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Enhancement of Bending Strength of Helical Gears by Using Asymmetric Involute Teeth Profiles

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Abstract

This paper presents an alternative method to enhance the bending stress for helical gears achieved that by using Asymmetric Involute Teeth Profiles. Theoretically, the bending stress of a symmetric gear tooth is calculated by classical Lewis equation (1892), then this equation is developed by M.Q. Abdullah (2012) to take into account the effect of asymmetry of tooth profiles. In this paper, the bending stress for helical gear teeth with asymmetric profile has been calculated analytically by modifying the M.Q. Abdullah equation. In this paper, analytically this equation has been modified to find the bending stress for helical gear with asymmetric teeth at a specified number of teeth (14 teeth) at any helix angle. Also this value has been verified by using numerical solution based on the finite element method technique that done by the software Autodesk simulation mechanical is package v2014. Where the generation process of helical gear have been modelled by using 3D CAD Graphics Software Autodesk inventor professional 2014. The results of this work indicate to the most important conclusion of a helical gear drive with asymmetric teeth profiles having loaded side pressure angle of (14.5°) and unloaded side pressure angle of (25°) is better than a helical gear drive with symmetric teeth profiles having standard pressure angles of (20°) from the point of view of tooth bending strength. There are significant enhancement in the results of maximum tooth bending stress for asymmetric involute of tooth gear compared with the standard gear. In addition, the results demonstrate optimum helix angle is (22.5°) which has the highest enhancement percentage.

Keywords: Gear, Helical, Involute profile, Asymmetric, Bending Stress, Autodesk inventor, Simulation.

1. Introduction

Helical gears are currently being used increasingly as a power transmitting gear owing to their relatively smooth and silent operation, large load carrying capacity and higher operating speed [1]. Where Sufficiency in bending load carrying capacity is a serious problem. It has been observed that most of the failures due to bending stress are at critical section of a gear tooth [2].

The Bending stress plays a significant role in gear design wherein its magnitude is controlled by the nominal bending stress and the stress concentration due to the geometrical shape of teeth [3]. In this work, asymmetric involute teeth profiles has been used to change in the form of the geometrical shape of teeth in order to obtain reduction-bending stress. This asymmetric involute tooth have load side of pressure angle (14.5°) and unloaded side of pressure angle (25°). In addition, the key geometrical parameters that be constant of cases studied such as module $m_o = 7$ mm, tooth face width $b = 60$ mm and same number of teeth on the pinion and gear $z_1 = z_2 = 14$ teeth (speed ratio $u = 1$). Where, these cases studied are the standard (symmetric) tooth profile of gear with pressure angle $(20^\circ/20^\circ)$ and asymmetric tooth profile of gear with pressure angle $(25^\circ/14.5^\circ)$, as shown in (Table 1) all the details of these cases studied. Where these cases studied, included spur and helical gears, also the helical gears included study effect several helix angles varied from 5° to 35° in each steps of 5° on the performance of these gear drives.

This work included theoretical analysis and numerical analysis, after that will be compared between them. Where all the results of analytical, numerical depended on applied the same external normal load (F_N) equal (2400 N), In addition, the material used for pinion and gear is steel with Poisson's ratio of 0.3 and Young modulus of $207000\ N/mm^2$.

2. Analytical Solution

2.1 Lewis equation.

The bending stress is one of the crucial parameters during the analysis of helical gears. When the total repetitive load acting on the gear tooth is greater than its strength then the gear tooth will fail in bending. Bending failure in gears is predicted by a formula developed by Wiltred Lewis [1]. Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth in which the tooth form entered into the formulation. The equation, announced in 1892, still remains the basis for most gear design today [4]. The formula uses the bending of cantilever beam to simulate the bending stress acting on the gear as shown in (Figure 1), the tangential load (F_t) induces bending stress which tends to break the tooth. The maximum bending stress induced by this force is given by [1]:

$$\sigma_{bending} = \frac{M.y}{I} \quad (1)$$

with $M = F_t . h$, $y = t/2$,and $I = b.t^3/12$, where M: is the bending moment at the weakest section of tooth, I: is the area moment of inertia, and b: is the face width of the gear.

Substituting the values for M, y and I in equation (1) the stress will be

$$\sigma_b = \frac{(F_t.h).t/2}{b.t^3/12} = \frac{6\,F_t.h}{b.t^2} \quad (2)$$

From similarity of the two triangles which is shown in (Figure 1),

$$\frac{k}{t/2} = \frac{t/2}{h} \quad (3)$$

Therefore; $h = \frac{t^2}{4k}$ (4)

Substituting Eq. (4) into Eq. (2) and multiplying by a term $(\frac{m_o}{m_o})$ get:

$$\sigma_b = \frac{6F_t\frac{t^2}{4k}}{b.t^2} = \frac{3F_t}{2b.k} = \frac{3F_t}{2b.k} * \frac{m_o}{m_o} \quad (5)$$

Therefore; $\sigma_b = \frac{F_t}{b.m_o.Y}$ (6)

Where $Y = \frac{2k}{3m_o}$ (7)

And from Eq. (6) and Eq. (7), it can be found that:

$$Y = \frac{1}{6m_o}\left(\frac{t^2}{k}\right) \quad (8)$$

Equation (6) is known as Lewis equation, and Equation (8) is represented (Y) the Lewis form factor which considers only for symmetric involute profile and its function of tooth geometry only, also Values of (Y) obtained from standard table as in ref. [4].

