

The Dynamic Modelling and Electrical Profiles of Three Phase Induction Motor Using MATLAB Program

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Abstract
This paper addresses the dynamic behaviour of 3-φ induction machines based on a quality mathematical model and computer program simulation in a stationary reference frame to avoid the complexity involved in the course of solving time-varying differential equations obtained from the dynamic model. Laplace transform of the dynamic equations was carried out to derive analytical solutions the current variables. 4th 0rder Runge-Kutta method is adopted to give numerical solution of the dynamic model. The basic electrical characteristics of the 3-φ induction machine are considered in the two-phase approximate equivalent model. The graphical curves were obtained from MATLAB program code developed.
Keywords: 3-ϕ Induction Machines, Dynamic Model, Reference Frame, αβ-Coordinate, Analytical and Numerical Solution, MATLAB program code

1. Introduction

With the advancement in motion control of an automation systems used in every section of industrial and commercial activities, induction motors AC drives are being preferred in contrast to their DC drives counterparts because of its low maintenance cost while offering equal and often superior dynamic performances in terms of speed and torque characteristics when compared with the DC drives.

However, squirrel cage rotor of induction motor is the most commonly used AC drives. The actual rotor cage contains a relatively large number of conductors (rotor bars) which are short circulated by the end rings.

It is clear that the dynamic model of an induction machine which comprises differential equations and algebraic expressions relating the machine variables (voltage, current and flux) and parameters during transient process and also, in the steady state is given by the voltage equations (Kirchoff's and Faraday's laws) and Newton's differential equations of motion that described the electrical and mechanical subsystem of the induction motor respectively. But these set of differential equations and expressions describing the dynamic behaviors of the induction motor are time-varying due to relative motion in the electric circuits of an induction motor (except for the locked rotors). Owing to complexity involved when solving these time-varying differential equations, even when they are being transformed to complex frequency (s) domain (i.e Laplace transformed which is shown later) a change of variables is often adopted to minimize such complexity arises from the differential equations in the course of electric machine analysis.

Several transformation of variables approach have been developed (park, Stanley, Clarke, Brereton and kron et al) to eliminates time varying inductances by referring the stator and rotor variables to a frame of reference that may rotate at an angular velocity θ_{ef} or remain stationary (i.e θ_{ef} =0) and this is known as arbitrary reference form.

Because of the uncertainty in the choice of reference frame to give accurate and precise results while investigating the electrical behaviors of induction machines, dynamic modeling and steady state analysis of an induction machines have gained popular research interests.

Therefore, one of the reference frames used in this paper to validate the dynamic response of three phase induction motor is a reference frame fixed in the stator (stationary reference frame). In this transformation method, the stator AC vector variables defined by radial and tangential space vectors of cylindrical coordinate are represented by rectangular coordinate system of two orthogonal axes denoted by α and β on the same plane (Clarke E). By this approach, the original variables f_{abcx} of the three phase machine model can be reduced to a sample, clear and intuitive (which is the basic purpose of any research wok) two-phase equivalent model having the same flux, torque and power as the original three phase induction machine. Note that the subscript *'x'* in the variable f_{abcx} could either be 's' or "r" denoting variables and parameters associated with either stator or rotor circuits respectively.

2. Induction motor mathematical model

It is generally known, that the three phase induction motor has three sets of windings, spatially displaced by $2\pi/3$ electrical degrees. If these stator windings star (Y) connected are being fed by a balanced three phase

supply voltage defined as;

$$\begin{aligned} V_{as} &= \sqrt{2}V_s \cos(\omega t) &&= V_{\max}\cos(\omega t) \\ V_{bs} &= \sqrt{2}V_s \cos\left(\omega t - 2\pi/3\right) &&= V_{\max}\cos\left(\omega t - 2\pi/3\right) \\ V_{cs} &= \sqrt{2}V_s \cos\left(\omega t + 2\pi/3\right) &&= V_{\max}\cos\left(\omega t + 2\pi/3\right) \end{aligned} \quad (1)$$

