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MHD Peristaltic Flow of a Couple Stress Fluids with Heat and Mass Transfer through a Porous Medium

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Abstract

In the present article, we have studied the effects of heat and mass transfer on the MHD flow of an incompressible, electrically conducting couple stress fluid through a porous medium in an asymmetric flexible channel over which a traveling wave of contraction and expansion is produced, resulting in a peristaltic motion. The flow is examined in a wave frame of reference moving with the velocity of the wave. Formulas of dimensionless velocity, temperature and concentration are obtained analytically under assumptions of long wavelength and low Reynolds number. The effects of various parameters of interest such as the couple stress fluid parameter, Darcy number, Hartmann number and Schmidt number on these formulas were discussed and illustrated graphically through a set of figures.

Key words: peristalsis, Couple stress fluid, Porous medium, MHD flow, Heat transfer, Mass transfer.

1. Introduction

Peristalsis is a form of transporting fluids in which an induced wave causes the propagation of the flexible walls of a channel/tube. This mechanism is seen in many biological systems such as the transportation of urine from kidney to bladder, movement of chyme in the gastroin testinal tract, blood circulation in the small blood vessels, and in the ducts efferentes of the male reproductive tract. Also in industry the phenomenon of peristaltic pumping are used in many useful applications like transportation of sanitary fluids, blood pump in heart lung machines and also in transporting of corrosive and toxic liquids to avoid contamination with the outside environment.

Recently, Elshehawy et al. [2] studied the peristaltic transport in an asymmetric channel through a porous medium. Srinivas and Kothandapani [12] examined the heat transfer analysis for peristaltic flow in an asymmetric channel. Mekheimer and Abdelmaboud [7] discussed the influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus.

Since most of physiological and industrial fluids are non-Newtonian fluids, Numerous attempts were done by researchers to study various models of non-Newtonian fluids. We might mention some of the recent studies that dealt with the peristaltic flow of non-Newtonian fluids. Hayat et al. [4] and they have analyzed the effect of an induced magnetic field on the peristaltic transport of a Carreau fluid. Haroun [3] has studied the peristaltic transport of fourth grade fluid in an inclined asymmetric channel. Wang et al. [15] have examined the magnetohydrodynamic peristaltic motion of a Sisko fluid in symmetric or asymmetric channel. Nadeem and Akram [9] have studied the peristaltic flow of Williamson fluid in an asymmetric channel. Eldabe et al. [1] have made an interesting analysis on the mixed convective heat and mass transfer in a non-Newtonian fluid obeying the biviscosity model at a peristaltic surface with temperature dependent viscosity. Abdelmaboud and Mekheimer [6] analyzed the transport of second order fluid through a porous medium. Nadeem et al. [8] have discussed the influence of heat and mass transfer on peristaltic flow of a third order fluid in a diverging tube.

The couple stress fluid is a special case of the non-Newtonian fluids where these fluids are consisting of rigid randomly oriented particles suspended in a viscous medium and their sizes are taken into account. This model can be used to describe human and animal blood, infected urine from a diseased kidney and liquid crystals. There have only few attempts for studying the peristaltic flow of a couple stress fluids, first discussed by Stokes [13]. From the recent attempts dealing with the couple stress model, we refer to Mekheimer [5], as he has investigated the problem of the peristaltic transport of a couple sterss fluid in a uniform and non-uniform channel. Also Nadeem and Akram [10] have investigated the peristaltic flow of a couple stress fluid under the effect of induced magnetic field in an asymmetric channel, and Sobh [11] has studied the effect of slip velocity on peristaltic flow of a couple stress fluid in uniform and non uniform channels.

In dealing with heat and mass transfer problems, we observe a phenomenon called diffusion - thermo effect (Duffor effect) in which an energy flux could be generated by the concentration gradients in addition to that generated by the temperature gradients, also on the other hand mass fluxes could be created by heat gradients which is known by themal- diffusion effect (Sort effect).

To the best of our knowledge no attempt has been reported yet to discuss the peristaltic transport of a couple stress fluid in the presence of heat and mass transfer. The aim of the present study is to investigate the effect of the mixed convective heat and mass transfer on the peristaltic transport of a couple stress fluid in an asymmetric channel through a porous medium in the presence of magnetic field while taking into consideration the viscous dissipation effect during this work. So that this study is useful in filling that gap.

2. Mathematical Analysis

Consider the flow of an incompressible, electrically conducting, couple stress fluid through a porous medium in an asymmetric two dimensional channel having width $d_1 + d_2$ with a sinusoidal waves travelling down its walls with a constant speed c. A rectangular coordinate system (X, Y) is choosen such that X-axis lies along the direction of wave propagation and Y-axis transverse to it. The fluid is subjected to a constant transverse magnetic field $\vec{B} = (0, B_0, 0)$. The effects of induced magnetic field is neglected by considering small magnetic Reynolds number, also the effect of Julian dissipation is neglected.

The geometry of the channel walls is given by

$$Y = H_1(X,t) = d_1 + a_1 \cos\left[\frac{2\pi}{\lambda}(X - ct)\right] \quad \text{upper wall} \quad (1)$$

$$Y = H_2(X,t) = -d_2 - b_1 \cos\left[\frac{2\pi}{\lambda}(X - ct) + \phi\right] \quad \text{lower wall} \quad (2)$$

Where a_1 and b_1 are amplitudes of the waves . λ is the wavelength, t is the time, $d_1 + d_2$ is the width of the channel and the phase difference ϕ varies in the range $0 \le \phi \le \pi$, where a_1, b_1, d_1, d_2 and ϕ satisfies $a_1^2 + b_1^2 + 2a_1 b_1 \cos\phi \le (d_1 + d_2)^2$ in order for the channel walls not to collide with each other. At the lower wall of the channel the temperature is T_1 and the concentration is C_1 while at the upper wall the temperature is T_0 and the concentration is C_0. See Fig. (20).

In Laboratory frame (X, Y), the Governing equations are given by:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (3)$$

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} + \mu\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \eta\left(\frac{\partial^4 U}{\partial X^4} + 2\frac{\partial^4 U}{\partial X^2 \partial Y^2} + \frac{\partial^4 U}{\partial Y^4}\right) - \sigma B_0^2 U - \frac{\mu}{k} U$$

(4)

$$\rho\left(\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = -\frac{\partial P}{\partial Y} + \mu\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \eta\left(\frac{\partial^4 V}{\partial X^4} + 2\frac{\partial^4 V}{\partial X^2 \partial Y^2} + \frac{\partial^4 V}{\partial Y^4}\right) - \frac{\mu}{k} V$$

(5)

$$\xi\left(\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y}\right) = \frac{\kappa_1}{\rho}\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) + \nu\left[2\left(\frac{\partial U}{\partial X}\right)^2 + 2\left(\frac{\partial V}{\partial Y}\right)^2 + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)^2\right] +$$

$$\frac{\eta}{\rho}\left[\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right)^2 + \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right)^2\right] + \frac{D_m k_T}{c_s}\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right) \quad (6)$$

$$\left(\frac{\partial C}{\partial t} + U\,\frac{\partial C}{\partial X} + V\,\frac{\partial C}{\partial Y}\right) = D_m\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right) + \frac{D_m\,k_T}{\bar{T}}\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right)$$

(7)

in which U, V are the velocity components in the laboratory frame (X, Y) , t is the time, P is the pressure, ρ is the density, T is the temperature of the fluid, C is the concentration of the fluid, $\bar{T}$ is the mean value of T_0 and T_1, D_m is the coefficient of mass diffusivity, μ is the coefficient of viscosity of the fluid, ν is the kinematic viscosity of the fluid, ξ is the specific heat at constant pressure, k is the permeability parameter, σ is the electrical conductivity of the fluid, $\kappa_1$is the thermal conductivity of the fluid, c_s is the concentration susceptibility and η is a constant associated with the couple stress.