2.2 M.Q. Abdullah equation.

A developed analytical method based on a trial graphical method has been achieved to find the solution of bending stress equation for symmetric and asymmetric involute gear teeth shapes. In order to find the developed bending stress formula for asymmetric involute tooth, the same assumptions that considered from Lewis for a symmetric tooth has been adopted. Therefore, by referring to (Figure 2), it could be shown that the bending stress formula for asymmetric tooth is [5]:

$$\sigma'_{bending} = \frac{M'.y'}{I'} \quad (9)$$

Where

$M' = F_t.h' - F_r.e$, $F_r = F_t.tan\,\beta_l$, $y' = t'/2$, $I' = b.t'^3/12$, $e = (t_u - t_l)/2$ and $t' = (t_u + t_l)$.

M': is the bending moment at the weakest section of asymmetric tooth, I': is the moment of inertia of the weakest section, t': is the total thickness of the weakest section.

t_l & t_u : are the weakest section thicknesses of the loaded and unloaded tooth sides respectively, and its calculated from ref. [5] with a radial load position (R) and with tooth design parameters of loaded side (ϕ_l & r_{fl}) and for unloaded side (ϕ_u & r_{fu}) respectively, with taking into account that the resulted value of (t) from this equation must be divided by (2).

h': is the load height from tooth critical section with a radial load position (R) and it calculated from ref. [5] with design parameters of loaded tooth side (ϕ_l &r_{fl}).

β_l : is the loading angle of loaded tooth side and it is calculated from ref. [5]

Therefore;

$$\sigma'_{bending} = \frac{[F_t.h_l - F_t.\tan\beta_l.(t_u - t_l)/2].(t_u + t_l)/2}{\frac{b.(t_u + t_l)^3}{12}} \quad (10)$$

$$\sigma'_{bending} = \frac{3F_t}{b}\left[\frac{2h_l - \tan\beta_l.(t_u - t_l)}{(t_u + t_l)^2}\right] * \frac{m_o}{m_o} \quad (11)$$

Finally $\sigma'_{bending} = \frac{F_t}{b.m_o.Y'}$ (12) Where, $Y' = \frac{1}{3\,m_o}\left[\frac{(t_u + t_l)^2}{2\,h_l - \tan\beta_l.(t_u - t_l)}\right]$ (13)

Equation (12) and Eq. (13) represent the developed Lewis bending stress and form factor equations of the asymmetric tooth respectively.

2.3 Modification on M.Q. Abdullah Equation.

Lewis equation (6) is used to calculate bending stress for symmetric involute tooth profile of spur and helical gear, and Lewis stress modified by M.Q. Abdullah equation (12) is used to find bending stress for asymmetric involute tooth profile for spur and developed this equation in order to find bending stress for asymmetric involute tooth profile for helical at a specified number of teeth used in paper. This development based on helix angle factor are based upon equations published in the literature of the Japanese Gear Manufacturer Association (JGMA) [6], where the helix Angle Factor is:

$$Y_\beta = 1 - \frac{\beta}{120} \quad (14)$$

After this Factor has is developed in order to fit with bending stress equation for symmetric and asymmetric involute tooth using trial and error method to access an approximation of the best relationship of the Lewis form factor, the developer factor is shown below :

$$K_\beta = \frac{1 - \frac{\beta}{170 - 2.05 \times \beta}}{\cos\beta} \quad (15)$$

Since the Lewis form factor product after Substituting Eq. (15) into Eq. (13) and multiplying by a term ($\frac{1}{K_\beta}$) get :

$$Y'_\beta = \frac{1}{3\,m_o}\left[\frac{(t_u + t_l)^2}{2\,h_l - \tan\beta_l.(t_u - t_l)}\right] \times \left(\frac{\cos\beta}{1 - \frac{\beta}{170 - 2.05 \times \beta}}\right) \quad (16)$$

Where this form factor represented modified Lewis form factor by M.Q. Abdullah, therefore made this form factor for symmetric and asymmetric involute tooth for helical gear at a specified number of teeth (z=14) only used in paper.

2.4 Verification of developed equation's M.Q. Abdullah

This developed equation is used to find bending stress for asymmetric involute tooth profile for helical gear at a specified number of teeth (z=14), the developed equation is represented by modifying Lewis form factor (Y'_β) of M.Q. Abdullah. Therefore, this modified Lewis form factor (Y'_β) is verified by compared at a specified the number of teeth (z=14) between the value of Lewis form factor (Y) by extracting values from the standard table for symmetric teeth at a helix angle certain and compared with the value of modified Lewis form factor (Y'_β) at the same of helix angle certain for symmetric teeth that calculated by same modified Lewis form factor (Y'_β). The comparison is performed in several different angle as shown in (Table 2).

According to these results, it is obvious that the verification of modified Lewis form factor for symmetric and asymmetric involute tooth profile for helical has been very successful. As well as the percentage of error in these results are very few and the results of the comparison are converging with each other.

3. Numerical analysis

Finite Element Analysis (FEA) is a numerical method to interpolate an approximate solution to boundary value problem. In this paper, the finite element method technique that done by the software Autodesk simulation mechanical package v2014. Where the generation process of helical gear have been modelled by using 3D CAD Graphics Software Autodesk inventor professional 2014.

3.1 Set-up for Numerical Bending Solution.

The procedure of calculating bending stress have been used 3D FEM analysis by Autodesk Simulation Mechanical software. The Analysis Type is chosen the Static Stress with Linear material properties. Also, model three teeth of gear have been prepared for this analysis of tooth bending stress. After that set-up of FE for this analyze the bending stress, the right of element type and number has been chosen according to the convergence test, so that right element type is Brick element and their element number is different for each model of helix angle. A selected Brick element which be content bricks and tetrahedral. Absolute mesh size of element have been specified from model mesh settings and generate 3D mesh for this model. The maximum bending stress will be occur in the root of the tooth, the mesh of region root of tooth is refined to get more accurate results by specify new mesh size from surface refinement commanded, as show (Figure 3) the absolute mesh size of element that equal (5.5 mm), total number of elements that equal (13609) and new mesh size of surface refinement on the root region that equal (0.9 mm).

