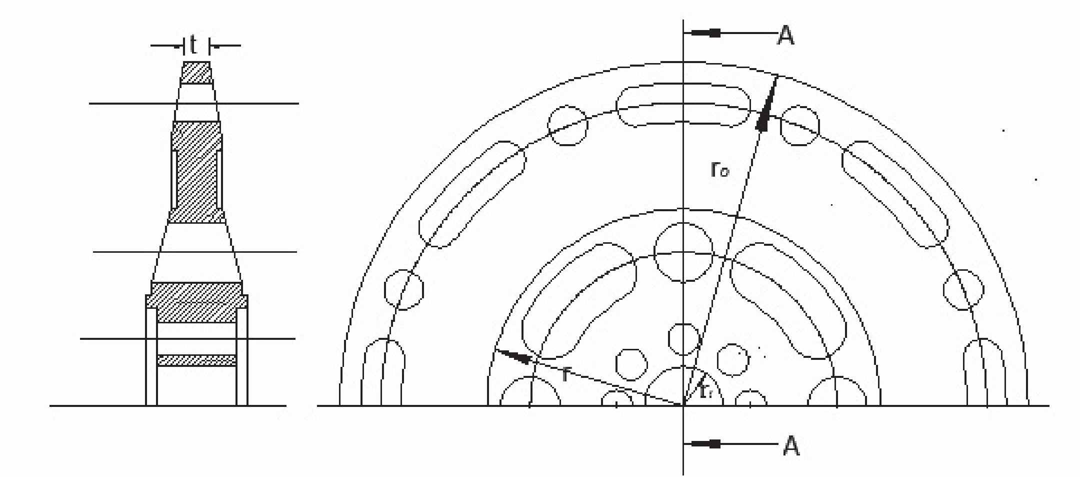


Figure 2:Sectional view A-A.

For a flywheel rotating at angular velocity ω, with uniform thickness the formulae for stress components can be obtained from the mechanics of elasticity as;

Radial stress: $\sigma_r = \frac{\rho}{g}\omega^2 + \frac{3+\upsilon}{8}\left\{r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right\}$ ………………..… ………. (1)

Tangential stress: $\sigma_t = \frac{\rho}{g}\omega^2 \frac{3+\upsilon}{8}\left\{r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\upsilon}{3+\upsilon}r^2\right\}$ ………………… …. (2)

Von Mises stress: $\sigma_y = \sqrt{\left(\sigma_r^2 - \sigma_r\sigma_t + \sigma_t^2\right)}$ ..……………….…..……… . (3)

$\sigma_z = 0$ for state of plane stress and small deformations.

For a varying thickness disc from von Mises and J_2 deformation theory of plasticity, it has been shown

$$h(r) = h_o\left[1 - n\left(\frac{r}{b}\right)^k\right] \quad \text{………..…} \quad (4)$$

Where; h_o is the thickness at the axis, *n* and *k* are the geometrical parameters, *a* and *b* correspond to inner and outer radius r_i and r_o respectively; ***h*** varies with the radius and takes values depending on either concave or convex shape. For concave n=0.4, k=0.7

Consider the strain displacement relations from mechanics;

$$\varepsilon_r = \frac{\partial u}{\partial r} \quad \text{……………………………………………..} \quad (5)$$

$$\varepsilon_t = \frac{u}{r} \qquad \text{..} \qquad (6)$$

For rotation at angular velocity ω, the acceleration lead to inertia force F= $-\rho r^2\omega^2$ which results to stresses in the component and due to rotation around axis $\frac{\rho}{g}\omega^2$[N/mm^4] for a uniform thickness.

Using (5) and (6), the equation of motion in the radial direction becomes;

$$\frac{\partial}{\partial r}\left(hr\sigma_r\right) - h\sigma_t = -\rho r^2\omega^2$$ Which can be expressed as; $$r\frac{\partial \sigma_r}{\partial r} = \sigma_t - \sigma_r - \rho r^2\omega^2 \qquad \text{.......} (7)$$

From the generalized Hooke's law for plane stress;

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - \nu\sigma_t\right) \qquad \text{..................................} \qquad (8)$$

$$\varepsilon_t = \frac{1}{E}\left(\sigma_t - \nu\sigma_r\right) \qquad \text{..................................} \qquad (9)$$

Equations (8) and (9) can be expressed as; $\sigma_r = \frac{E}{1-\nu^2}\left[\varepsilon_r + \nu\varepsilon_t\right]$ and $\sigma_t = \frac{E}{1-\nu^2}\left[\varepsilon_t - \nu\varepsilon_r\right]$

Eliminating u in (5), (6) and using (7) we get the compatibility displacement equation;

$$\frac{\partial}{\partial r}\left(\sigma_r + \partial_t\right) = -\left(1+\nu\right)\rho\omega^2 r \qquad \text{.....................................} \qquad (10)$$

Integrating equation (10);

$$\sigma_r = A + B\frac{b^2}{r_2} - \frac{3+\nu}{8}\rho\omega^2 r^2 \qquad \text{..} \qquad (11)$$

Similarly;

$$\sigma_t = A - B\frac{b^2}{r^2} - \frac{1+3\nu}{8}\rho\omega^2 r^2 \qquad \text{.......................................} \qquad (12)$$

A and B are dependent on ω.

