Investigating the Convergence and Bit Error Rate of Adaptive Algorithms over Time Varying Rayleigh Fading Channel

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Abstract
The fastest growing segment of the communication industry is the mobile wireless communication system. However, the systems faced a lot of challenges such as delay in the propagation of signals due to time-varying channel and effect of high speed transmission over Rayleigh fading which result into Inter-Symbol Interference (ISI) distortion. Least Mean Square (LMS) and Normalized Least Mean Square (NLMS) have been previously used to adapt the system using the step size, and Eigen value. In this paper, the adaptive Algorithms over a time-varying channel were compared using convergence level, Bit Error Rate (BER), and Mean Square Error (MSE). The system model consists of bits to symbol converter, 16-QAM modulator and Raised Cosine transmit filter, all at the transmitter, time-varying Rayleigh fading with Additive White Gaussian Noise added, and at the receiver are Raised Cosine Receive filter, 16-QAM demodulator, then each of the Adaptive LMS and NLMS filters which received delay from the Random integer generator, and the integer/symbol to bit converter at the output. The system model was simulated using MATLAB/SIMULINK software package. The algorithms were evaluated using convergence MSE at SNR of 10, 20 and 30dB over different number of iterations to determine the convergence rate, constellation diagram and BER. The results obtained showed that the flat convergence level of LMS and NLMS at SNR of 10dB are obtained with 300 and 200 iterations respectively, while 200 and 150 iterations are obtained at SNR of 20 and at SNR 30, the convergence level are obtained at 150 and 100 iterations respectively. BER values of 0.1598 and 0.0858 are obtained for LMS and NLMS respectively. Therefore, LMS algorithm took more iterations than NLMS algorithm to achieve the same error, and also lower BER value of NLMS is also in agreement with the result.
Keywords: Convergence, MSE, LMS algorithm, NLMS algorithm, Intersymbol interference (ISI).

1. Introduction

Wireless communication is the transmission of baseband signal without physical connection between the users which may be fixed or in motion. The Radio frequency (Rf) transmitted signals are governed by the mechanisms such as scattering, reflection, diffraction and refraction or their combinations according to types of environment. These mechanisms cause the RF signal to propagate in multiple paths. The multipath manifested in several ways depending on the degree of path difference relative to the signaling rate. [2, 3]

The multiple copies of the transmitted RF signals arrived at the receiver at different time due to the delay of the successive signals which may add up constructively or destructively under unfavourably conditions. The resultant effect is the fluctuation of the received signal known as fading. When the delays are large compared with symbol period of the digital modulation due to the time-varying native of the channel, the net effect is the distortion at the receiver known as inter-symbol interference distortion (ISI).

Various wireless channels have time varying nature which makes their transfer functions change with time. In high speed digital communication, time-varying multipath interference and multiuser are its associated problem. Since the changes in transmitted signals are sudden, there is need for adaptive algorithms to track that converges fast and stable [7 – 10]. In addition to time-varying nature of the channels, high speed digital transmissions also suffer from ISI.

There are many equalization to combat ISI distortion such as linear and non-linear equalizer. Linear equalizers are computationally simple. In a time varying channel, adaptive algorithms adapts to the changes in the channel to recover the transmitted signals. The adaptive algorithms widely in use are Least Mean Square (LMS) and Normalized Least Mean Square (NLMS) [5, 6] because of their plainness in the implementation. The Normalized Least Mean Square (NLMS) takes into consideration the variation in the signal level at the filter output but at the expense of cost of performance. The simplicity of linear equalizers comes at the cost of performance. Non-linear equalizers are complex, computationally intensive and offer better performance.

A lot of researches have been carried out focusing on the comparison of the LMS and NLMS algorithms. Some of the authors focusing on convergence behavior using adjustable step-size variations, studying the cross-correlation of the original and the filtered signals, human voice signals contaminated with interference signals cancellation in terms of the correlation coefficient of the input and output signals. Other approaches by researchers are adjusting the time-varying step-size based on the square of the time-averaged estimate of autocorrelation variation of step-size to obtain fast convergence, and studying the effect of filter length and step size parameters. [5 – 12] Some of these past researches proved that NLMS was faster in convergence than LMS algorithms when setting the step-size to one. [5, 8, 10, 11, 12, 13]

In this paper, the convergence rate of these adaptive algorithms over time varying Rayleigh fading channel with 16 – Quadrative Amplitude Modulation (16-QAM) signaling using BER at different SNR, MSE performance at different SNR for different number iterations and constellation diagram of 16 QAM were investigated. The results obtained showed that LMS algorithm took 300, 200 and 150 iterations to converge at SNR of 10, 20 and 30 dB respectively while NLMS took 200, 150 and 100 iterations respectively to have optimum convergence.

The results obtained showed that NLMS algorithm has a faster convergence rate, lower BER, lower MSE than LMS over the time varying Rayleigh fading in the presence of AWGN. Hence, the distortion which can occur as a result of the delay in receiving the signal and high speed transmission can be eliminated with this algorithm.

2. Adaptive Equalizer

This is a system which adjusts its parameters when received the distorted signal from the time-varying Rayleigh channel in order to correct or eliminate the distortion. The output signal $x(n)$ from an unknown time-varying Rayleigh channel is given by [3, 6, 8]

$$x(n) = \sum h(k)p(n-k) + q(n) \tag{1}$$

where h(k) is the channel response

$p(n-k)$ is the input to the channel at time n-k

$q(n)$ represents the Additive White Gaussian Noise (AWGN) within the channel

$x(n)$ is the output signal and served as the input signal to the equalizer.