In this work, there are two types of applied transmitted load, one of these loads at the tip of tooth and another load acts near the middle of tooth. The models were constrained with boundary conditions, as following, which are shown in (Figure 3):

- The surface on the two sides of the three teeth of the gear rim are considered as fixed constraint.
- Apply surface pin constraints to cylindrical surfaces for model of the three teeth for gear as fixed pin constraint.

In addition, all the modeling of gear tooth are analyzed with same the loading conditions and same constraints and boundary conditions.

4. Result and Discussions

4.1 Results of Analytical solution

The theoretical results have been identified for tooth bending stresses that achieved all cases studied of (Table 1), and to complete the solution bending stresses must be determine the Lewis form factor by tooth loading angle, tooth weakest section thickness and load height ,as shown in (Table 3) that shows the geometrical properties for the gear tooth shape.

(Table 4) shows the analytical results for nominal tooth bending stress for both types of gear, the standard gear (symmetric) and asymmetric tooth of gear for helical gears with several different helix angle, as well as the enhancement percentage for each step that compare with symmetric of gear for same step. Where all these results have been calculated for nominal tooth bending induced the applied load at the tip point of tooth. It's clear from this table that there is an enhancement in the tooth bending stresses for asymmetric tooth of gear for each type of helical gear at all the helix angle. It was noted that the bending stresses were decreased and improved with increasing the helix angle for both types of gear.

4.2 Results FEM for Tooth Bending Stress.

In (Figure 4) and (Figure 5) demonstrates the bending stress distribution inside a gear tooth for spur gear and selected sample of helical gears, respectively. The bending stress is represented by the von mises stress on the root of tooth. Where the maximum tooth bending stress due to the tangential force acts at the tip point of tooth. In addition, (Table 5) present more detailed results of FE for all the helix angles of helical gear for both type of gears, the standard gear and asymmetric involute of tooth gear. With the enhancement percentage for each step that based on the reference the value of standard gear for same this step. Also (Figure 6) shows the relationship between the enhancement percentage with helix angles related to its.

(Figure 7) and (Figure 8) show maximum tooth bending stress for spur gear and selected sample of helical gears when the tangential force acts near the middle tooth, the bending stress is represented by tensor stress in perpendicular direction which on applied tangential force. In addition, (Table 6) shows more detailed results of FE for all the helix angles of helical gear for both types of gears, with the enhancement percentage for each step that based on the reference the value of standard gear for same this step. Also (Figure 9) shows the relationship between the enhancement percentage with helix angle related to its.

It is clear that these (Tables 5) and (Table 6) show a clear enhancement in asymmetric involute of tooth gear compared with the standard gear, as well as shown that the best improvement occur when the load is applied near the middle of tooth. Also shows that the optimum helix angle was (22.5°) which has the highest enhancement percentage as shown (Figure 6) and (Figure 9). The optimum helix angle (22.5°) for both cases when applied force at the tip point of tooth and near middle of tooth, as shows maximum enhancement percentage (1.727 %) and (16.874 %), respectively.

The (Figure 10) and (Figure 11) shows the relationship between bending stress with helix angles for both type of gears when applied force at the tip point of tooth and near middle of tooth, Respectively. These figures shows decreases and improve bending stresses with increasing the helix angle for both types of gear.

Generally, enhancement in tooth bending stress is due to increase in the tooth weakest section thickness caused by using asymmetric tooth profiles.

5. The comparison between Results of numerical and analytical

Investigation the analytical results for maximum tooth bending stresses value based on the value of nominal tooth bending stress of analytical method then used stress concentration factor ($k_t = 1.38967$), where the theoretical value of stress concentration factor calculated from empirical relation for gears with (20°) pressure angle had been derived by Dolan and Broghamer of Ref.[4,7].

The Comparison between the analytical and numerical results of maximum tooth bending stresses have been done for standard gears with both spur gear and helical gears for all helix angles, when the load acts at the tip of the tooth, are listed in (Table 7). In addition, (Figure 12) shows the Comparison between the analytical and numerical results of the bending stresses with helix angle, when the load acts at the tip of tooth.

According to the comparison results of (Table 7), it is clear that there is a very convergence between the theoretical and numerical results in all angles. Where that the maximum of percentage difference of (1.332 %) at the helix angle (25°). And these results shows the validity of the theoretical equation developed for this case.

6. Conclusions

1) A developed analytical equation has been achieved to find the solution of bending stress for symmetric and asymmetric of involute tooth profile for helical gears at a specified number of teeth, and verification of this equation was very successful with a high accuracy.
2) The maximum tooth bending stress was decreased and improved with increasing the helix angle for each types of gear, the standard gear (symmetric) and asymmetric tooth of gear.
3) There are significant enhancement in the FE results of maximum tooth bending stress for the type of the asymmetric involute of tooth gear compared with the standard gear, as well as clear that the best improvement occur when the load is applied to near middle of tooth compared with this load is applied at the tip point of tooth. Maximum enhancement percentage were (1.727 %) and (16.874 %) when applied force at the tip point of tooth and near middle of tooth, respectively.
4) Optimum helix angle of (22.5°) has the highest enhancement percentage of maximum tooth bending stress for both cases when the force applied at the tip point of tooth and near the middle of tooth with enhancement percentage (1.727 %) and (16.874 %), respectively.