Introducing a wave frame (x, y) moving with the velocity c away from the fixed frame (X, Y) by the transformation

$$x = X - ct\,,\ y = Y\,,\ u = U - c\,,\ v = V\,,\ p(x) = P(X, t) \tag{8}$$

in which (u, v) are the components of the velocity in the wave frame.

and defining the following non dimensional quantities

$$\bar{x} = \frac{x}{\lambda}\,,\ \bar{y} = \frac{y}{d_1}\,,\ \bar{u} = \frac{u}{c}\,,\ \bar{v} = \frac{v}{c},\ \bar{t} = \frac{c}{\lambda}t,\ h_1 = \frac{H_1}{d_1}\,,\ h_2 = \frac{H_2}{d_1}\,,\ \delta = \frac{d_1}{\lambda},\ \ M = \sqrt{\frac{\sigma}{\mu}}B_0 d_1\,,\ \varphi = \frac{C - C_0}{C_1 - C_0}.$$

$$d = \frac{d_2}{d_1}\,,\ a = \frac{a_1}{d_1}\,,\ b = \frac{b_1}{d_1}\,,\ \bar{p} = \frac{d_1^2 p}{\mu c \lambda}\,,\ Re = \frac{\rho c d_1}{\mu}\,,\ \theta = \frac{T - T_0}{T_1 - T_0}\,,\ S_r = \frac{\rho D_m k_T (T_1 - T_0)}{\bar{T}\,\mu (C_1 - C_0)}\,,\ D_a = \frac{k}{d_1^2}\,,\ \gamma^2 = \frac{\mu}{\eta} d_1^2$$

$$P_r = \frac{\rho\,\nu\,\xi}{\kappa_1}\,,\ S_c = \frac{\mu}{\rho\,D_m}\,,\ D_f = \frac{\rho D_m k_T (C_1 - C_0)}{\mu\,\xi\,c_s\,(T_1 - T_0)}\ \ ,\ E_c = \frac{c^2}{\xi\,(T_1 - T_0)} \tag{9}$$

where Re is the Reynolds number, M is the Hartmann number, D_a is the Darcy number, P_r is the Prandtl number, S_c is the Schmidt number, S_r is the Soret number, E_c is the Eckert number, D_f is the Dufour number and γ is the couple stress fluid parameter.

The non-dimensional time averaged flows q and Q in the wave and in the laboratory frames respectively are related by

$$Q = 1 + d + q \tag{10.a}$$

in which $$q = \int_{h_1}^{h_2} u\,dy \tag{10.b}$$

3. Analytic Solutions

Using the above transformations (8) and the non-dimensional quantities (9) with the assumptions of long wave length and low Reynolds number. Eqs. (4) - (7) can be written after dropping the bars in the following form:

$$-\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{\gamma^2}\frac{\partial^4 u}{\partial y^4} - \left(M^2 + \frac{1}{D_a}\right)(u + 1) = 0 \tag{11}$$

$$\frac{\partial p}{\partial y} = 0 \qquad (12)$$

$$\frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 + \frac{E_c}{\gamma^2}\left(\frac{\partial^2 u}{\partial y^2}\right)^2 + D_f\,\frac{\partial^2 \varphi}{\partial y^2} = 0$$

(13)

$$\frac{1}{S_c}\frac{\partial^2 \varphi}{\partial y^2} + S_r\frac{\partial^2 \theta}{\partial y^2} = 0 \qquad (14)$$

and assuming that the components of the couple stress tensor at the walls to be zero [14], the corresponding boundary conditions in dimensionless form are given by

$$u = -1,\quad \frac{\partial^2 u}{\partial y^2} = 0,\quad \theta = 0\,,\quad \varphi = 0 \qquad \text{at}\quad y = h_1 = 1 + a\cos(2\,\pi\,x) \qquad (15)$$

$$u = -1\,,\quad \frac{\partial^2 u}{\partial y^2} = 0\,,\quad \theta = 1\,,\quad \varphi = 1 \qquad \text{at}\quad y = h_2 = -d - b\cos(2\,\pi\,x\, + \phi) \qquad (16)$$

From Eq. (12) $p \neq p(y)$ and therefore $\frac{\partial p}{\partial x} = \frac{dp}{dx}$.

The closed form solution of Eq. (11) is given by

$$u = A_1\,e^{-\frac{A_5 y}{\sqrt{2}}} + A_2 e^{\frac{A_5 y}{\sqrt{2}}} + A_3 e^{-\frac{A_6 y}{\sqrt{2}}} + A_4 e^{\frac{A_6 y}{\sqrt{2}}} - \frac{1+\left(\frac{dp}{dx}+M^2\right)D_a}{1+M^2\,D_a} \qquad (17)$$

where $A_5 = \sqrt{\gamma^2 - \frac{A_7}{D_a}}$, $A_6 = \sqrt{\gamma^2 + \frac{A_7}{D_a}}$ and $A_7 = \sqrt{\gamma^2\,D_a(-4 + (-4\,M^2 + \gamma^2)\,D_a}$

(18)

and using appropriate boundary conditions (15) and (16) to evaluate A_1, A_2, A_3 and A_4 , we get

$$A_1 = \frac{\frac{dp}{dx} e^{\frac{A_5(h_1+h_2)}{\sqrt{2}}} D_a\,(\gamma^2\,D_a + A_7)}{2\left[e^{\frac{A_5 h_1}{\sqrt{2}}} + e^{\frac{A_5 h_2}{\sqrt{2}}}\right](1+M^2\,D_a)\,A_7} \,,\qquad A_2 = \frac{\frac{dp}{dx}\,D_a\,(\gamma^2\,D_a + A_7)}{2\left[e^{\frac{A_5 h_1}{\sqrt{2}}} + e^{\frac{A_5 h_2}{\sqrt{2}}}\right](1+M^2\,D_a)\,A_7}$$

$$A_3 = \frac{\frac{dp}{dx} e^{\frac{A_6(h_1+h_2)}{\sqrt{2}}} D_a\,(-\gamma^2\,D_a + A_7)}{2\left[e^{\frac{A_6 h_1}{\sqrt{2}}} + e^{\frac{A_6 h_2}{\sqrt{2}}}\right](1+M^2\,D_a)\,A_7} \,,\qquad A_4 = \frac{\frac{dp}{dx}\,D_a\,(-\gamma^2\,D_a + A_7)}{2\left[e^{\frac{A_6 h_1}{\sqrt{2}}} + e^{\frac{A_6 h_2}{\sqrt{2}}}\right](1+M^2\,D_a)\,A_7}$$

(19)

Using Eqs. (10.b) , (17) and (19) to get an explicit form for $\frac{dp}{dx}$, we get

$$\frac{dp}{dx} = -\frac{((1+M^2\,D_a)(q+h_1-h_2))}{D_a\,(h_1-h_2)\left(1 - \frac{\sqrt{2}\,(\gamma^2\,D_a + A_7)\,Tanh\left[\frac{A_5}{2\sqrt{2}}(h_1-h_2)\right]}{A_7\,A_5\,(h_1-h_2)} + \frac{\sqrt{2}\,(\gamma^2\,D_a - A_7)\,Tanh\left[\frac{A_6}{2\sqrt{2}}(h_1-h_2)\right]}{A_7\,A_6\,(h_1-h_2)}\right)}$$

(20)

Using Eqs. (14) and (17) into Eq. (13), the solution of Eq. (13) in terms of θ in closed form is given by

$$\theta = A_8 + A_9\, y - \frac{E_c\, P_r\, y^2\, A_{14}}{(-1+A_{13})\, D_a} - \left(\frac{e^{\sqrt{2}\, y\, A_5}\, E_c\, P_r\, A_2^2\, (A_7 - 3\,\gamma^2\, D_a) - e^{\sqrt{2}\, y\, A_6}\, E_c\, P_r\, A_4^2\, (A_7 + 3\,\gamma^2\, D_a)}{8\,(-1+A_{13})\,\gamma^2\, D_a}\right)$$

$$+ \frac{E_c\, P_r\, \left(\left(-\, e^{-\sqrt{2}\, y\, A_5}\, A_1^2 + e^{-\sqrt{2}\, y\, A_6}\, A_3^2\right) A_7 + 3\,\gamma^2 \left(e^{-\sqrt{2}\, y\, A_5}\, A_1^2 + e^{-\sqrt{2}\, y\, A_6}\, A_3^2\right) D_a\right.}{8\,(-1+A_{13})\,\gamma^2\, D_a}$$

$$-\left[\frac{2\left(e^{\frac{y\,(A_5 - A_6)}{\sqrt{2}}}\, A_2\, A_3 + e^{\frac{y\,(-A_5 + A_6)}{\sqrt{2}}}\, A_1\, A_4\right) E_c\, P_r\, A_{10}}{(-1+A_{13})\,(A_5 - A_6)^2\, D_a}\right] + \left[\frac{2\left(e^{-\frac{y\,(A_5 + A_6)}{\sqrt{2}}}\, A_1\, A_3 + e^{\frac{y\,(A_5 + A_6)}{\sqrt{2}}}\, A_2\, A_4\right) E_c\, P_r\, A_{11}}{(-1+A_{13})\,(A_5 + A_6)^2\, D_a}\right].$$