Considering the flywheel, Let the inner radius $r_i = a$ and external $r_o = b$,

Apply BC's, $\sigma_r = 0$ at a and b, hence $A = \left(\frac{3+\nu}{8}\right)\rho\omega^2\left(a^2+b^2\right)$ and $B = -\left(\frac{3+\nu}{8}\right)\rho\omega^2 a^2$

Substituting in (12) and (13),

$$\sigma_r = \left(\frac{3+\nu}{8}\right)\rho\omega^2\left(a^2 + b^2 - \frac{a^2b^2}{r^2} - r^2\right) \qquad \text{....................................} \qquad . (13)$$

$$\sigma_t = \left(\frac{3+\nu}{8}\right)\rho\omega^2\left(a^2+b^2-\frac{a^2b^2}{r^2}-\frac{1+3\nu}{3+\nu}r^2\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

From (7) and (13), displacement can be obtained as;

$$u = \frac{(3+\nu)(1-\nu)}{8E}\rho\omega^2 r\left\{a^2+b^2-\left(\frac{1-\nu}{3+\nu}\right)r^2-\left(\frac{1+\nu}{1-\nu}\right)\frac{a^2b^2}{r^2}\right\} \quad \ldots\ldots\ldots (15)$$

Hence radial stress, $\boldsymbol{\sigma_r}$ is a maximum at r = $\sqrt{ab}$ i.e.

$$\sigma_r = \frac{(3+\nu)}{8}\rho\omega^2(b-a)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

This is the value of maximum radial stress for the rotation at angular velocity ω rad/s, ρ-is the density, υ-Poisson's ratio.

3.2 Finite Element Analysis using ANSYS

The overall aim of a FEA is to recreate mathematically the behavior of an actual engineering model of a physical prototype. This model is comprised of nodes, elements, material properties, real constants, boundary conditions and any other feature used to represent the physical system. Thus analysis reflects the performance of a design to meet specifications as per its manufacture and construction.

Being axisymmetric, a 3D quarter solid model for analysis was generated by describing the geometry, establishing controls over the size in a command file and then instruct ANSYS program to generate all the nodes and elements automatically as shown in Figure 3(a). Then, once analysis is completed a user specified symmetry expansion is applied to view the whole model behavior.

3.2.1 Building Geometry

3.2.1.1 Element type

Solid272; this is preferred to model axisymmetrical solid structures, defined by four nodes and nodes created automatically in the circumferential direction. Each node has 3-DOF; translations in the nodal x, y and z directions. In addition, it has mixed formulation capabilities for simulating deformations, plasticity, hyper elasticity, stress stiffening, large deflection and large strain capabilities.

3.2.1.2 Modeling Assumptions:

- **i.** The material is isotropic.
- **ii.** Vibration effects are ignored.
- **iii.** Steady state conditions.
- **iv.** There is rigid connection on the drive shaft with no keyway for notch effects and no chamfering.

3.2.1.3 Meshing Method

Tetrahedral free meshing operation with smart element sizing was adopted to mesh the volume automatically as shown in Figure 3b.