The results of this work indicate to the most important conclusion of a helical gear drive having asymmetric teeth profiles with loaded side pressure angle of (14.5°) and unloaded side pressure angle of (25°) is better than a standard helical gear drive having symmetric teeth profiles with standard pressure angles of (20°) for a certain helix angle that have been from the point of view of tooth bending strength.

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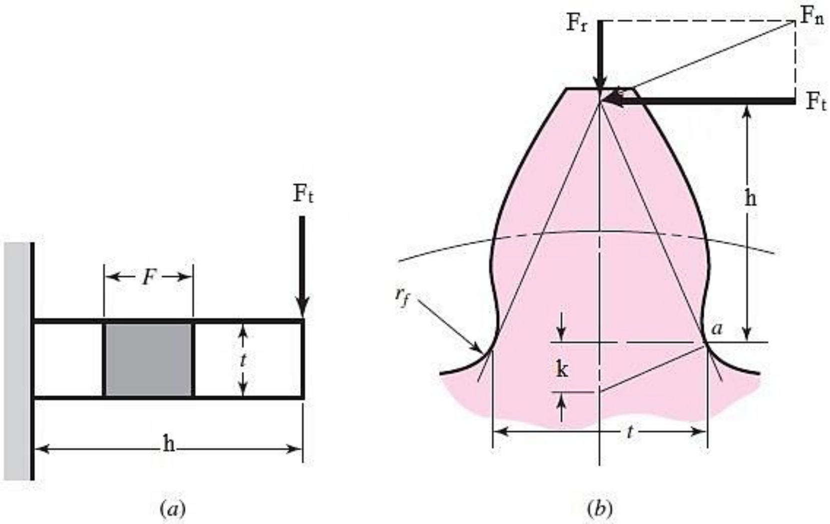


Figure 1. Gear Tooth as Cantilever Beam.

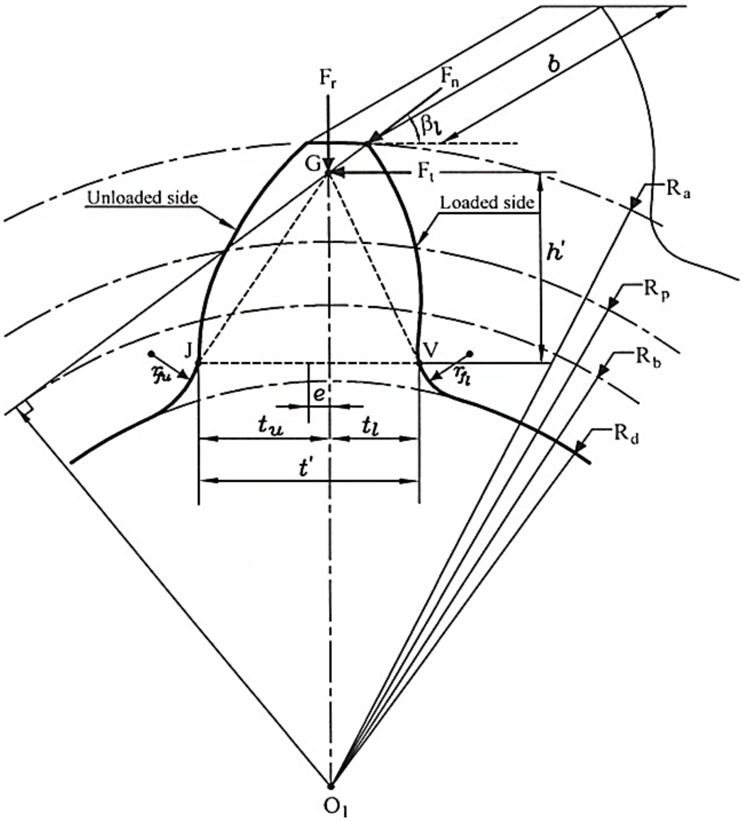


Figure 2. Determination of weakest section for asymmetric tooth.

Table 1. Cases Studied

Cases Studied	Standard Gear	Asymmetric Gear
ϕ_u/ϕ_l	$20^{\circ}/20^{\circ}$	$25^{\circ}/14.5^{\circ}$
h_a	m_o	m_o
h_d	1.166 m_o	1.25 m_o
r_f	0.3 m_o	0.39 m_o

Table 2. Verification of modified Lewis form factor for symmetric tooth for helical gear.

	Y	(Y'_{β})	
10°	0.2923	0.29	0.809 %
15°	0.2999	0.296	1.3 %
20°	0.308	0.303	1.67 %
22.5°	0.3127	0.309	1.2 %
25°	0.31799	0.314	1.27 %
30°	0.3316	0.331	0.1711 %
35°	0.3525	0.346	1.867 %

Where:

* The value of Lewis form factor (Y) obtained from standard table at every helix angle at equivalent number of teeth $z_{\beta} = \frac{z}{(\cos\beta)^3}$, the number of teeth (z=14).

** The value of modified Lewis form factor (Y'_{β}) obtained from Eq. (16) at every helix angle and the number of teeth (z=14).

Table 3. The geometrical properties for the gear tooth shape.

Cases Studied	β_l $(degree)$	t (mm)	h (mm)	Y $(-)$
Standard Gear	32.378	10.933	12.689	0.224
Asymmetric Gear	29.194	11.193	12.605	0.242

Table 4. The analytical results for nominal tooth bending stress for helical gears with the enhancement percentage that compare with the standard gear, when the load acts at the tip of tooth.