(21)

and upon substituting Eq. (21) into Eq. (14) and solving ,we obtain a solution for φ in closed form as

$$\varphi = A_{15} + A_{16}\, y + \frac{A_{12}\, A_{14}\, y^2}{(-1+A_{13})\, D_a} + \frac{e^{\sqrt{2}\, y\, A_5}\, A_{12}\, A_2^2\, (A_7 - 3\,\gamma^2\, D_a)}{8\,(-1+A_{13})\,\gamma^2\, D_a} - \frac{A_{12}\, D_a\, e^{-\frac{y\,(3\, A_5 + A_6)}{\sqrt{2}}}}{8\,(-1+A_{13})\, A_7^3}\, [-4\,(\,4\, e^{2\sqrt{2}\, y\, A_5}\, A_2\, A_3$$

$$(\gamma^2 - A_5\, A_6)\; A_7 +\; 4\, e^{\sqrt{2}\, y\,(A_5 + A_6)}\, A_1\, A_4\, (\gamma^2 - A_5\, A_6)\; A_7 + 4\, e^{\sqrt{2}\, y\, A_5}\, A_1\, A_3\, (\gamma^2 + A_5\, A_6)\, A_7 +$$

$$4\, e^{\sqrt{2}\, y\,(2\, A_5 + A_6)}\, A_2\, A_4\, (\gamma^2 + A_5\, A_6)\, A_7 + e^{\frac{y\,(3\, A_5 - A_6)}{\sqrt{2}}}\, \gamma^2 A_3^2\, (-4 + 3\; A_7) +\; e^{\frac{y\,(3\, A_5 + A_6)}{\sqrt{2}}}\, \gamma^2\, A_4^2\, (-4 + 3\; A_7) +$$

$$e^{\frac{y\,(A_5 + A_6)}{\sqrt{2}}}\, \gamma^2\, A_1^2\, (4 + 3\; A_7)\,) + (\,-8\, e^{2\sqrt{2}\, y\, A_5}\, A_2\, A_3\, A_7\; A_{17} - \; 8\, e^{\sqrt{2}\, y\,(A_5 + A_6)}\, A_1\, A_4\, A_7\; A_{17} -$$

$$8\, e^{\sqrt{2}\, y\, A_5} A_1\, A_3\, A_7\, A_{19} - \; 8\, e^{\sqrt{2}\, y\,(2\, A_5 + A_6)}\, A_2\, A_4\, A_7\, A_{19} + \; e^{\frac{y\,(3\, A_5 - A_6)}{\sqrt{2}}}\, \gamma^2\, (-4\, M^2 +\; \gamma^2)\, A_3^2\, (-8 + 3\, A_7) +$$

$$e^{\frac{3\, y\,(A_5 + A_6)}{\sqrt{2}}}\, \gamma^2\, (-4\, M^2 +\; \gamma^2)\, A_4^2\; (-8 + 3\, A_7) +\; e^{\frac{y\,(A_5 + A_6)}{\sqrt{2}}}\, \gamma^2\, (-4\, M^2 +\; \gamma^2)\, A_1^2\; (8 + 3\, A_7)\,)\, D_a +$$

$$\gamma^2\, (-4\, M^2 +\; \gamma^2)^2 \left(-\, e^{\frac{y\,(A_5 + A_6)}{\sqrt{2}}}\, A_1^2 + e^{\frac{y\,(3\, A_5 - A_6)}{\sqrt{2}}}\, A_3^2 \;+ e^{\frac{3\, y\,(A_5 + A_6)}{\sqrt{2}}}\, A_4^2\right) D_a^2\;\;]\;.$$

(22)

where, $A_{10} = (-2 + (-2\, M^2 +\; A_5\, A_6\,) D_a\,)$, $A_{11} = (2 + (2\, M^2 +\; A_5\, A_6\,) D_a\,)$, $A_{12} = E_c\, P_r\, S_c\, S_r$

$A_{13} = D_f\; P_r\, S_c\, S_r$, $A_{14} = (A_1\, A_2 +\; A_3\, A_4)\, (1 + M^2\, D_a)$, $A_{17} = (\gamma^2\, A_5\, A_6 - \; 2M^2(-\gamma^2 + A_5\, A_6))$, $A_{18} = (\gamma^2\, A_5\, A_6 - \; 2M^2(\gamma^2 + A_5\, A_6))$, $A_{19}(-\gamma^2\, A_5\, A_6 +\; 2M^2(\gamma^2 + A_5\, A_6))$.

(23)

Using Appropriate boundary conditions from (15) and (16) into (21) and (22) and solving, we get

$$A_8 = -\,\frac{1}{h_1 - h_2}\, \left[\frac{-1}{(2(\,-1 + A_{13}\,)\, D_a}\right[\frac{-\, e^{\sqrt{2}\, A_5\, h_1}\, E_c\, P_r\, A_2^2\, (A_7 - 3\,\gamma^2\, D_a) +\; e^{\sqrt{2}\, A_6\, h_1}\, E_c\, P_r\, A_4^2\, (A_7 + 3\,\gamma^2\, D_a)}{4\,\gamma^2} -$$

$$\frac{(4\,e^{\frac{(A_5-A_6)\,h_1}{\sqrt{2}}}A_2\,A_3+4\,e^{\frac{(-A_5+A_6)\,h_1}{\sqrt{2}}}A_1\,A_4)\,E_c\,P_r\,A_{10}}{(A_5-A_6)^2}+\frac{(4\,e^{-\frac{(A_5+A_6)\,h_1}{\sqrt{2}}}A_1\,A_3+4\,e^{\frac{(A_5+A_6)\,h_1}{\sqrt{2}}}A_2\,A_4)\,E_c\,P_r\,A_{11}}{(A_5+A_6)^2}-$$

$$\frac{e^{-\sqrt{2}\,(A_5+A_6)}\,E_c\,P_r\,(\,e^{\sqrt{2}\,A_6\,h_1}\,A_1^2\,(A_7-3\,\gamma^2\,D_a)-\,e^{\sqrt{2}\,A_5\,h_1}\,A_3^2\,(A_7+3\,\gamma^2\,D_a))}{4\,\gamma^2}-\,2\,E_c\,P_r\,A_{14}\,h_1^2\,]\,h_2\;+$$

$$h_1[-1-\frac{e^{\sqrt{2}\,A_5\,h_2}\,E_c\,P_r\,A_2^2\,(A_7-3\,\gamma^2\,D_a)+\,e^{\sqrt{2}\,A_6\,h_2}\,E_c\,P_r\,A_4^2\,(A_7+3\,\gamma^2\,D_a)+\left(e^{-\sqrt{2}\,A_5\,h_2}\,A_1^2+e^{-\sqrt{2}\,A_6\,h_2}\,A_3^2\right)(\,3\,\gamma^2\,D_a-E_c\,P_r\,A_7\,)}{(8\,(-1+A_{13}\,)\,\gamma^2\,D_a)}-$$

$$\frac{(2\,e^{\frac{(A_5-A_6)\,h_2}{\sqrt{2}}}E_c\,P_r\,A_2\,A_3\,A_{10}+2\,e^{\frac{(-A_5+A_6)\,h_2}{\sqrt{2}}}E_c\,P_r\,A_1\,A_4\,A_{10}\,)}{(-1+A_{13}\,)\,(A_5-A_6)^2\,D_a}+\frac{2\,e^{-\frac{(A_5+A_6)\,h_2}{\sqrt{2}}}E_c\,P_r\,A_1\,A_3\,A_{11}+2\,e^{\frac{(A_5+A_6)\,h_2}{\sqrt{2}}}E_c\,P_r\,A_2\,A_4\,A_{11}}{(-1+A_{13}\,)\,(A_5+A_6)^2\,D_a}-$$