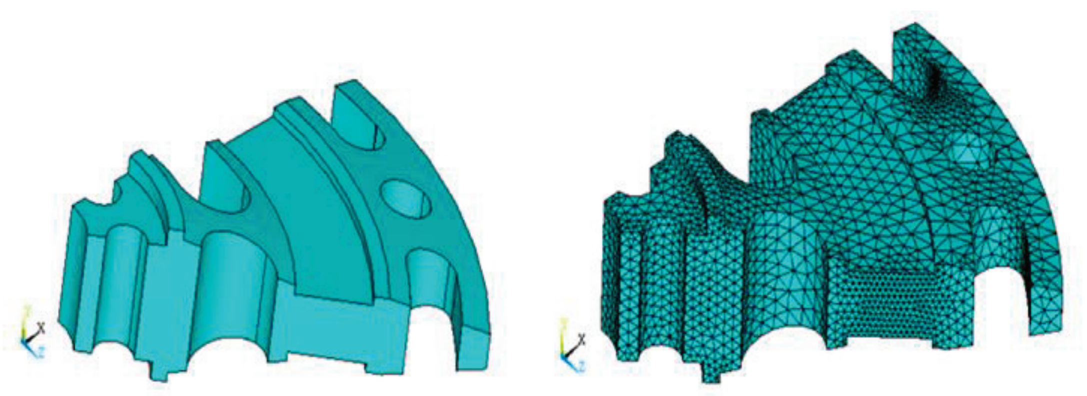


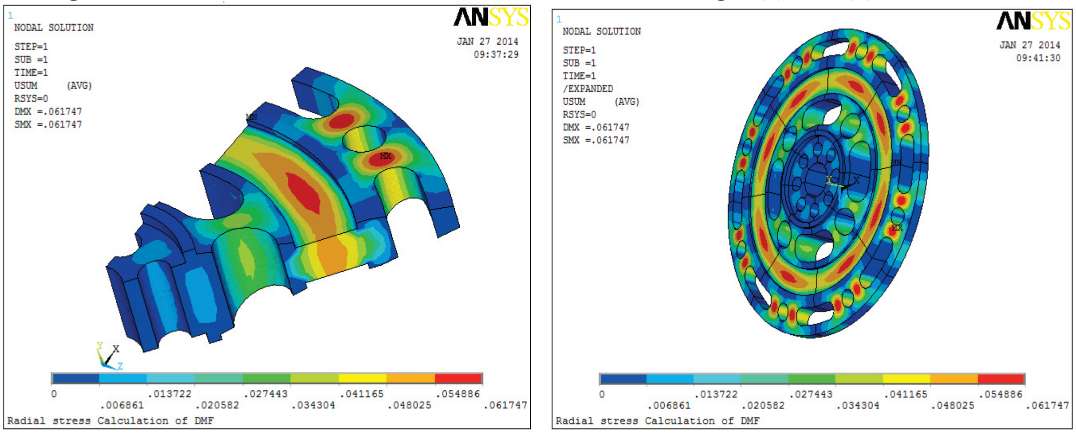
Figure 3: a) Model b) Meshed model

3.2.1.4 Boundary conditions and Load

The displacement was symmetry BC constrained on radial and axial directions and UY set to zero. Then inertia load applied on the rotational y-axis and the centre of gravity pre-conditioned. Then solution solver applied. By using mechanical ANSYS apdl, it has the benefit of solution control options. Then the stress distribution is investigated when its rotating at a given speed.

3.2.1.5 General Nodal Results

The displacement deformed results for the model are as shown in Fig **4 (a) and (b)**



a) Model **(b) Expanded model**

Figure 4: Stress distribution.

4.0 Discussion of results

The DMF concave shape in thickness on outer is divided into 30625elements with a total number of 46804 nodes. Then the radial stress distribution is investigated when rotating around the y-axis at 6000rpm. The stress component σ_r at shaft radius r_i, outer radius r_o and at selected point r was calculated and compared with ANSYs results as shown in table 1.

Table 1: Value of Radial Stress

Position		Radial stress(σ_r)		Difference
	Node no.	Calculated	ANSYS	
r_i	7	0.00	0.006861	0.00
$r = \sqrt{r_i r_o}$	41846	0.06934	0.062747	0.7%
r_o	29	0.00	0.0010099	0.00

Due to DMF symmetry, stress components can be obtained at any point along the radius and thickness at any particle position. The above presented results in table 1 are from selected nodal points at different positions of r, with the aim of illustrating where maximum stress occurs. The very much approximation obtained by finite element method through element averaging for accuracy purposes, hence it is achievable which adds the reliability to obtain values at any desired nodal point. This kind of analysis can be applied in situations which include;

- Illustration of dangerous sections of flywheel rotation with higher stresses.
- A way of optimizing the structure of the design of the DMF.

5.0 Conclusion

Radial stress analysis for the complex geometry DMF has been accomplished. It can be shown that the approximation obtained is closely accurate as depicted in two decimal figures. This can be shown that during rotation on a non-uniform thickness flywheel despite its design, the stresses are concentrated at the middle with very minimal stresses on the interior and outer boundaries of the flywheel. This design with less mass ensures optimal inertia to ensure that it would not be sluggish and unresponsive in operation. However, it’s meant to achieve the right amount of inertia. This shows that the type of engineering analysis test is very significant in determining results using FEM which would have been challenging by using semi-analytical numerical solutions for such complex designs.

Additional static analysis for such rotating elements using high accuracy and time saving FEA methods such as ANSYS is much useful due to their geometry. These significant elements have the ability of storing and or transferring energy hence necessary since during actuation process there is temperature rise.

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