A desired magnetic field within the three phase induction motor is established by a set of three-phase currents of the same amplitude and angular frequency *'ω'* expressed as;

$$\begin{aligned} I_{as} &= \sqrt{2}I_s \cos(\omega t) &&= I_{\max}\cos(\omega t) \\ I_{bs} &= \sqrt{2}I_s \cos\left(\omega t - 2\pi/3\right) &&= I_{\max}\cos\left(\omega t - 2\pi/3\right) \\ I_{cs} &= \sqrt{2}I_s \cos\left(\omega t + 2\pi/3\right) &&= I_{\max}\cos\left(\omega t + 2\pi/3\right) \end{aligned} \quad (2)$$

Since the stator windings are considered to be three identical, sinusoidally distributed windings displaced by $2\pi/3$ with N_s equivalent turns and resistances R_s per phase. Likewise, rotor windings are also considered as there identical simusoidally distributed windings displaced by $2\pi/3$, with N_r equivalent turns and resistances R_r.

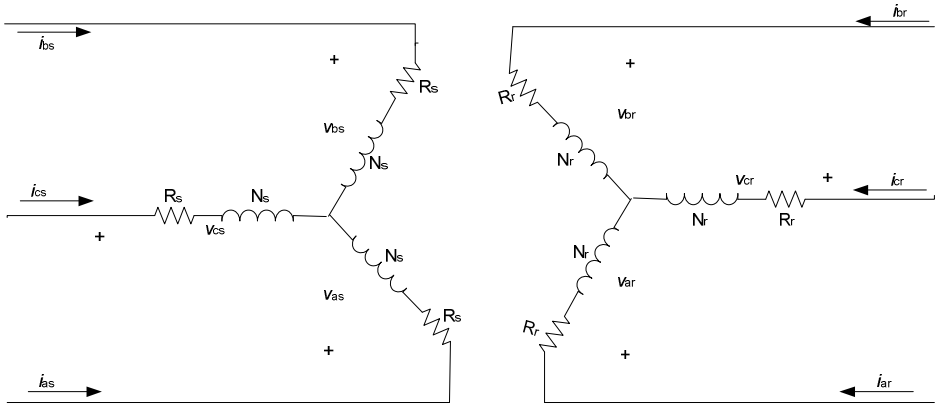

Fig. 1: Three-Phase, Wye-Connected Symmetrical Induction Motor

The voltage equation in the motor variables may be expressed as;

$$V_{abcx} = R_x i_{abcx} + \frac{d}{dt}\left(N_x \phi_{abcx}\right) \quad (3)$$

Since; $N_x \phi_{abcx} = \psi_{abcx}$

Therefore; the voltage equations are;

$$\begin{aligned} V_{abcs} &= R_s i_{abcs} + \frac{d}{dt}\psi_{abcs} \\ V_{abcr} &= R_r i_{abcr} + \frac{d}{dt}\psi_{abcr} \end{aligned} \quad (4)$$

Where; variable *'ψ'* is the flux linkages in the windings.
The proceeding sections summarized the conversion of three phase winding model to its two phase equivalent model.

2.1 Two – Phase Equivalent Model of the 3-Φ Induction Motor

In order to obtain the two-phase equivalent model of the three phase induction motor model variables; stationary reference fame of *αβ* – plane is used as shown in equation (5a).

$$f_{\alpha\beta x} = K\, f_{abcx} \quad (5a)$$

Where; $f_{\alpha\beta x} = \left[f_{\alpha x} \quad f_{\beta x}\right]^T$, $f_{abcx} = \left[f_{ax}\ f_{bx}\ f_{cx}\right]^T$ and

$$k = {}^{2}/_{3}\begin{bmatrix} 1 & -{}^{1}/_{2} & -{}^{1}/_{2} \\ 0 & {}^{\sqrt{3}}/_{2} & -{}^{\sqrt{3}}/_{2} \end{bmatrix}$$