$$\frac{E_c\,P_r\,A_{14}\,h_2^2}{(-1+A_{13}\,)\,D_a}]]\,.$$

(24)

$$A_9=\frac{-1}{2\,h_1}\,[\frac{e^{\sqrt{2}\,A_5\,h_1}\,E_c\,P_r\,A_2^2\,(1+A_7+(M^2-\gamma^2)\,D_a\,)}{(-1+A_{13}\,)(A_7-\gamma^2\,D_a)}+\frac{e^{\sqrt{2}\,A_6\,h_1}\,E_c\,P_r\,A_4^2\,(-1+A_7+(-M^2+\gamma^2)\,D_a\,)}{(-1+A_{13}\,)(A_7+\gamma^2\,D_a)}-\frac{2\,E_c\,P_r\,A_{14}\,h_1^2}{(-1+A_{13})\,D_a}+$$

$$\frac{[E_cP_rA_7(\left(-e^{-\sqrt{2}\,A_5\,h_1}A_1^2+e^{-\sqrt{2}\,A_6\,h_1}A_3^2\right)+3\gamma^2D_a\left(e^{-\sqrt{2}\,A_5\,h_1}A_1^2+e^{-\sqrt{2}\,A_6\,h_1}A_3^2\right))]}{4\,(-1+A_{13})\,\gamma^2\,D_a}-\frac{(A_2A_3e^{\frac{(A_5-A_6)\,h_1}{\sqrt{2}}}+A_1A_4\,e^{-\frac{(A_5-A_6)\,h_1}{\sqrt{2}}})4E_cP_rA_{10}}{(-1+A_{13}\,)\,(A_5-A_6)^2\,D_a}+$$

$$\frac{(A_1A_3\,e^{-\frac{(A_5+A_6)\,h_1}{\sqrt{2}}}+A_2A_4\,e^{\frac{(A_5+A_6)\,h_1}{\sqrt{2}}})4E_cP_rA_{11}}{(-1+A_{13}\,)\,(A_5+A_6)^2\,D_a}-\frac{2}{h_1-h_2}\,[\frac{-1}{2(-1+A_{13})\,D_a}\,(\frac{e^{\sqrt{2}\,A_5\,h_1}\,E_c\,P_r\,A_2^2\,(-1-A_7+(-M^2+\gamma^2)\,D_a\,)}{-A_7+\gamma^2\,D_a}+$$

$$\frac{e^{\sqrt{2}\,A_6\,h_1}\,E_c\,P_r\,A_4^2\,(-1+A_7+(-M^2+\gamma^2)\,D_a\,)}{A_7+\gamma^2\,D_a}-\frac{(A_2\,A_3\,e^{\frac{(A_5-A_6)\,h_1}{\sqrt{2}}}+A_1A_4\,e^{-\frac{(A_5-A_6)\,h_1}{\sqrt{2}}})4\,E_c\,P_r\,A_{10}}{(A_5-A_6)^2}-2\,E_c\,P_r\,A_{14}\,h_1^2\;+$$

$$\frac{(A_1A_3\,e^{-\frac{(A_5+A_6)\,h_1}{\sqrt{2}}}+A_2A_4\,e^{\frac{(A_5+A_6)\,h_1}{\sqrt{2}}})4E_cP_rA_{11}}{(A_5+A_6)^2}-\frac{1}{4\,\gamma^2}\;\;e^{-\sqrt{2}\,(A_5+A_6)\,h_1}\,E_cP_r(\,e^{\sqrt{2}\,A_6\,h_1}\,A_1^2\,(A_7-\,3\,\gamma^2\,D_a)-$$

$$e^{\sqrt{2}\,A_5\,h_1}A_3^2(A_7+3\gamma^2D_a))h_2+h_1(-1+\frac{e^{\sqrt{2}\,A_5\,h_2}\,E_cP_rA_2^2\,(1+A_7+(M^2-\gamma^2)\,D_a\,)}{2\,(-1+A_{13}\,)(A_7-\gamma^2\,D_a)}+\frac{e^{\sqrt{2}\,A_6\,h_2}E_cP_rA_4^2\,(-1+A_7+(-M^2+\gamma^2)D_a)}{2\,(-1+A_{13}\,)(A_7+\gamma^2\,D_a)}+$$

$$\frac{E_c\,P_r\,((-\,e^{-\sqrt{2}\,A_5\,h_2}\,A_1^2+\,e^{-\sqrt{2}\,A_6\,h_2}\,A_3^2)\,A_7+3\,\gamma^2\,E_c\,P_r\,(\,e^{-\sqrt{2}\,A_5\,h_2}\,A_1^2+\,e^{-\sqrt{2}\,A_6\,h_2}\,A_3^2\,)\,D_a\,)}{8\,(-1\,+A_{13}\,)\,\gamma^2\,D_a}-\frac{E_c\,P_r\,A_{14}\,h_2^2}{(-1+A_{13})\,D_a}-$$

$$\frac{(A_2\,A_3\,e^{\frac{(A_5-A_6)\,h_2}{\sqrt{2}}}+\,A_1A_4\,e^{-\frac{(A_5-A_6)\,h_2}{\sqrt{2}}}\,)\,2\,E_c\,P_r\,A_{10}}{(-1+A_{13}\,)\,(A_5-A_6)^2\,D_a}+\frac{(A_1\,A_3\,e^{-\frac{(A_5+A_6)\,h_2}{\sqrt{2}}}+\,A_2A_4\,e^{-\frac{(A_5+A_6)\,h_2}{\sqrt{2}}}\,)\,2\,E_c\,P_r\,A_{11}}{(-1+A_{13}\,)\,(A_5+A_6)^2\,D_a}\;)]]\quad.$$

(25)

$$A_{15}\;=\frac{1}{h_1-h_2}\,[[-\,\frac{e^{\sqrt{2}\,A_5\,h_1}\,A_2^2\,A_{12}\,(\,A_7-3\,\gamma^2\,D_a)}{8\,(-1+A_{13}\,)\,\gamma^2\,D_a}+\frac{D_a\,A_{12}\,e^{-\frac{(3\,A_5+A_6)\,h_1}{\sqrt{2}}}}{8\,(-1+A_{13}\,)\,A_7^3}\,(-4\,(\,4\,e^{2\,\sqrt{2}\,A_5\,h_1}\,A_2\,A_3\,(\gamma^2-A_5\,A_6)\,A_7\,+$$

$$4e^{\sqrt{2}\,(A_5+A_6)\,h_1}\,A_1A_4(\gamma^2-A_5A_6)A_7+4\,e^{\sqrt{2}\,A_5\,h_1}\,A_1A_3(\gamma^2+A_5A_6)A_7+4e^{\sqrt{2}(2\,A_5+A_6)h_1}\,A_2A_4(\gamma^2+A_5A_6)A_7\,+$$

$$e^{\frac{(3\,A_5-A_6)\,h_1}{\sqrt{2}}}\,\gamma^2A_3^2(-4+3\,A_7)+e^{\frac{3\,(\,A_5+A_6)\,h_1}{\sqrt{2}}}\,\gamma^2\,A_4^2\,(-4+3\,A_7)+e^{\frac{(\,A_5+A_6)\,h_1}{\sqrt{2}}}\,\gamma^2A_1^2\,(4+3\,A_7))\,+$$

$$(-8\, e^{2\sqrt{2} A_5 h_1} A_2 A_3 A_7 A_{17} - 8\, e^{\sqrt{2}(A_5+A_6) h_1} A_1 A_4 A_7 A_{17} - 8\, e^{\sqrt{2}(2 A_5+A_6) h_1} A_2 A_4 A_7 A_{19} -$$

$$8\, e^{\sqrt{2} A_5 h_1} A_1 A_3 A_7 A_{19} + e^{\frac{(3 A_5-A_6) h_1}{\sqrt{2}}} \gamma^2 A_3^2 (-8+3 A_7)(-4 M^2+\gamma^2) +$$

$$e^{\frac{3(A_5+A_6) h_1}{\sqrt{2}}} \gamma^2 A_4^2 (-8+3 A_7)(-4 M^2+\gamma^2) + e^{\frac{(A_5+A_6) h_1}{\sqrt{2}}} \gamma^2 A_1^2 (8+3 A_7)(-4 M^2+\gamma^2)) D_a +$$