The output of the system $y(n) = w^T x(n)$

$y(n)$ is the equalizer output, w^T is the tap weight vector.

2.1 Adaptive Distortion Cancellation

An adaptive equalizer consists of a transversal filter coupled with an adaptive control mechanism that is being controlled by LMS and NLMS algorithms. Figure 1 shows an adaptive equalizer. An input signal $x(n)$ when passed through a transversal filter produced an output signal $y(n)$, the output signal $y(n)$ is then subtracted from the training sequence response $d(n)$ to produce an error signal $e(n)$. The input signal $x(n)$ and error signal $e(n)$ are combined together in an adaptive algorithm control mechanism to adjust a weight of the transversal filter so as to minimize Mean Square Error MSE value. This process is repeated for a number of iterations until the filter reached a steady-state or convergence flat value. [1, 3, 5].

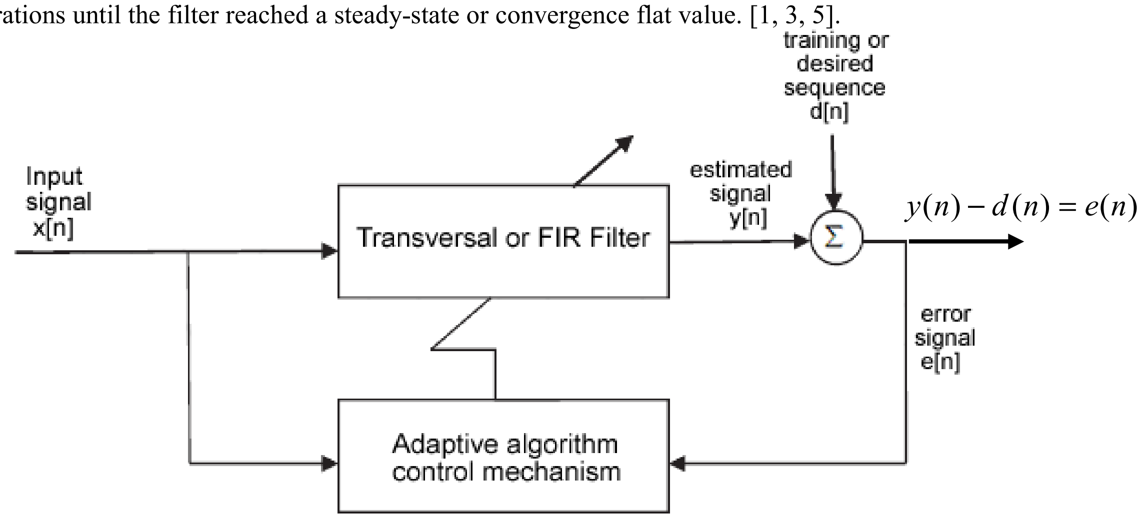

Figure 1: Adaptive equalizer

2.2 LMS Algorithm

LMS algorithm is one of the adaptive control algorithms that changes the filter tap weights to have minimum $e(n)$ i.e. finding the filter coefficients that give least mean square of the error.

According to [14, 15, 16] LMS algorithm for Lth order is given as follows:

Parameters: L = filter length
S = step size

Initialization: $w_n = 0$

Computation: For n = 0, 1, 2, 3, 4.......

Input to the equalizer $'x(n)'$ is given as

$$x(n) = [x(n), x(n-1), \ldots.., x(n-L+1)^T \quad (2)$$

Output signal from the equalizer $'y(n)'$ is expressed as

$$y(n) = w_n^T x(n) \quad (3)$$

Error signal estimation $'e(n)'$ is

$$e(n) = d(n) - y(n) \quad (4)$$

The expression for updating the coefficient is given by

$$w(n+1) = w(n) + Se(n)\ x\ (n) \quad (5)$$

Input signal $x(n)$ through the tap vectors to the transversal filter are convolved with the coefficient matrix of the filter which are updated by the LMS algorithm, this gives the output of the equalizer. An error signal $e(n)$ depicted in equation (4) is the difference between the training sequence or desired signal $d(n)$ and the filter output [$w_n^T x(n)$]

Variable step-size LMS algorithms was used to improve the performance of the LMS algorithm by using large step-size at the early stages of the adaptive process to have rapid convergence rate. High level measurement noise or input data tends to deteriorate the convergence rate.

Equation (5) is the weight $w(n)$ update function for the LMS algorithm. The convergence condition imposed on step size is given by

$$0 \leq S \leq 1/\lambda max \quad (6)$$

where λmax is the largest eigen value of autocorrelation matrix. If it is chosen to be very small, then convergence becomes slow, if it is kept large, then convergence becomes fast, but stability becomes a problem. Therefore it is better to select within bounded conditions as defined in equation (6). The drawback of the LMS algorithm is its sensitivity to the scaling of its input $x(n)$ that is if the input $x(n)$ is large, then the problem of noise gradient amplification rises. This make it very hard to choose a learning rate 's' that guarantees stability of the algorithm. [1, 4]

2.3 Normalized Least Mean Square Algorithm

The Normalized least mean squares filter (NLMS) is a variant of the LMS algorithm that solved the problem of sensitive to the scaling of input $x(n)$ by normalizing with the power of the input as depicted in Equation (9) [5].