The instantaneous values of the three-phase system variables can be obtained from the stationary reference frame using:

$$f_{abcx} = k^{T} f_{\alpha\beta x} \qquad (5b)$$

$$\text{Where; } k^{T} = \begin{bmatrix} 1 & 0 \\ -{}^{1}/_{2} & {}^{\sqrt{3}}/_{2} \\ -{}^{1}/_{2} & -{}^{\sqrt{3}}/_{2} \end{bmatrix}$$

2.2 Dynamic Equations of the Two Phase Equivalent Model

The α-β transformation applied to the three-phase stator voltages of equation (1) is;

$$\begin{bmatrix} V_{\alpha s} \\ V_{\beta s} \end{bmatrix} = {}^{2}/_{3}\begin{bmatrix} 1 & -{}^{1}/_{2} & -{}^{1}/_{2} \\ 0 & {}^{\sqrt{3}}/_{2} & -{}^{\sqrt{3}}/_{2} \end{bmatrix}\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix}$$

$$\Rightarrow \quad V_{\alpha s} = V_{\max}\cos(\omega t)$$
$$V_{\beta s} = V_{\max}\sin(\omega t) \qquad \textbf{(6)}$$

To obtain the dynamic equations of the motor, it is convenient to refer all rotor variables to the stator side by scaling rotor variables with appropriate transformation ratios (similar to transformer circuit). It is very important to note that all voltage terms due to angular speed of reference frame circuit are exempted since a stationary reference frame is adopted in this paper. The equivalent circuit of the motor referred to the stator is shown in Fig.2.

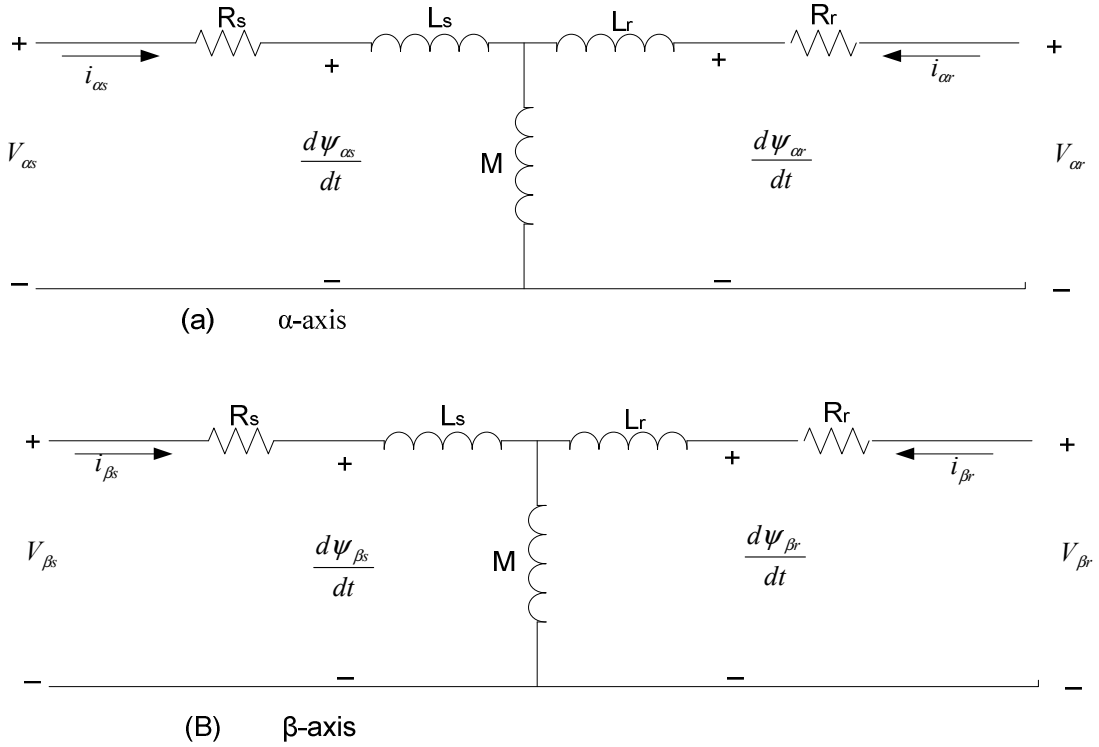

Fig.2: α-β (Stationary) Reference Frame Equivalent of a 3-Phase Symmetrical Induction Motor.