$$\gamma^2 (-4 M^2+\gamma^2)\left(-e^{\frac{(A_5+A_6) h_1}{\sqrt{2}}} A_1^2 + e^{\frac{(3 A_5-A_6) h_1}{\sqrt{2}}} A_3^2 + e^{\frac{3(A_5+A_6) h_1}{\sqrt{2}}} A_4^2\right) D_a^2) - \frac{A_{14} h_1^2}{(-1+A_{13}) D_a}] h_2 +$$

$$h_1(-1 + \frac{e^{\sqrt{2} A_5 h_2} A_2^2 A_{12} (A_7 - 3\gamma^2 D_a)}{8(-1+A_{13})\gamma^2 D_a} - \frac{D_a A_{12}\, e^{-\frac{(3 A_5+A_6) h_2}{\sqrt{2}}}}{8(-1+A_{13}) A_7^3} (-4(4\, e^{2\sqrt{2} A_5 h_2} A_2 A_3 (\gamma^2 - A_5 A_6) A_7 +$$

$$4e^{\sqrt{2}(A_5+A_6) h_2} A_1 A_4 (\gamma^2 - A_5 A_6) A_7 + 4e^{\sqrt{2} A_5 h_2} A_1 A_3 (\gamma^2 + A_5 A_6) A_7 + 4e^{\sqrt{2}(2A_5+A_6) h_2} A_2 A_4 (\gamma^2 + A_5 A_6) A_7 +$$

$$e^{\frac{(3 A_5-A_6) h_2}{\sqrt{2}}} \gamma^2 A_3^2 (-4+3 A_7) + e^{\frac{3(A_5+A_6) h_2}{\sqrt{2}}} \gamma^2 A_4^2 (-4+3 A_7) + e^{\frac{(A_5+A_6) h_2}{\sqrt{2}}} \gamma^2 A_1^2 (4+3 A_7)) +$$

$$(-8\, e^{2\sqrt{2} A_5 h_2} A_2 A_3 A_7 A_{17} - 8\, e^{\sqrt{2}(A_5+A_6) h_2} A_1 A_4 A_7 A_{17} - 8\, e^{\sqrt{2}(2 A_5+A_6) h_2} A_2 A_4 A_7 A_{19} -$$

$$8\, e^{\sqrt{2} A_5 h_2} A_1 A_3 A_7 A_{19} + e^{\frac{(3 A_5-A_6) h_2}{\sqrt{2}}} \gamma^2 A_3^2 (-8+3 A_7)(-4 M^2+\gamma^2) +$$

$$e^{\frac{3(A_5+A_6) h_2}{\sqrt{2}}} \gamma^2 A_4^2 (-8+3 A_7)(-4 M^2+\gamma^2) + e^{\frac{(A_5+A_6) h_2}{\sqrt{2}}} \gamma^2 A_1^2 (8+3 A_7)(-4 M^2+\gamma^2)) D_a +$$

$$\gamma^2 (-4 M^2+\gamma^2)^2\left(-e^{\frac{(A_5+A_6) h_2}{\sqrt{2}}} A_1^2 + e^{\frac{(3 A_5-A_6) h_2}{\sqrt{2}}} A_3^2 + e^{\frac{3(A_5+A_6) h_2}{\sqrt{2}}} A_4^2\right) D_a^2) - \frac{A_{14} h_2^2}{(-1+A_{13}) D_a}] \quad .$$

(26)

$$A_{16} = \frac{A_{12}\, e^{-\frac{(3 A_5+A_6) h_1}{\sqrt{2}}}}{8(-1+A_{13}) A_7^2 (h_1-h_2)} (-4\left(-e^{\frac{(A_5+A_6) h_1}{\sqrt{2}}} A_1^2 - e^{\frac{(5 A_5+A_6) h_1}{\sqrt{2}}} A_2^2 + e^{\frac{(3 A_5-A_6) h_1}{\sqrt{2}}} A_3^2 + e^{\frac{3(A_5+A_6) h_1}{\sqrt{2}}} A_4^2\right) A_7 +$$

$$(16e^{2\sqrt{2} A_5 h_1} A_2 A_3 (-\gamma^2 + A_5 A_6) + (16e^{\sqrt{2}(A_5+A_6) h_1} A_1 A_4 (-\gamma^2 + A_5 A_6) - (16e^{\sqrt{2}(A_5+A_6) h_1} A_1 A_4 (\gamma^2 +$$
$$A_5 A_6) -$$

$$(16\, e^{\sqrt{2}(2 A_5+A_6) h_1} A_2 A_4 (\gamma^2 + A_5 A_6) + e^{\frac{(3A_5-A_6) h_1}{\sqrt{2}}} A_3^2 A_{15} + e^{\frac{3(A_5+A_6) h_1}{\sqrt{2}}} A_4^2 A_{15} - e^{\frac{(A_5+A_6) h_1}{\sqrt{2}}} A_1^2 A_{16} -$$

$$e^{\frac{(5 A_5+A_6) h_1}{\sqrt{2}}} A_2^2 A_{16}) D_a + (3\, e^{\frac{(A_5+A_6) h_1}{\sqrt{2}}} \gamma^2 (-4 M^2+\gamma^2) A_1^2 + 3\, e^{\frac{(5 A_5+A_6) h_1}{\sqrt{2}}} \gamma^2 (-4 M^2+\gamma^2) A_2^2 +$$

$$3\, e^{\frac{(3 A_5-A_6) h_1}{\sqrt{2}}} \gamma^2 (-4 M^2+\gamma^2) A_3^2 + 3\, e^{\frac{3(A_5+A_6) h_1}{\sqrt{2}}} \gamma^2 (-4 M^2+\gamma^2) A_4^2 - 8\, e^{2\sqrt{2} A_5 h_1} A_2 A_3 A_{17} -$$

$$8\, e^{\sqrt{2}(A_5+A_6) h_1} A_1 A_4 A_{17} + 8\, e^{\sqrt{2} A_5 h_1} A_1 A_3 A_{18} + 8\, e^{\sqrt{2}(2 A_5+A_6) h_1} A_2 A_4 A_{18}) D_a^2) +$$

$$+ \frac{A_{12} A_{14} h_1^2}{(-1+A_{13}) D_a (-h_1+h_2)} + \frac{e^{-\frac{(3 A_5+A_6) h_2}{\sqrt{2}}}}{8(-1+A_{13}) A_7^2 (-h_1+h_2)} (4 A_{12}((e^{\frac{(A_5+A_6) h_2}{\sqrt{2}}} A_1^2 + e^{\frac{(5 A_5+A_6) h_2}{\sqrt{2}}} A_2^2 - e^{\frac{(3 A_5-A_6) h_2}{\sqrt{2}}} A_3^2 -$$

$$e^{\frac{3(A_5+A_6)h_2}{\sqrt{2}}} A_4^2)\, A_7 + 8e^{\frac{(3A_5+A_6)h_2}{\sqrt{2}}} \gamma^2(A_1 A_2 + A_3 A_4)\, h_2^2) - D_a(-16e^{2\sqrt{2}A_5 h_1} A_2 A_3(-\gamma^2 + A_5A_6)A_{12} -$$

$$16e^{\sqrt{2}(A_5+A_6)h_2} A_1A_4\, (-\gamma^2 + A_5A_6)\, A_{12} + 16e^{\sqrt{2}(A_5+A_6)h_2} A_1A_3(\gamma^2 + A_5A_6)\, A_{12} +$$

$$16\, e^{\sqrt{2}(2A_5+A_6)h_2} A_2 A_4 (\gamma^2 + A_5 A_6)\, A_{12} + 32\, e^{\frac{(3A_5+A_6)h_2}{\sqrt{2}}} \gamma^2 (-1 + A_{13}) - e^{\frac{(3A_5-A_6)h_2}{\sqrt{2}}} A_3^2 A_{12} A_{15} -$$