Helix angle, β	$\sigma_{bending}/_{nominal}$ (N/mm^2)		Enhancement Percentage
	Standard Gear	Asymmetric Gear	
0° (spur)	21.516	20.592	4.2945 %
5°	20.922	20.024	4.2936 %
10°	20.386	19.511	4.2921 %
15°	19.874	19.021	4.2944 %
20°	19.347	18.516	4.2947 %
22.5°	19.059	18.24	4.2972 %
25°	18.742	17.937	4.2944 %
30°	17.975	17.203	4.2952%
35°	16.909	16.183	4.2945%

Where:

$$\text{Enhancement Percentage} = \left(\frac{(\sigma_{bending})_{standard\ case} - (\sigma_{bending})_{asymmetric\ case}}{(\sigma_{bending})_{standard\ case}}\right) * 100\%$$

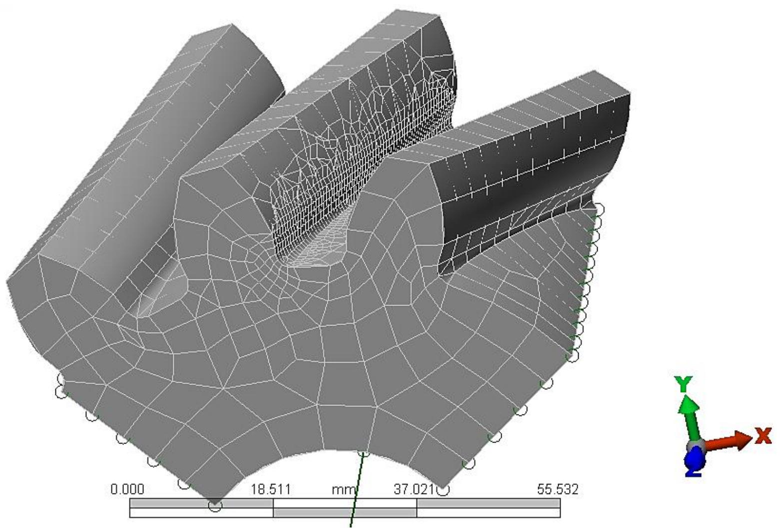
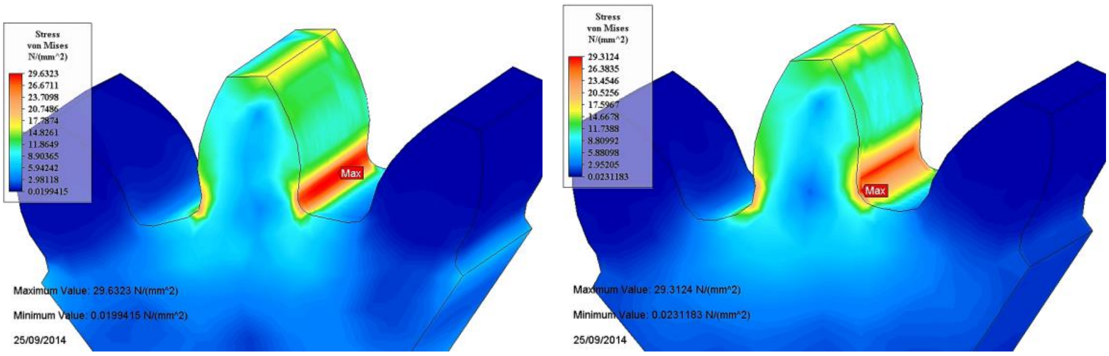
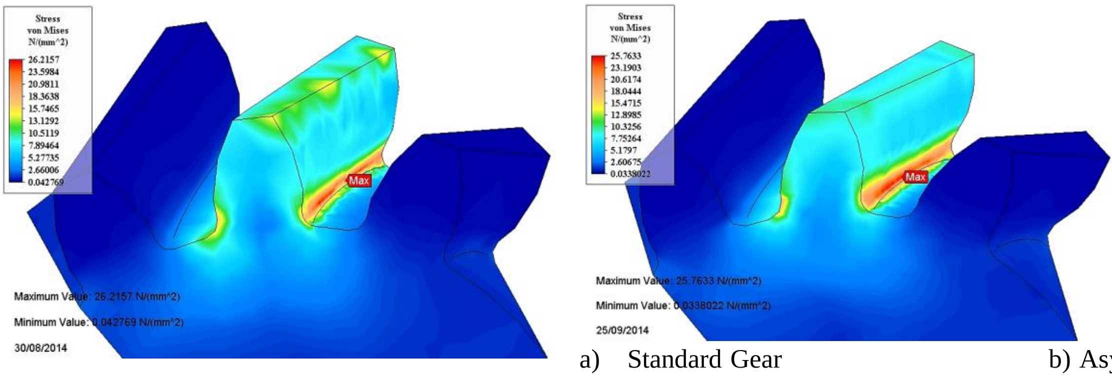


Figure 3. Three Dimensional Model with Mesh for Helical gear of helix angle (22.5°).



a) Standard Gear b) Asymmetric Gear

Figure 4. Three Dimensional Von Mises Stress distributions inside tooth of spur Gear, when the load acts at the tip of tooth.



a) Standard Gear b) Asymmetric Gear

Figure 5. Three Dimensional Von Mises Stress distributions inside tooth of Helical Gear (helix angle = 22.5°), when the load acts at the tip of tooth.

Table 5. FE results of maximum tooth bending stress for helical gears with the enhancement percentage that compare with the standard gear, when the load acts at the tip of tooth.