A.) Voltage equations:

Applying KVL and Faraday's law around the stator and rotor loops respectively.

Stator Loops:

$$V_{\alpha s} = R_s i_{\alpha s} + \frac{d}{dt}\psi_{\alpha s}$$
$$V_{\beta s} = R_s i_{\beta s} + \frac{d}{dt}\psi_{\beta s} \quad (7a)$$

Rotor Loops:

(8a)

$$V_{\alpha r} = R_r i_{\alpha r} + \frac{d}{dt}\psi_{\alpha r}$$
$$V_{\beta r} = R_r i_{\beta r} + \frac{d}{dt}\psi_{\beta r}$$

But, rotor windings are short circuited by the end rings at of squirrel cage rotor. Therefore, $V_{\alpha r} = V_{\beta r} = 0$. Thus, rotor voltage equations can be reduced to;

$$0 = R_r i_{\alpha r} + \frac{d}{dt}\psi_{\alpha r}$$
$$0 = R_r i_{\beta r} + \frac{d}{dt}\psi_{\beta r} \quad (8b)$$

Hence; the transformed dynamic voltage equation can be expressed in matrix form as;

$$\begin{bmatrix} V_{\alpha s} \\ V_{\beta s} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \\ R_r & 0 \\ 0 & R_r \end{bmatrix} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{\alpha r} \\ i_{\beta r} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi_{\alpha s} \\ \psi_{\beta s} \\ \psi_{\alpha r} \\ \psi_{\beta r} \end{bmatrix} \quad (9)$$

B.) Flux Linkage Equation

The flux linkages within the stator and rotor of the motor equivalent circuit on αβ-plane are given by:

$$\psi_{\alpha s}(i_{\alpha s}, i_{\beta s}, i_{\alpha r}, i_{\beta r}, \theta) = L_s i_{\alpha s} + M(i_{\alpha r}\cos\theta - i_{\beta r}\sin\theta)$$
$$\psi_{\beta s}(i_{\alpha s}, i_{\beta s}, i_{\alpha r}, i_{\beta r}, \theta) = L_s i_{\beta s} + M(i_{\alpha r}\sin\theta + i_{\beta r}\cos\theta)$$
$$\psi_{\alpha r}(i_{\alpha s}, i_{\beta s}, i_{\alpha r}, i_{\beta r}, \theta) = L_r i_{\alpha r} + M(i_{\alpha s}\cos\theta + i_{\beta s}\sin\theta) \quad (10)$$
$$\psi_{\beta r}(i_{\alpha s}, i_{\beta s}, i_{\alpha r}, i_{\beta r}, \theta) = L_r i_{\beta r} + M(-i_{\alpha s}\sin\theta + i_{\beta s}\cos\theta)$$

Re-writing the flux linkages of equation (10) in a state-variable form:

$$\psi_{\alpha\beta} = L[i_{\alpha\beta}]$$

Where;

$$\psi_{\alpha\beta} = [\psi_{\alpha s} \;\; \psi_{\beta s} \;\; \psi_{\alpha r} \;\; \psi_{\beta r}]^T, \quad i_{\alpha\beta} = [i_{\alpha s} \;\; i_{\beta s} \;\; i_{\alpha r} \;\; i_{\beta r}]^T$$ and the inductance matrix L is defined as;

$$L = \begin{bmatrix} L_s & 0 & M\cos\theta & -M\sin\theta \\ 0 & L_s & M\sin\theta & M\cos\theta \\ M\cos\theta & M\sin\theta & L_r & 0 \\ -M\sin\theta & M\cos\theta & 0 & L_r \end{bmatrix} \quad \text{such that;} \quad L = L^T$$