$$e^{\frac{3(A_5+A_6)h_2}{\sqrt{2}}} A_4^2 A_{12}A_{15} + e^{\frac{(A_5+A_6)h_2}{\sqrt{2}}} A_1^2 A_{12}A_{16} + e^{\frac{(5A_5+A_6)h_2}{\sqrt{2}}} A_2^2 A_{12}A_{16} + 8\, e^{\frac{(3A_5+A_6)h_2}{\sqrt{2}}} \gamma^2(-8M^2 + \gamma^2)$$

$$(A_1 A_2 + A_3 A_4)\, A_{12}\, h_2^2) - D_a^2 (-3\, e^{\frac{(A_5+A_6)h_2}{\sqrt{2}}} \gamma^2 (-4M^2 + \gamma^2)\, A_1^2 A_{12} - 3\, e^{\frac{(5A_5+A_6)h_2}{\sqrt{2}}} \gamma^2$$

$$(-4M^2 + \gamma^2)\, A_2^2 A_{12} - 3\, e^{\frac{(3A_5-A_6)h_2}{\sqrt{2}}} \gamma^2 (-4M^2 + \gamma^2)\, A_3^2 A_{12} - 3\, e^{\frac{3(A_5+A_6)h_2}{\sqrt{2}}} \gamma^2 (-4M^2 + \gamma^2)\, A_4^2 A_{12} -$$

$$8\, e^{\frac{(3A_5+A_6)h_2}{\sqrt{2}}} \gamma^2 (-4M^2 + \gamma^2)(-1 + A_{13}) + 8\, e^{2\sqrt{2}A_5 h_2} A_2 A_3 A_{17} A_{12} + 8\, e^{\sqrt{2}(A_5+A_6)h_2} A_1A_4A_{17}A_{12} -$$

$$8\, e^{\sqrt{2}A_5 h_2} A_1 A_3 A_{12} A_{18} - 8\, e^{\sqrt{2}(2A_5+A_6)h_2} A_2 A_4 A_{12} A_{18} +)D_a^2) + 8\, e^{\frac{(3A_5+A_6)h_2}{\sqrt{2}}} M^2 \gamma^2(-4M^2 + \gamma^2)$$

$$(A_1 A_2 + A_3 A_4)\, A_{12}\, h_2^2 \ .$$

(27)

4. Results and discussions

In this section, we shall discuss the influence of various physical parameters of interest on the pressure gradient dp/dx, the pressure rise ΔP, the temperature profile θ and the concentration profile φ. For this purpose Figures (1) - (19) were prepared. In all these figures, as $\gamma \to \infty$, this corresponds to the case of considering a Newtonian fluid.

Figures (1) - (4) illustrate the variations of dp/dx for a given wavelength versus x, where $x \in [0,1]$. Figure (1) shows that by increasing M, dp/dx increases in the narrow part of the channel $x \in [0.27,0.64]$ and decreases in the wider part of the channel $x \in [0,0.27] \cup [0.64,1]$. Figure (2) indicates that the effect of D_a on dp/dx is quite opposite to that of M. From Figure (3) it can be seen that an increase in γ decreases dp/dx in the narrow part of the channel $x \in [0.27,0.64]$ while in the wider part of the channel $x \in [0,0.27] \cup [0.64,1]$ there is no noticeable difference. Figure (4) indicates that as ϕ increases, a lesser amount of pressure gradient is required in order to pass the flow in the narrow part of the channel.

Figures (5) - (7) present the variation of the pressure rise ΔP per wavelength against the time averaged flux Q.When pressure difference $\Delta P = 0$ which is the case of free pumping, the corresponding time averaged flux Q is denoted by $Q^* > 0$. Here we subdivide the graph into four regions, (I) $\Delta P = 0$ and

$Q = Q^* > 0$ (free pumping region), (II) $\Delta P > 0$ and $Q < 0$ (backward pumping region), (III) $\Delta P > 0$ and $Q^* > Q > 0$ (peristaltic pumping region), (IV) $\Delta P < 0$ and $Q^* < Q$ (co-pumping region). Figure (5) depicts that with increasing γ, ΔP decreases in the backward, peristaltic and free pumping regions till it reaches a critical value $Q = 1.6$ in the co-pumping region where ΔP starts to increase by increasing γ. From Figure (6) it is noticed that by increasing D_a , ΔP decreases in the backward pumping region till it reaches a critical value $Q = 0.62$ in the peristaltic pumping region after which ΔP increases with decreasing D_a . From Figure (7) we observe that the effect of M is quite opposite to that of D_a in all pumping regions, however that critical value $Q = 0.62$ remains unchanged.

Figures (8) - (13) describe the variation of the temperature profile with y for several values of D_a , E_c , P_r , γ and M. From Figures (8), (9), (12) and (13) it is clear that by increasing E_c , P_r, D_a and S_c the temperature profile increases, while from Figures (10) and (11) we observe that the temperature profile decreases with the increase in γ and M.

Figures (14) - (19) are plotted to study the effects of S_r , D_f , D_a , γ, S_c and M on the concentration profile. Here we have chosen the values of S_r and D_f such that their product is a constant value, as we assume that the mean temperature is kept constant. Figure (15) shows that by decreasing D_f and increasing S_r the concentration profile decreases, while in Figure (16) it is clear that by increasing D_f and decreasing S_r the concentration profile increases. Figures (17) and (19) show that concentration profile decreases with the increase in D_a and S_c . Figures (14) and (18) illustrates that by increasing M and γ the concentration profile increases.

5. Conclusion

In this article, we have presented a mathematical model that describes a MHD peristaltic flow of a couple stress fluid through a porous medium in an asymmetric channel in presence heat and mass transfer. The governing equations of the problem were solved analytically under assumptions of long wavelength and small Reynolds number. A set of graphs were plotted in order to analyze the effects of various physical parameters on these solutions. The main findings can be summarized as follows:

- The peristaltic pumping region increases as the couple stress parameter γ decreases.
- By decreasing the couple stress parameter γ, the longitudinal pressure gradient dp/dx increases in the narrow part of the channel while in the wider part there is no appreciable difference.
- By increasing the couple stress parameter γ the temperature profile decreases and the concentration profile increases.
- Increasing the value of S_c leads to an increase in the temperature profile whereas it causes a decrease in the concentration profile.

- The concentration profile of the fluid decreases with decrease of D_f (or increase in S_r) and vise versa.
- By letting $\gamma \to \infty,\ D_a \to \infty,\ D_f \to 0,\ S_r \to 0,\ S_c \to 0$, we can get the results obtained for the temperature profiles by Srinivas and Kothandapani [12].

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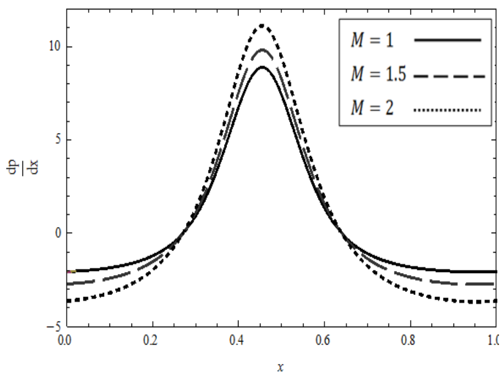

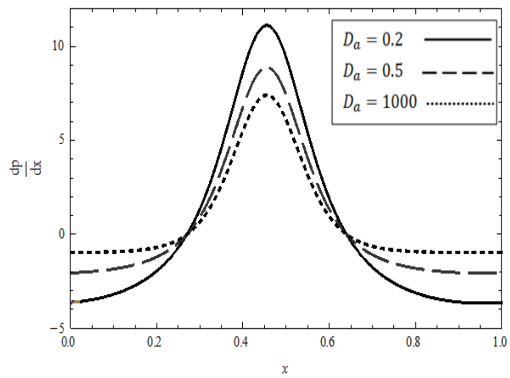

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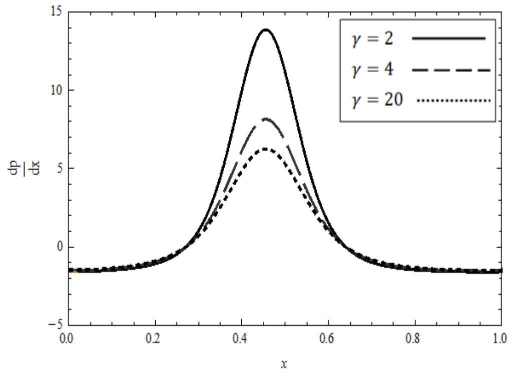
Figure (1). Pressure gradient versus x for $a = 0.6$, $b = 0.7, d = 1.5, \phi = \frac{\pi}{6}, q = -2, \gamma = 4, D_a = 0.5$.