Helix angle, β	$\sigma_{bending}/max$ (N/mm^2)		Enhancement Percentage
	Standard Gear	Asymmetric Gear	
0° (spur)	29.632	29.312	1.079 %
5°	28.797	28.448	1.212 %
10°	28.153	27.709	1.576 %
15°	27.375	26.943	1.577 %
20°	26.629	26.199	1.614 %
22.5°	26.216	25.763	1.727 %
25°	25.699	25.356	1.335 %
30°	24.724	24.404	1.294 %
35°	23.257	22.947	1.331 %

Where:

Enhancement Percentage $= \left(\frac{(\sigma_{bending})_{standard\ case} - (\sigma_{bending})_{asymmetric\ case}}{(\sigma_{bending})_{standard\ case}}\right) * 100\%$

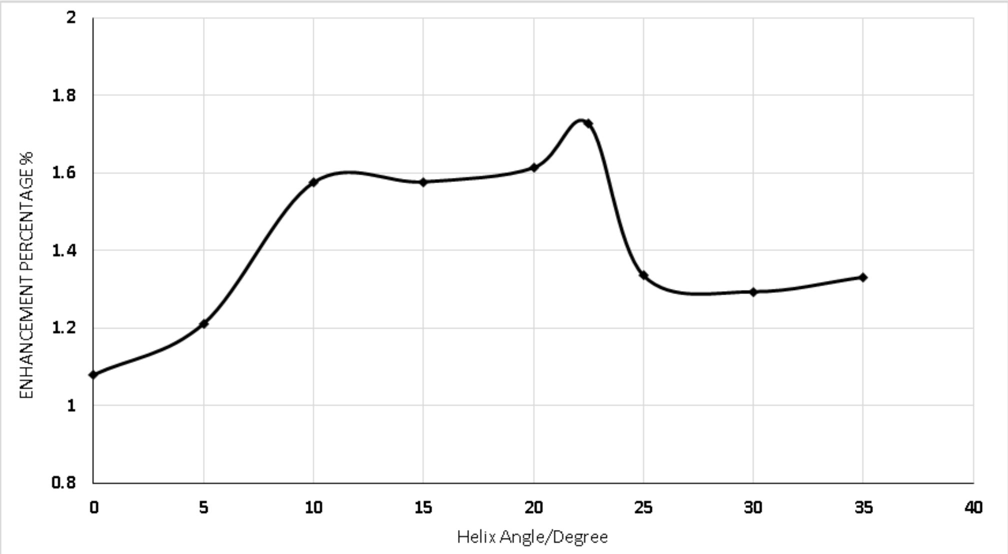
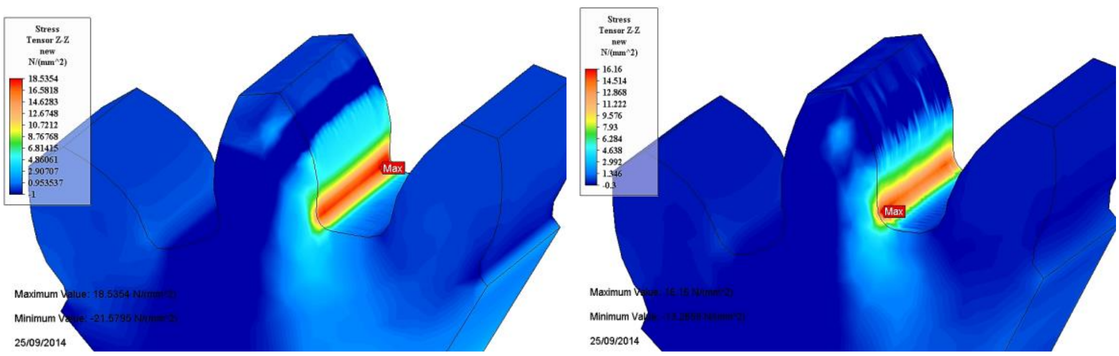
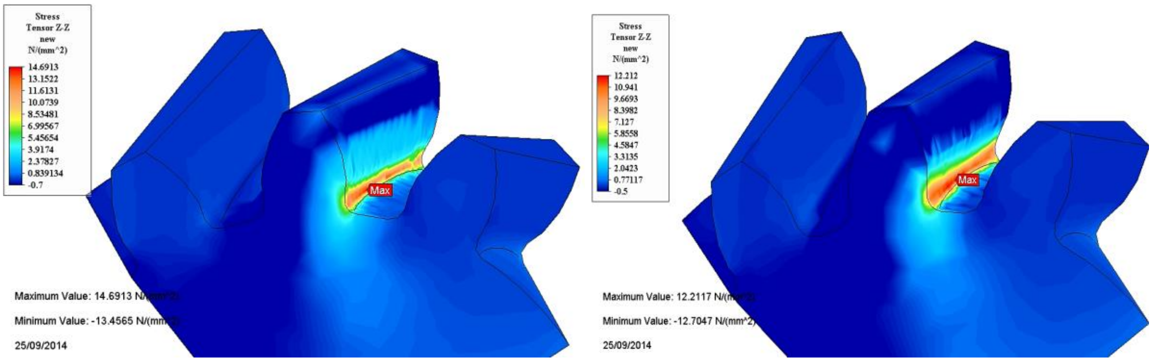


Figure 6. The relationship between the enhancement percentage with helix angle related to it's, when the load acts at the tip of tooth.



a) Standard Gear b) Asymmetric Gear

Figure 7. Three Dimensional Von Mises Stress distributions inside tooth of spur Gear, when the load acts near the middle of tooth.



a) Standard Gear b) Asymmetric Gear

Figure 8. Three Dimensional Von Mises Stress distributions inside tooth of Helical Gear (helix angle = 22.5°), when the load acts near the middle of tooth.

Table 6. FE results of maximum tooth bending stress for helical gears with the enhancement percentage that compare with the standard gear, when the load acts near the middle of tooth.