C.) Torque Equation

The total torque on the rotor is;

$$T_r = M[i_{\beta s}(i_{\alpha r}\cos\theta - i_{\beta r}\sin\theta) - i_{\alpha s}(i_{\alpha r}\sin\theta + i_{\beta r}\cos\theta)] \quad (11)$$

With the load torque T_L, the Newton's equations are:

$$J\frac{d\omega}{dt} = T_r - T_L \tag{12}$$

And $$\omega = \frac{d\theta}{dt}$$

Where *'θ'* is the transformation angle due to position changes of the motor motion with respect to the stator.
N.B: All the dynamic equations derived above are for a paired of poles (i.e one North pole and one South Pole).

2.3 Dynamic Equations of a Multiple Pairs of Poles Induction
If the rotor has 'P' pairs of poles, the flux linkages are modified as;

$$\begin{aligned}
\psi_{\alpha s}(i_{\alpha s}, i_{\beta s}, i_{\alpha r}, i_{\beta r}, \theta) &= L_s i_{\alpha s} + M[i_{\alpha r}\cos(P\theta) - i_{\beta r}\sin(P\theta)] \\
\psi_{\beta s}(i_{\alpha s}, i_{\beta s}, i_{\alpha r}, i_{\beta r}, \theta) &= L_s i_{\beta s} + M[i_{\alpha r}\sin(P\theta) + i_{\beta r}\cos(P\theta)] \\
\psi_{\alpha r}(i_{\alpha s}, i_{\beta s}, i_{\alpha r}, i_{\beta r}, \theta) &= L_r i_{\alpha r} + M[i_{\alpha s}\cos(P\theta) + i_{\beta s}\sin(P\theta)] \\
\psi_{\beta r}(i_{\alpha s}, i_{\beta s}, i_{\alpha r}, i_{\beta r}, \theta) &= L_r i_{\beta r} + M[-i_{\alpha s}\sin(P\theta) + i_{\beta s}\cos(P\theta)]
\end{aligned} \tag{12}$$

Since, N_s and N_r were defined as the stator and rotor windings per a pole paired respectively. Therefore, the total number of windings in a stator phase windings are; $\boldsymbol{PN_s}$ and $\boldsymbol{PN_r}$ respectively.
Then, the dynamic equations for P-multiple pairs of poles are;

$$\begin{aligned}
V_{\alpha s} &= R_s i_{\alpha s} + \frac{d}{dt}[L_s i_{\alpha s} + M[i_{\alpha r}\cos(P\theta) - i_{\beta r}\sin(P\theta)]] \\
V_{\beta s} &= R_s i_{\beta s} + \frac{d}{dt}[L_s i_{\beta s} + M[i_{\alpha r}\sin(P\theta) + i_{\beta r}\cos(P\theta)]] \\
0 &= R_r i_{\alpha r} + \frac{d}{dt}[L_r i_{\alpha r} + M[i_{\alpha s}\cos(P\theta) + i_{\beta s}\sin(P\theta)]] \\
0 &= R_r i_{\beta r} + \frac{d}{dt}[L_r i_{\beta r} + M[-i_{\alpha s}\sin(P\theta) + i_{\beta s}\cos(P\theta)]]
\end{aligned} \tag{13}$$

$$J\frac{d\omega}{dt} = PM[i_{\beta s}(i_{\alpha r}\cos(P\theta) - i_{\beta r}\sin(P\theta)) - i_{\alpha s}(i_{\alpha r}\sin(P\theta) + i_{\beta r}\cos(P\theta))] - T_L \quad \text{and} \tag{14}$$

$$\omega = \frac{d\theta}{dt}$$

3. Mathematical model solution using Laplace transformation

Expanding equation (13) above;