Figure (2). Pressure gradient versus x for $a = 0.6$, $b = 0.7, d = 1.5, \phi = \frac{\pi}{6}, q = -2, \gamma = 4, M = 1$.



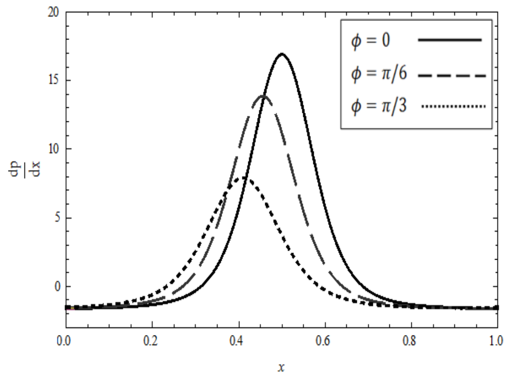
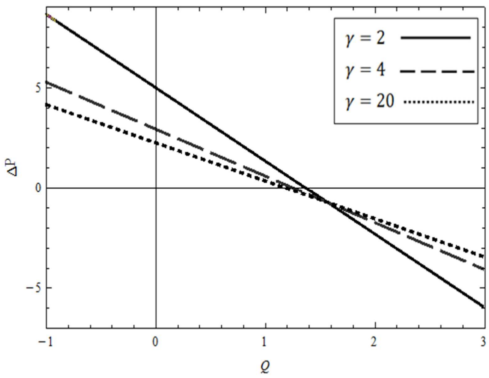


Figure (3). Pressure gradient versus x for $a = 0.6$, $b = 0.7, d = 1.5, \phi = \frac{\pi}{6}, q = -2, M = 1, D_a = 1.$

Figure (4). Pressure gradient versus x for $a = 0.6$, $b = 0.7, d = 1.5, \gamma = 2, q = -2, M = 1, D_a = 1.$



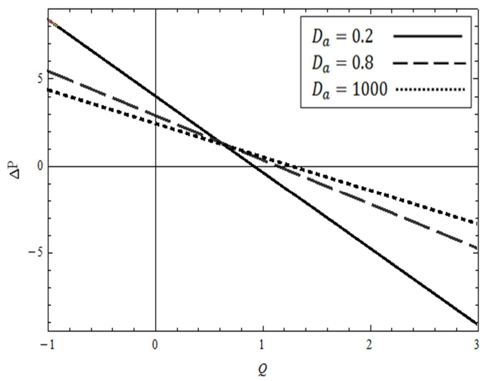


Figure (5). Pressure rise versus Q for $a = 0.7$, $b = 1.2,\ d = 2, M = 0.5, \phi = \frac{\pi}{4}, D_a = 2.$

Figure (6). Pressure rise versus Q for $a = 0.7$, $b = 1.2\ , d = 2, M = 0.5, \phi = \frac{\pi}{4}, \gamma = 5.$

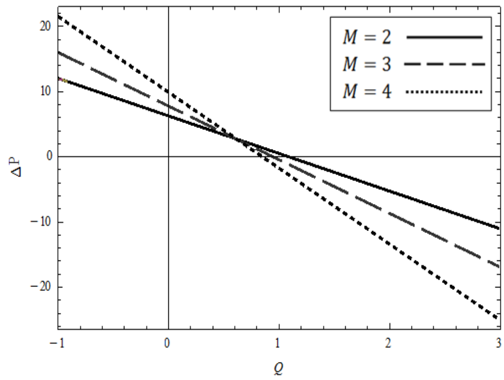


Figure (7). Pressure rise versus Q for $a = 0.$ $b = 1.2, d = 2, \gamma = 2, \phi = \frac{\pi}{4}, D_a = 1.$

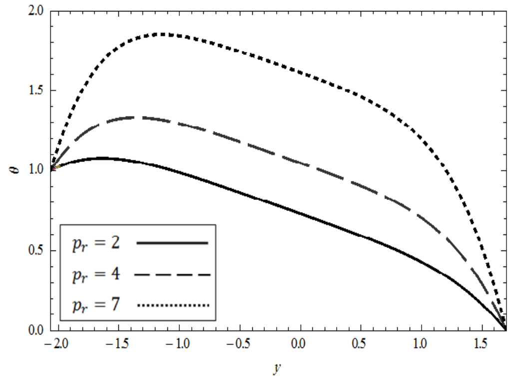


Figure (8). Temperature profile for $a = 0.7, b = 0.8,$ $d = 1.5, q = -1.5, \phi = \frac{\pi}{4}, M = 1, D_a = 2, \gamma = 4,$ $E_c = 0.5, S_r = 0.6, D_f = 0.1, S_c = 0.5, x = 0.$

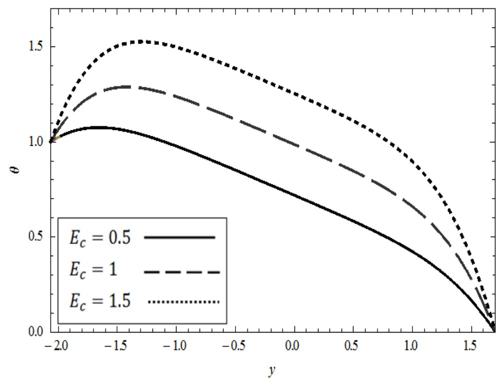


Figure (9). Temperature profile for $a = 0.7, b = 0.8,$ $d = 1.5, q = -1.5, \phi = \frac{\pi}{4}, M = 1, \gamma = 4, D_a = 1$ $P_r = 2, S_r = 0.6, D_f = 0.1, S_c = 0.5, x = 0.$

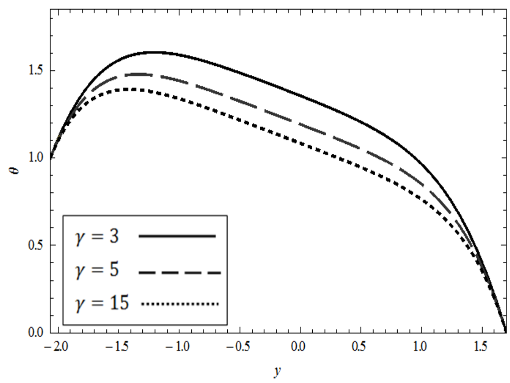


Figure (10). Temperature profile for $a = 0.7, b = 0.8,$ $d = 1.5, q = -1, \phi = \frac{\pi}{4}, M = 1, D_a = 1, E_c = 1,$ $P_r = 2, S_r = 0.6, D_f = 0.1, S_c = 0.5, x = 0.$