Helix angle, β	$\sigma_{bending}/_{max}$ (N/mm^2)		Enhancement Percentage
	Standard Gear	Asymmetric Gear	
0° (spur)	18.535	16.16	12.815 %
5°	17.612	15.271	13.292 %
10°	16.898	14.594	13.635 %
15°	16.12	13.57	15.819 %
20°	15.63	13.14	15.931 %
22.5°	14.691	12.212	16.874 %
25°	13.523	11.55	14.5899 %
30°	12.747	10.941	14.168 %
35°	11.931	10.231	13.249 %

Where:

Enhancement Percentage $= \left(\frac{\sigma_{bending})_{standard\ case} - \sigma_{bending})_{asymmetric\ case}}{\sigma_{bending})_{standard\ case}}\right) * 100\%$

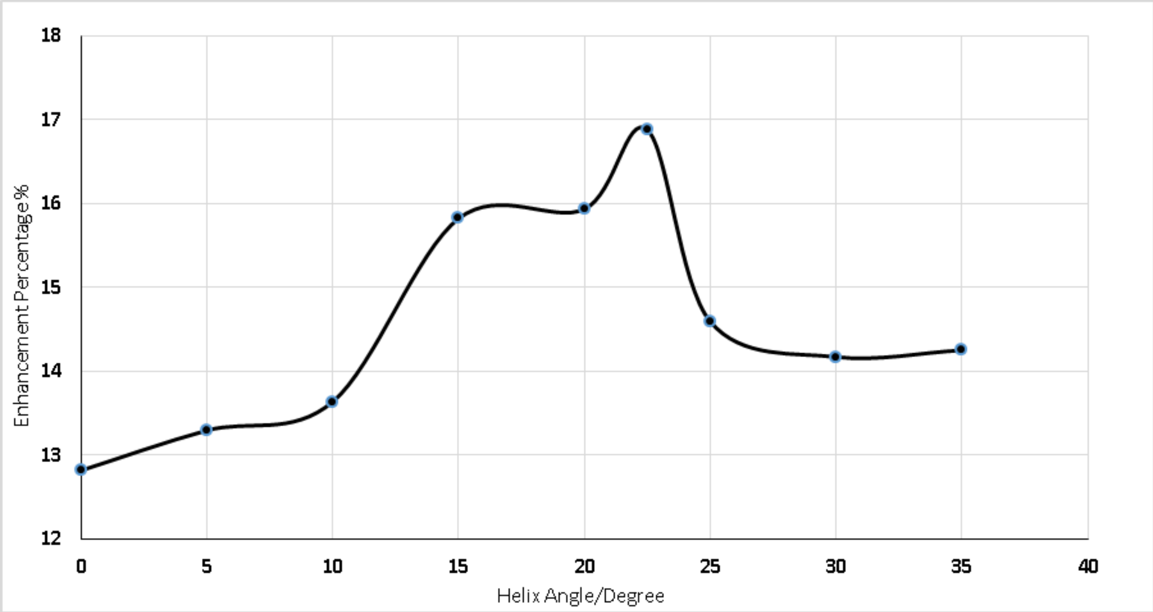


Figure 9. The relationship between the enhancement percentage with helix angle related to its, when the load acts near the middle of tooth.

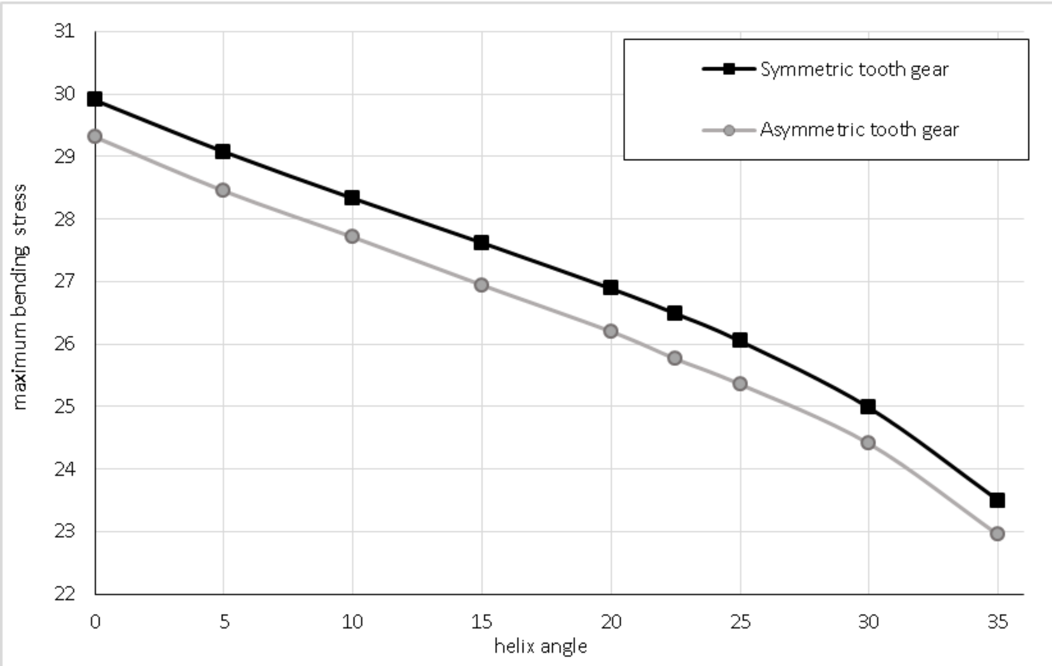


Figure 10. The relationship between the bending stress with helix angle, when the load acts at the tip of tooth.