$$\begin{aligned}
V_{\alpha s} &= R_s i_{\alpha s} + L_s\frac{di_{\alpha s}}{dt} + M\cos(P\theta)\frac{di_{\alpha r}}{dt} - M\sin(P\theta)\frac{di_{\beta r}}{dt} \\
V_{\beta s} &= R_s i_{\beta s} + L_s\frac{di_{\beta s}}{dt} + M\sin(P\theta)\frac{di_{\alpha r}}{dt} + M\cos(P\theta)\frac{di_{\beta r}}{dt} \\
0 &= R_r i_{\alpha r} + L_r\frac{di_{\alpha r}}{dt} + M\cos(P\theta)\frac{di_{\alpha s}}{dt} + M\sin(P\theta)\frac{di_{\beta s}}{dt} \\
0 &= R_r i_{\beta r} + L_r\frac{di_{\beta r}}{dt} - M\sin(P\theta)\frac{di_{\alpha s}}{dt} + M\cos(P\theta)\frac{di_{\beta s}}{dt}
\end{aligned} \tag{15}$$

If in equation (15): $M\cos(P\theta) = M^{I}$ and $M\sin(P\theta) = M^{II}$. Then, taking the Laplace transform of the resulting equations, we have;

$$
\begin{aligned}
V_{\alpha s} &= R_s I_{\alpha s} + L_s[sI_{\alpha s} - i_{\alpha s}(0)] + M^I[sI_{\alpha r} - i_{\alpha r}(0)] - M^{II}[sI_{\beta r} - i_{\beta r}(0)] \\
V_{\beta s} &= R_s I_{\beta s} + L_s[sI_{\beta s} - i_{\beta s}(0)] + M^{II}[sI_{\alpha r} - i_{\alpha r}(0)] + M^I[sI_{\beta r} - i_{\beta r}(0)] \\
0 &= R_r I_{\alpha r} + L_r[sI_{\alpha r} - i_{\alpha r}(0)] + M^I[sI_{\alpha s} - i_{\alpha s}(0)] + M^{II}[sI_{\beta s} - i_{\beta s}(0)] \\
0 &= R_r i_{\beta r} + L_r\left[sI_{\beta r} - i_{\beta r}(0)\right] - M^{II}\left[sI_{\alpha s} - i_{\alpha s}(0)\right] + M^I\left[sI_{\beta s} - i_{\beta s}(0)\right]
\end{aligned}
\tag{16}
$$

Where; $I_{\alpha\beta} = \mathcal{L}\{i_{\alpha\beta}\} = I_{\alpha\beta}(s)$

If the initial conditions $i_{\alpha s}(0) = i_{\beta s}(0) = i_{\alpha r}(0) = i_{\beta r}(0) = 0$ are substituted in equation (16) and collecting the like terms;

$$
\begin{aligned}
V_{\alpha s} &= [R_s + sL_s]I_{\alpha s} + M^I sI_{\alpha r} - M^{II} sI_{\beta r} \\
V_{\beta s} &= [R_s + sL_s]I_{\beta s} + M^{II} sI_{\alpha r} + M^I sI_{\beta r} \\
0 &= M^I sI_{\alpha s} + M^{II} sI_{\beta s} + [R_r + sL_r]I_{\alpha r} \\
0 &= -M^{II} sI_{\alpha s} + M^I sI_{\beta s} + [R_r + sL_r]I_{\beta r}
\end{aligned}
\tag{17}
$$

Writing this in a state matrix form;

$$
\begin{bmatrix} V_{\alpha s} \\ V_{\beta s} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + sL_s & 0 & M^I s & -M^{II} s \\ 0 & R_s + sL_s & M^{II} s & M^I s \\ M^I s & M^{II} s & R_r + sL_r & 0 \\ -M^{II} s & M^I s & 0 & R_r + sL_r \end{bmatrix} \begin{bmatrix} I_{\alpha s} \\ I_{\beta s} \\ I_{\alpha r} \\ I_{\beta r} \end{bmatrix} \tag{17}
$$

This could be represented by;

$$
\left[V_{\alpha\beta}(s)\right] = Z\left[I_{\alpha\beta}(s)\right] \tag{18}
$$