The NLMS algorithm can be summarized as:

Parameters: L = filter length
S= step size

Initialization: $w_n = 0$

Computation: For n = 0, 1, 2, 3, 4.......

$$x(n) = [x(n), x(n-1), \ldots., x(n-p+1)]^T \quad (7)$$

$$y(n) = w_n^T x(n) \quad (8)$$

$$e(n) = d(n) - y(n) \quad (9)$$

$$w(n+1) = w(n) + \frac{se(n)\ x\ (n)}{x^T(n)\ x\ (n)} \tag{10}$$

this In this case, (10) has been normalized by the conjugate transpose $x^T(n)\ x\ (n)$ of the input vector x(n), this conjugate transpose represents the square deviations (errors) of the output thus eliminating the effect of distortion making the algorithm to be relatively stable in nature. [3]
The convergence conditions imposed on step size of the NLMS is given by

$$0 \le S \le 2 \tag{11}$$

3. System Model

The system model consists of random integer generator, bits to symbol converter, 16-QAM modulator, and a raised cosine filter at the transmitting end, Rayleigh fading channel with AWGN added to form a channel. At the receiver are raised cosine filter, 16-QAM modulator, adaptive algorithm module (LMS/NLMS filter), Integer to symbols converter, and an error calculator which are all implemented using MATLAB communications tool.
A random stream of bits was generated by the random integer generator block x[n] to imitate a generalized message bit stream. The steam of bits $x(n)$ was converted into integer symbols from 0 to M-1 before the 16-Mary QAM modulator performed its function.

With M=16, four bits are grouped together to form an integer symbol in the range of 0 to 15. The signal input x`[n] is a sequence of 4 bits pair, the next process is serial to parallel conversion by placing two MSB bits in Quadrature channel while others are placed within the In phase I-channel. The parallel bit information was encoded using Gray code which was then mapped to 16QAM rectangular constellation.

The data stream was converted to an analog format using a raised cosine filter implemented units. (Transmit raised cosine filter block in MATLAB) before the stream was transmitted across the channel. Pulse shaping centralizes the bandwidth in a signal and limits the sideband frequencies hence reducing the transmitted power leading to a reduced bandwidth in transmitting over the channel. This eliminates ISI distortion of the sideband frequencies which were suppressed so as to avoid distortion of the preceding transmitted symbols. A roll off factor (α) of 0.5 was used within the filter for more efficient use of the spectrum. The pulsed shaped data stream $x(t)$ was transmitted across the time varying channel and was subjected to distortion thereby introducing error.

Signals subjected to time-varying Rayleigh fading channel to mimic real life situation was simulated, noise was added to the transmission. This contributed to the non-ideality of the communication process, thus accounting for environmental factors. This was all achieved with the AWGN function of MATLAB in the channel stage of signal transmission. After the message data stream had gone through the channel, it was filtered by the raised cosine filter (Receive raised cosine filter block in MATLAB) and the down sampled to return to the original pulse shape.

The down sampled received message $y_1(n)$ was QAM demodulated from the constellation structure into a stream of symbols $y_2(n)$ which went through the adaptive equalizer.

The delay block supplied the desired response after a suitable delay which was applied to the adaptive equalizers (LMS/NLMS) in the form of a training sequence which in turn trained the filter coefficients by the weight update equations of the normalized update block LMS algorithm. The equalizer block provides option for the selection of the LMS and NLMS algorithms for simulation.

The stream of symbols from the equalizer was then mapped back from integer back to bits. At this point, an error calculator was connected between the transmitter bits to symbol converter and the receiver symbol to bits converter which estimated the bit error rate of the whole simulation. The system simulation model is shown in Figure 2.