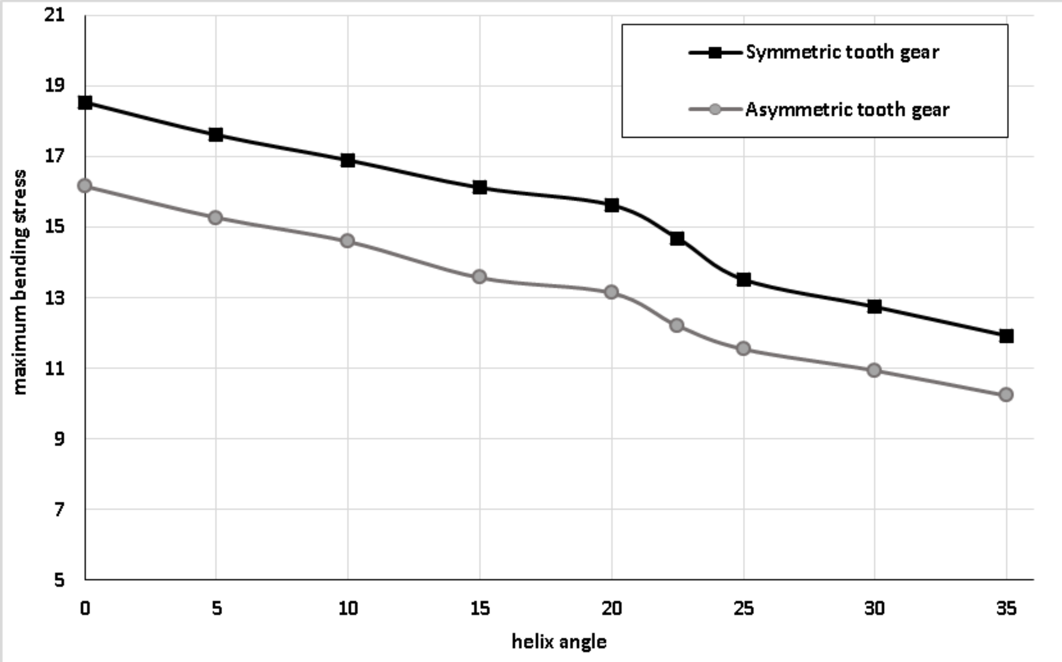


Figure 11. The relationship between the bending stress with helix angle, when the load acts near the middle of tooth.

Table 7. Comparison between the analytical and numerical values of maximum tooth bending stresses for standard gears, when the load acts at the tip of tooth.

Helix angle, β	Standard Gear $\sigma_{bending}/max$ (N/mm^2)		The Percentage difference %
	Numerical results	Analytical results	
0° (spur)	29.632	29.9	0.896 %
5°	28.797	29.075	0.954 %
10°	28.153	28.33	0.623 %
15°	27.375	27.619	0.883 %
20°	26.629	26.8859	0.956 %
22.5°	26.216	26.4857	1.019 %
25°	25.699	26.0456	1.332 %
30°	24.724	24.979	1.023 %
35°	23.257	23.498	1.028 %

Where:

$$\text{Difference Percentage} = \left(\frac{\sigma_{bending})_{analytical} - \sigma_{bending})_{numerical}}{\sigma_{bending})_{analytical}}\right) * 100\%$$

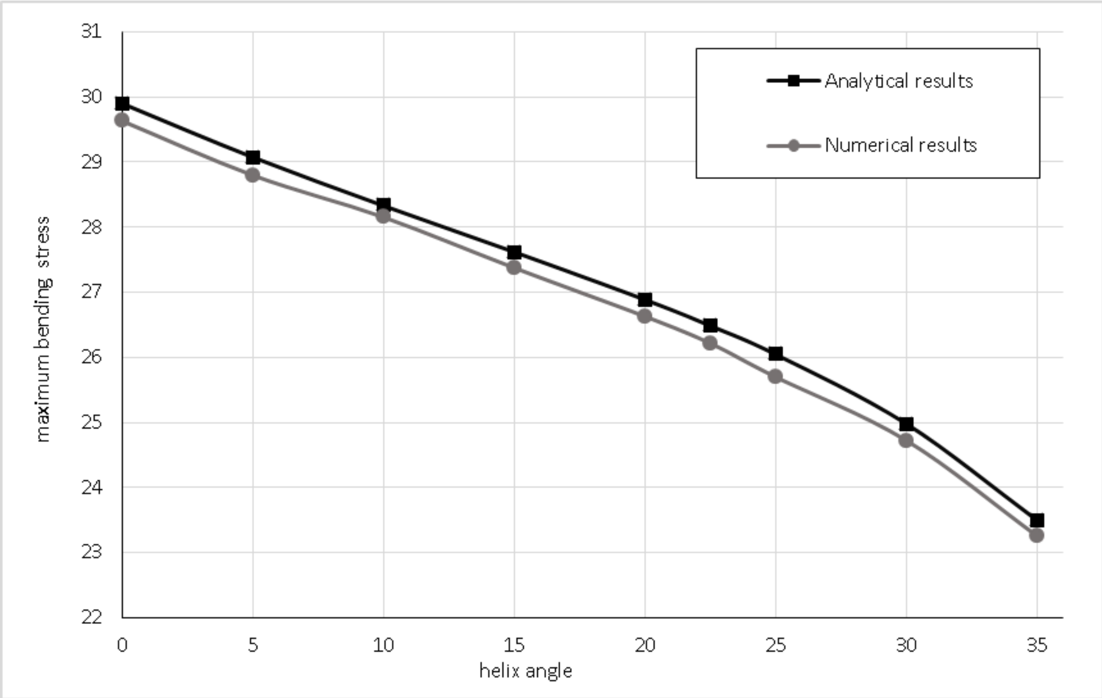


Figure 12. Comparison between the analytical and numerical results of the bending stresses with helix angles, when the load acts at the tip of tooth.