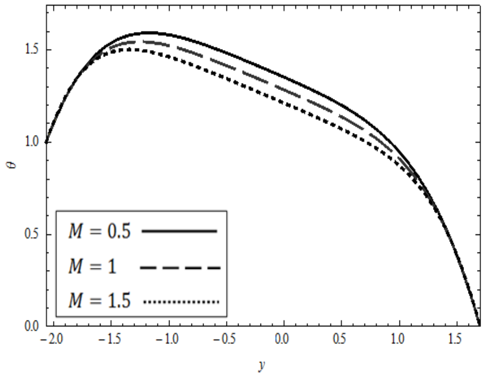


Figure (11). Temperature profile for $a = 0.7, b = 0.8,$

$d = 1.5, q = -1, \phi = \frac{\pi}{4}, D_a = 2, \gamma = 4, E_c = 1,$

$P_r = 2,\ S_r = 0.6, D_f = 0.1,\ S_c = 0.5, x = 0.$

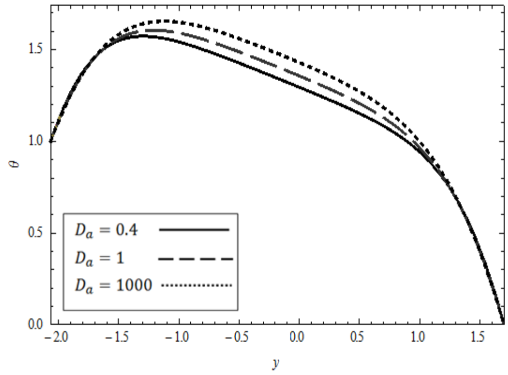


Figure (12). Temperature profile for $a = 0.7, b = 0.8,$

$d = 1.5, q = -1, \phi = \frac{\pi}{4},\ \gamma = 4, M = 1, P_r = 2,$

$E_c = 1, S_r = 0.6, D_f = 0.1, S_c = 0.5, x = 0.$

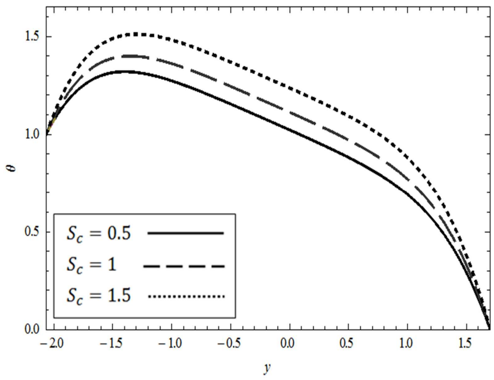


Figure (13). Temperature profile for $a = 0.7, b = 0.8,$

$d = 1.5, q = -1.5, \phi = \frac{\pi}{4}, D_a = 1, \gamma = 4, M = 1,$

$E_c = 0.5, P_r = 4,\ S_r = 0.6, D_f = 0.1, x = 0.$

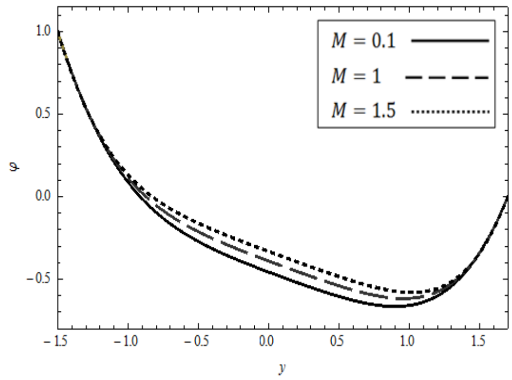


Figure (14). concentration profile for $a = 0.7, b = 1.2,$

$d = 1.5, q = -1.5, \phi = \frac{\pi}{2}, D_a = 1, \gamma = 4, P_r = 4,$

$E_c = 0.8, S_r = 1, D_f = 0.06, S_c = 1, x = 0.$

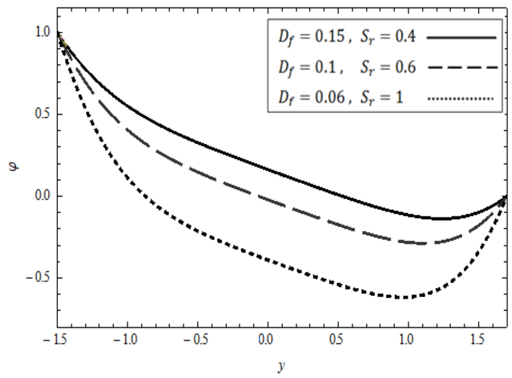
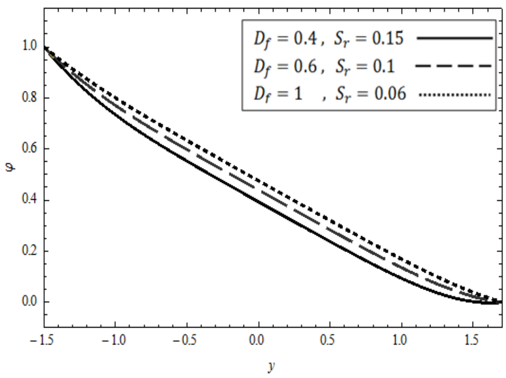


Figure (15). Concentration profile for $a = 0.7, b = 1.2,$

$d = 1.5, q = -1.5, \phi = \frac{\pi}{2}, D_a = 1, \gamma = 4, E_c = 0.8$

$P_r = 4, M = 1, S_c = 1, x = 0.$



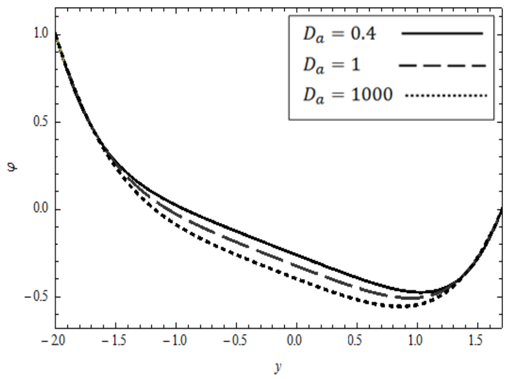


Figure (16). concentration profile for $a = 0.7, b = 1.2,$

$d = 1.5, q = -1.5, \phi = \frac{\pi}{2}, D_a = 1, \gamma = 4, E_c = 0.8,$

$P_r = 4, M = 1, S_c = 1, x = 0.$

Figure (17). Concentration profile for $a = 0.7, b = 1.2,$

$d = 2, q = -1.5, \phi = \frac{\pi}{2}, \gamma = 4, E_c = 1, M = 1$

$P_r = 4, S_r = 0.6, D_f = 0.1, S_c = 1, x = 0.$

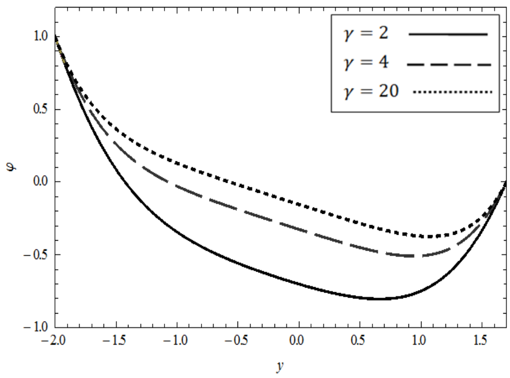


Figure (18). concentration profile for $a = 0.7, b = 1.2,$ $d = 2, q = -1.5, \phi = \frac{\pi}{2}, M = 1,\ D_a = 1, P_r = 4,$ $E_c = 1,\ S_r = 0.6,\ D_f = 0.1, S_c = 1, x = 0$.

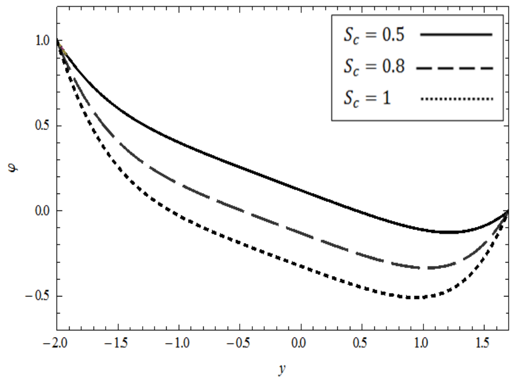


Figure (19). Concentration profile for $a = 0.7, b = 1.2,$ $d = 2, q = -1.5, \phi = \frac{\pi}{2}, \gamma = 4, E_c = 1, M = 1,$ $P_r = 4,\ D_a = 1, S_r = 0.6, D_f = 0.1, x = 0$.

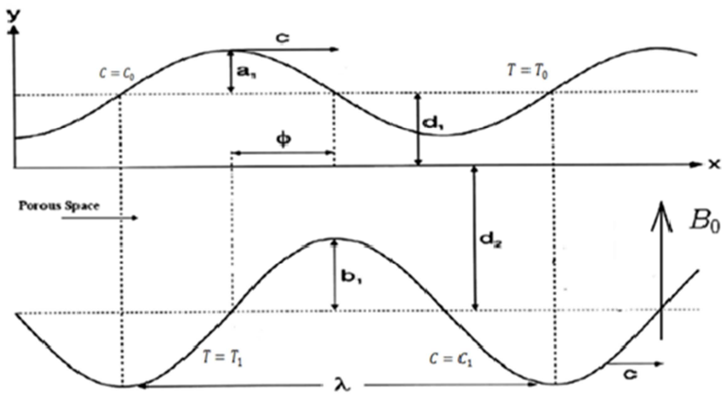


Figure (20).
Geometry of the problem