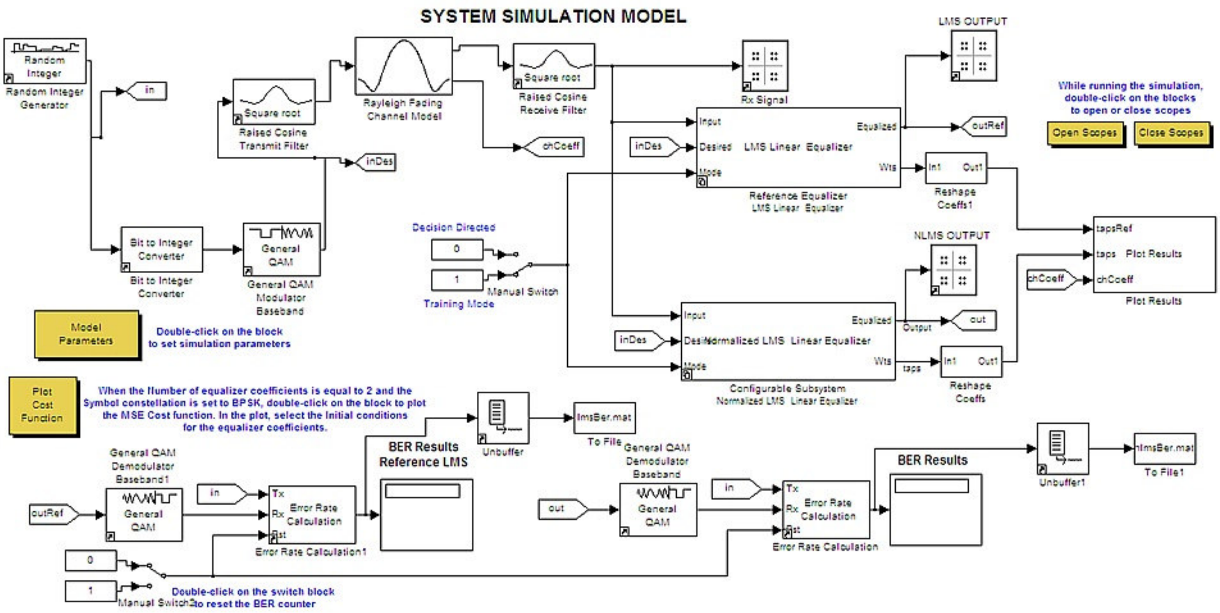

Figure 2. System Simulation Model

3.1 System Simulation Parameters

The choice of 16-QAM was made because of the applicability to microwave digital radio which forms the basis for digital communication system, possible to transmit more bits per symbol and susceptibility to noise thus obtaining higher data rates and maintaining an acceptable bit error rate for radio communication systems. The simulation parameters are contained in Table 1.

Table 1. Simulation Parameters

Parameters	**Value/ type**
Noise	AWGN
Fading	Time Varying Rayleigh fading
Receive Filter	Raised Cosine Filter
Transmit Filter	Raised Cosine Filter
Equalizer algorithms	LMS and NLMS
Step size for Algorithm (μ)	0.001, 0.01 and 0.1
Roll off factor (α)	0.5

4. Results and Discussion

The convergence or stability level in term of MSE of the LMS and NLMS for 50 to 500 iterations at interval of 50 iterations using the combination of SNR of 10, 20, 30dB over the time-varying Rayleigh fading channel are presented in Figures 3 and 4.

Figure 3 showed the convergence or stability value using MSE for LMS algorithm in time- varying Rayleigh fading channel at different number of iterations. It can be observed that the convergence values are obtained at 300, 200 and 150 iterations for 10dB, 20dB and 30dB respectively. Similarly for NLMS algorithm shown in Figure 4, the convergence values obtained are 200, 150 and 100 iterations for 10dB, 20dB and 30dB respectively. These showed that the worst convergence level was at SNR of 10dB for LMS and NLMS. Then at SNR of 20dB there was fair convergence level for the two adaptive algorithms. The best convergence level was obtained at SNR of 30dB but remained unstable due to poor eigen values that the algorithm possessed for LMS while little instability level arose during the training period of the filter within the equalizer employing NLMS before switching to tracking.

Figures 5 – 7 showed the combination of convergence MSE against iterations for LMS and NLMS algorithms at different SNR for the purpose of comparison. Figure 5 is the plot for the two algorithms at SNR of 10dB while Figure 6 is for SNR of 20dB and Figure 7 showed the plot at SNR of 30dB for the two algorithms. The results obtained are justifiable in that NLMS algorithm tracked the distorted signal received from time-varying Rayleigh fading channel by updating its filter coefficient in a limited time.

Figures 8 – 10 showed the constellation diagrams for 16-QAM. The unequalized received signal for 16-QAM at 30dB is shown in Figure 8. It was clear that uneven distances among the bits were observed indicating poor constellation points. Figure 9 showed the constellation points of the output of the LMS algorithm at SNR of 30dB run for 3secs over 150 iterations. It was observed that the constellation points were closer than that of the received signal before equalizing. The distance among the corresponding bits indicate that LMS equalizer has converged at a faster time than unequalized. Figure 10 depicts the constellation points of 16-QAM at the output of NLMS equalizer at 30dB over 100 iteration ran for 2secs. It was observed that the spaces among the equalized 16 bits were evenly distributed and converged at shorter time for the same number of iterations. In summary, it is clear from the result that LMS algorithm took more iteration than required to achieve the steady state error but in NLMS is less number of iteration along with mean square error was observed. This implied that the bits converged at a faster rate than LMS.

BER performance as a function of SNR with 16-QAM signaling scheme over the time varying channel is shown in Figure 11 for both LMS and NLMS. BER values of 0.1598 and 0.0858 are obtained for LMS and NLMS algorithms respectively at SNR of 4dB. Therefore, LMS algorithm took more iterations than NLMS algorithm to achieve the same error. The lower BER values of NLMS also indicate good performance. All the performance metrics considered are in agreement with one another and with the results obtained literature, though 16-QAM modulation and time-varying Rayleigh fading were considered in this paper.

The results obtained are justifiable because in NLMS the power of the input has been normalized and hence becomes independent of the input signal (tracking mode) as depicted in equation 10, hence leading to a better eigen value parameter in form of the step size being used, lower BER value as compared to the LMS after simulation, lower value of Mean Square Error (MSE), a higher value of SNR. Also, there exist training modes within the filter that depends on the delay signal for information about the bits being transmitted and later on becomes independent on the training signal as it would have learnt and trained the filter coefficients to predict the delayed signal hence reducing or eliminating an overhead (which can cause slowness of the system) within the communication system hence leading to a faster convergence of the time varying channel in a reduced number of iterations. Therefore, the effect of ISI distortion will be more reduced with NLMS than with LMS.

Table 2 depicts the convergence level of the LMS and the NLMS Algorithm with their corresponding Signal to Noise ratio values. It can be seen that a higher SNR value leads to a faster optimum convergence level (lower convergence value). This makes the NLMS algorithm best suitable for tracking time-varying channels due to its convergence at the 100^{th} iteration.

Table 3 shows the BER values for NLMS and LMS algorithms with corresponding SNR values. Higher values of SNR values lead to a corresponding decrease in BER values. The BER parameters of the NLMS algorithm show reduced values as compared to the BER parameters of the LMS algorithm, this plot is depicted in Figure 11.

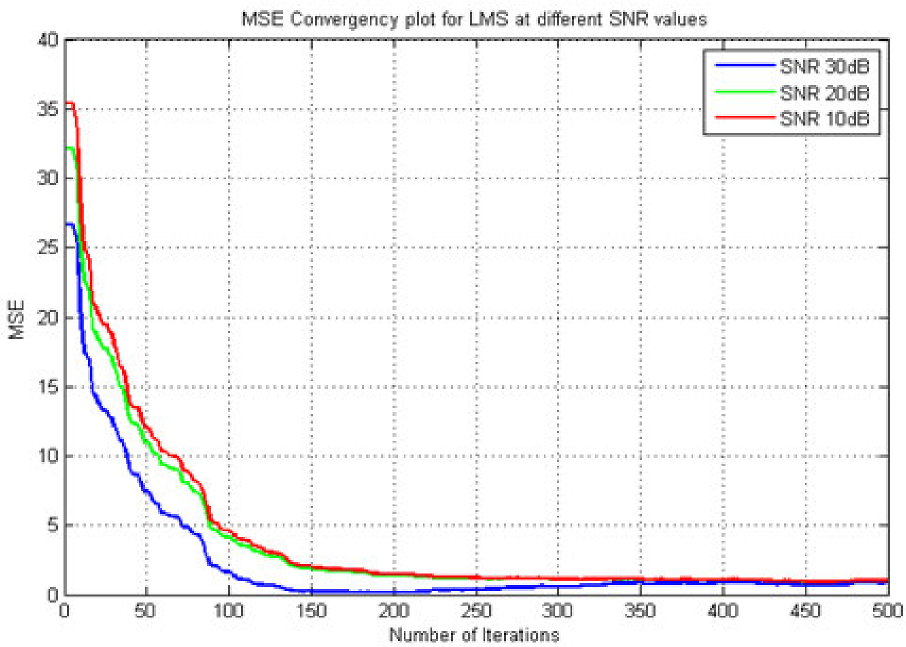

Figure 3. Convergence plot for LMS at different SNR values

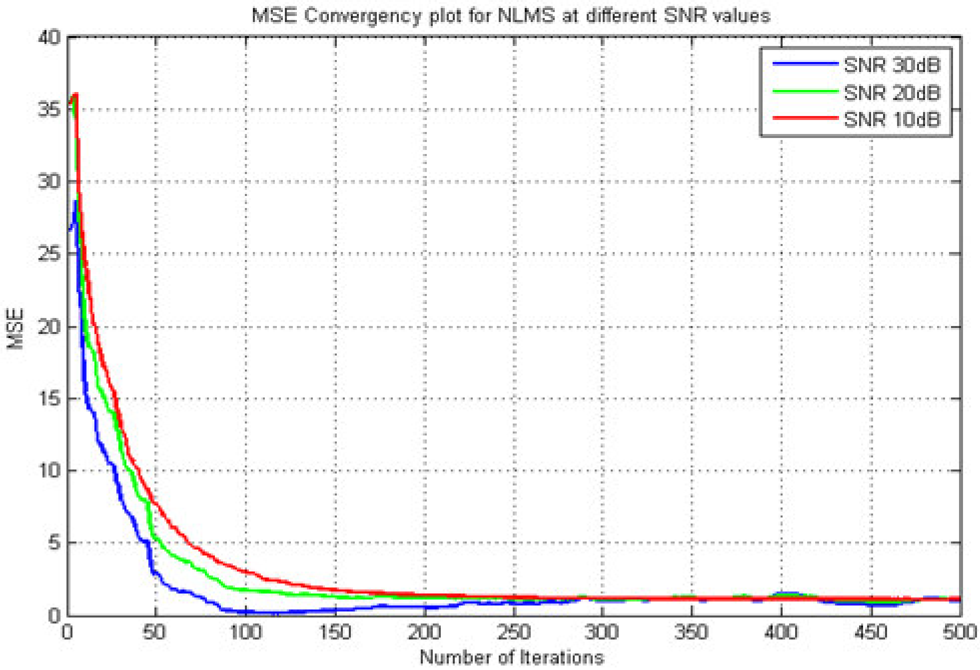

Figure 4. Convergence plot for NLMS at different SNR values

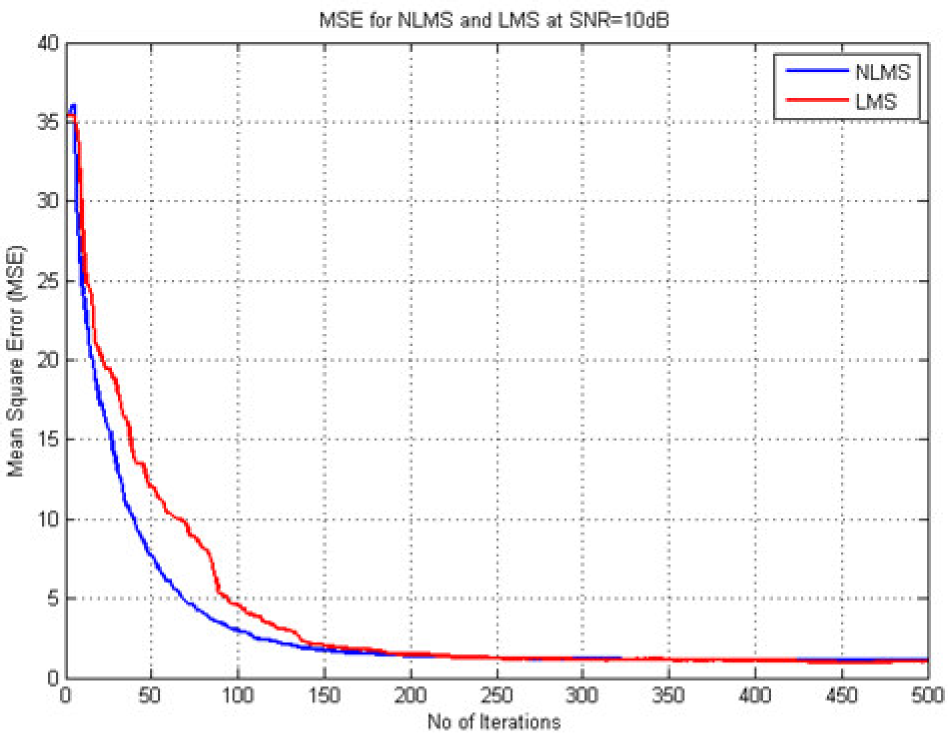

Figure 5. Convergence values for NLMS and LMS at a SNR of 10db

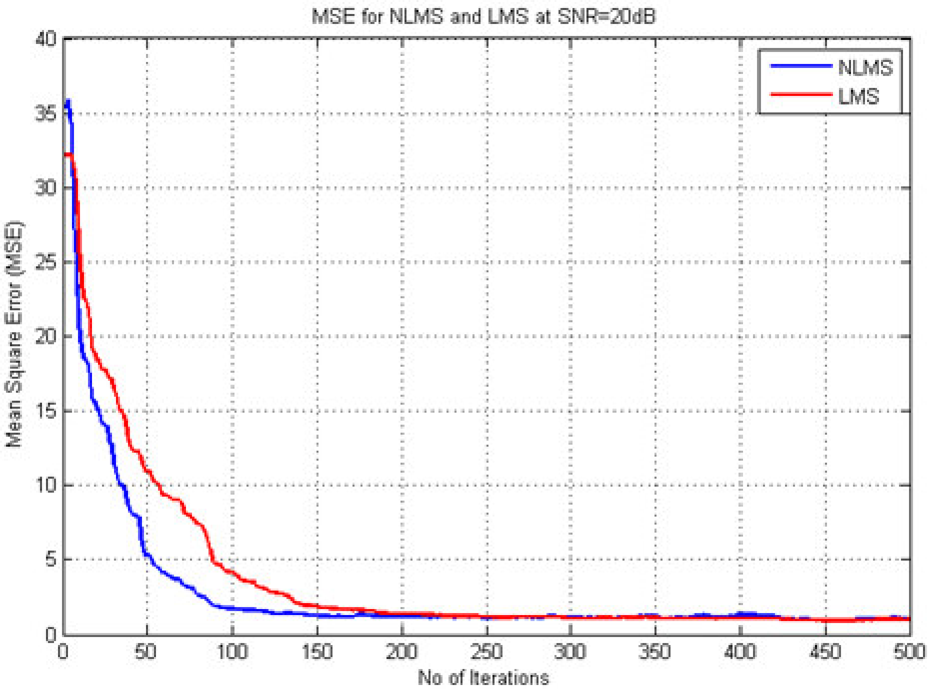

Figure 6. Convergence values for NLMS and LMS at a SNR of 20dB

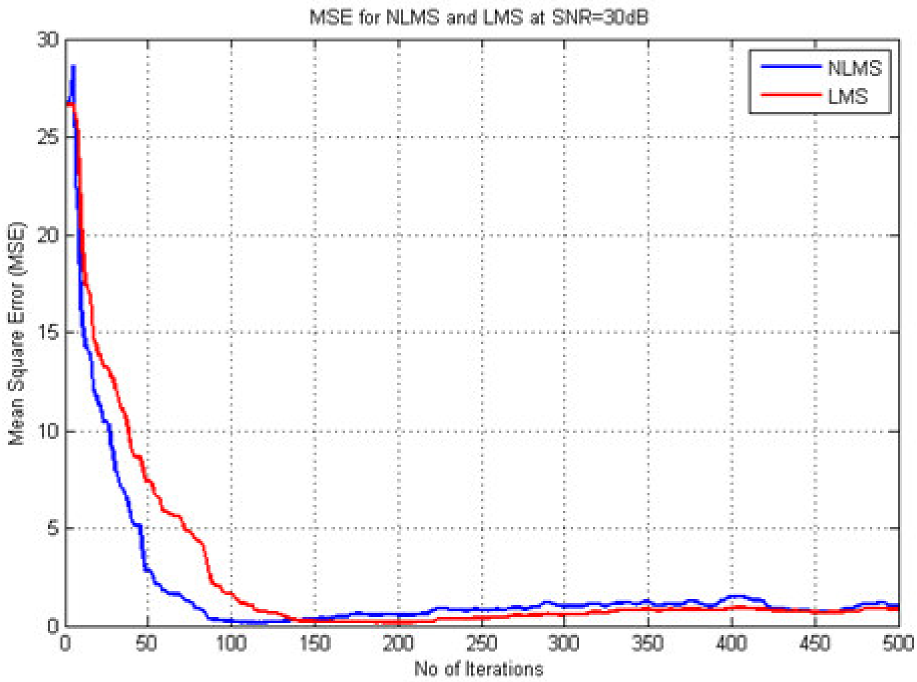

Figure 7: Convergence values for NLMS and LMS at a SNR of 30dB

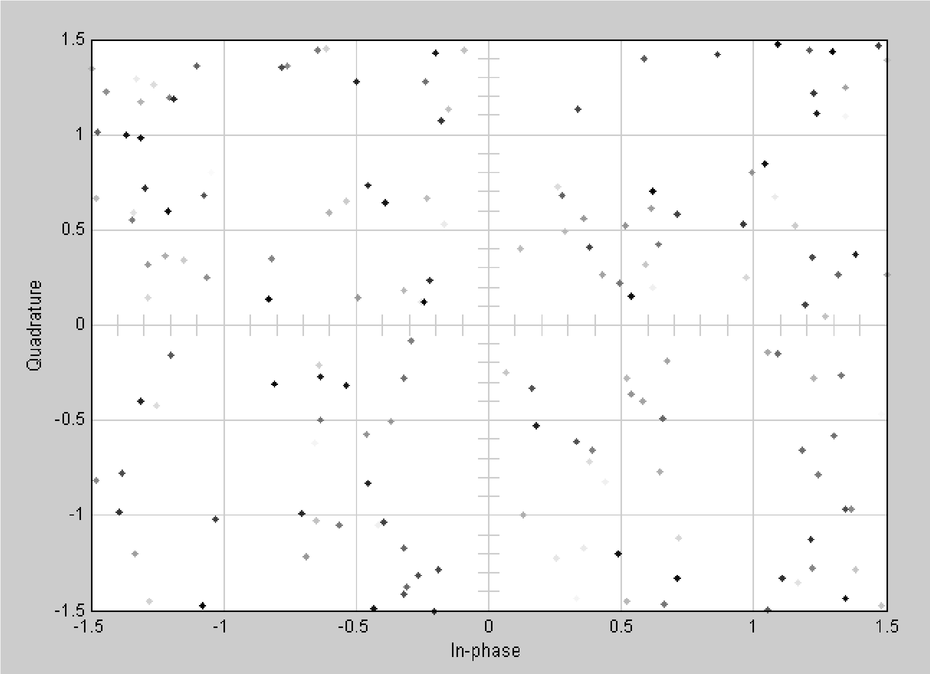

Figure 8: Constellation diagram for the received unequalized signal.

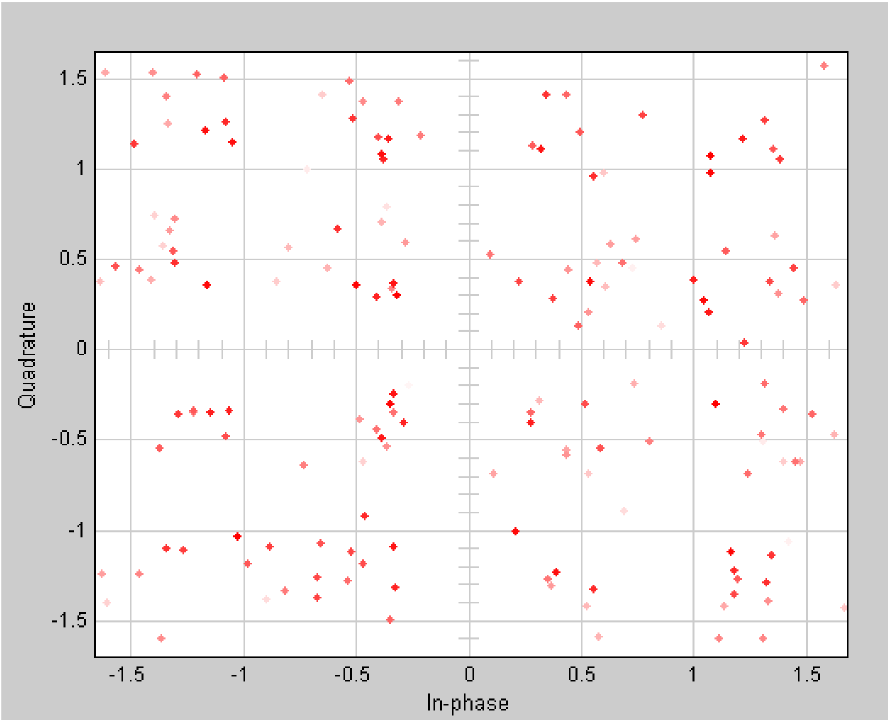

Figure 9: Constellation diagram for the output of LMS equalizer.

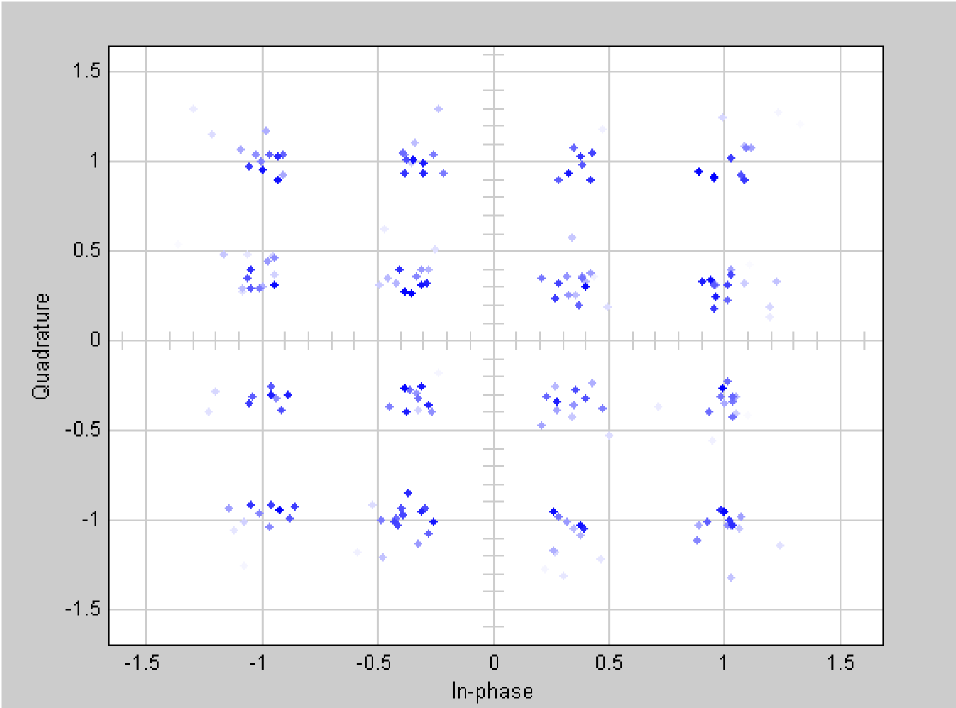

Figure 10: Constellation diagram for the output of NLMS equalizer.

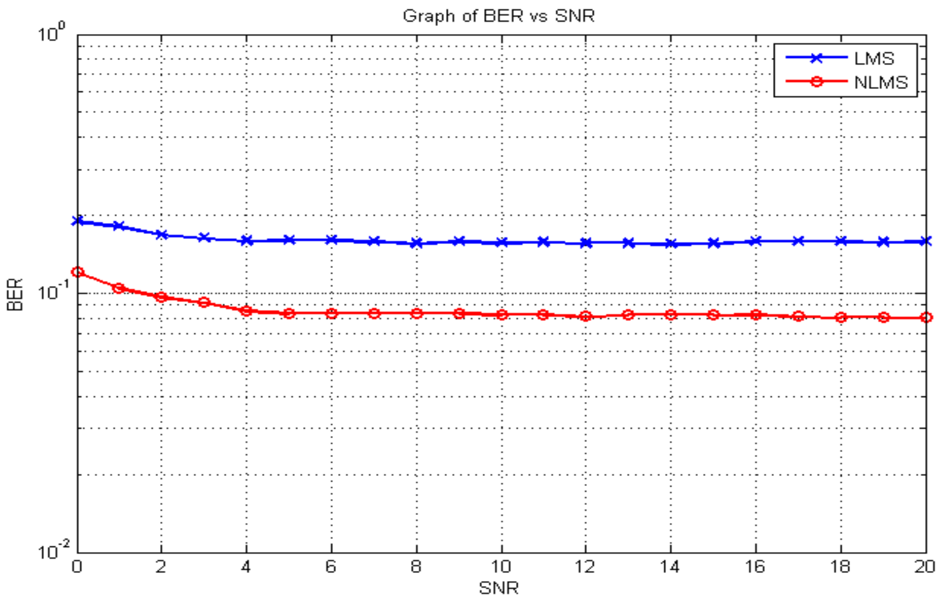

Figure 11: BER values versus SNR values for both algorithms

Table 2. Convergence values against the SNR for the algorithms.

Signal to Noise Ratio (dB)	**LMS Iterations**	**NLMS Iteration**
10	300	200
20	200	150
30	150	100

Table 3. BER values for Adaptive algorithms

SNR(dB) Values	BER Values for LMS	BER Values for NLMS
10	0,1571	0.0831
8	0.1562	0.0840
6	0.1607	0.0840
4	0.1598	0.0858
2	0.1689	0.0968
0	0.1900	0.1205

5. Conclusion

In this paper, the convergence level using Mean Square Error (MSE) with a number of iterations and BER in NLMS and LMS algorithms over time-varying Rayleigh fading channel have been investigated using Randomly data generated. The model for the system incorporating each of the two adaptive algorithms has been developed and simulated using MATLAB/SIMULINK software tools. Each of the NLMS and LMS adaptive algorithms processed the distorted signal.

The NLMS algorithm has shown to have higher convergence speed in terms of MSE, number of iterations and constellation diagram. Less error values and even distribution constellation points with NLMS are also observed as against the LMS with partially even constellation diagram. This is in agreement with the results obtained for MSE. This paper has shown that the effect of time-varying Rayleigh channel known as Intersymbol interference ISI distortion in wireless communication can be drastically reduced with the use of the developed system model incorporating NLMS and 16-QAM signaling Scheme.

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