Where the impedance matrix $Z=Z^T$ is given by;

$$
Z = \begin{bmatrix} R_s + sL_s & 0 & M^I s & -M^{II} s \\ 0 & R_s + sL_s & M^{II} s & M^I s \\ M^I s & M^{II} s & R_r + sL_r & 0 \\ -M^{II} s & M^I s & 0 & R_r + sL_r \end{bmatrix} = Z^T
$$

Solving equation (17) using Cramer's rule;

$$
\Delta = \begin{vmatrix} R_s + sL_s & 0 & M^I s & -M^{II} s \\ 0 & R_s + sL_s & M^{II} s & M^I s \\ M^I s & M^{II} s & R_r + sL_r & 0 \\ -M^{II} s & M^I s & 0 & R_r + sL_r \end{vmatrix}
$$

$$
\Delta_1 = \begin{vmatrix} V_{\alpha s} & 0 & M^I s & -M^{II} s \\ V_{\beta s} & R_s + sL_s & M^{II} s & M^I s \\ 0 & M^{II} s & R_r + sL_r & 0 \\ 0 & M^I s & 0 & R_r + sL_r \end{vmatrix}
$$

$$\Delta_2 = \begin{vmatrix} R_s + sL_s & V_{\alpha s} & M^I s & -M^{II} s \\ 0 & V_{\beta s} & M^{II} s & M^I s \\ M^I s & 0 & R_r + sL_r & 0 \\ -M^{II} s & 0 & 0 & R_r + sL_r \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} R_s + sL_s & 0 & V_{\alpha s} & -M^{II} s \\ 0 & R_s + sL_s & V_{\beta s} & M^I s \\ M^I s & M^{II} s & 0 & 0 \\ -M^{II} s & M^I s & 0 & R_r + sL_r \end{vmatrix}$$

$$\Delta_4 = \begin{vmatrix} R_s + sL_s & 0 & M^I s & V_{\alpha s} \\ 0 & R_s + sL_s & M^{II} s & V_{\beta s} \\ M^I s & M^{II} s & R_r + sL_r & 0 \\ -M^{II} s & M^I s & 0 & 0 \end{vmatrix}$$

Such that; analytical solutions to the currents are;

$$\begin{aligned} I_{\alpha s} &= \frac{\Delta_1}{\Delta} \\ I_{\beta s} &= \frac{\Delta_2}{\Delta} \\ I_{\alpha r} &= \frac{\Delta_3}{\Delta} \\ I_{\beta r} &= \frac{\Delta_4}{\Delta} \end{aligned} \qquad (19)$$

In sequel to the above Laplace transform which could only provide analytical solution to the unknown variables in the dynamic equations, there is a need to develop a quantitative method that can discretize the solutions of the equations (13) & (14) instead of analytical solution. Since this work is aimed at investigating the dynamic behavior of the motor. Therefore, the numerical values of the variables obtained in a discrete form could be used to display this in graphical form. The numerical method adopted in solving the differential equations in this paper is Fourth (4^{th})-Order Runge-Kutta method because it is an excellent general purpose differential equation solver [Greenberg, 1998]. Here we present 4^{th} order Runge Kutta method for generating approximate numerical solution values to an initial value problem; $y^I = f(x, y)$ with initial condition $y(x_0) = y_0$ at selected points x_o, x_n……x_N. If the distance between successive points at which approximate values of the solutions are computed *(step size)=h*. To compute point (x_n, y_n) require gradient of $f = f(x_n, y_n)$ which can be summarized as follows;

If $x_n = x_{n-1} + h$ for n = 1,2,3,.... .....N

Then;

$$k_1 = hf\,(x_{n-1},\ y_{n-1});$$

$$k_2 = hf\,(x_{n-1} + \tfrac{1}{2}h,\ y_{n-1} + \tfrac{1}{2}k_1);$$

$$k_3 = hf\,(x_{n-1} + \tfrac{1}{2}h,\ y_{n-1} + \tfrac{1}{2}k_2);$$

$$k_4 = hf\,(x_n,\ y_{n-1} + k_3);$$

Finally,

$$y_n = y_{n-1} + \Delta y_{n-1}$$

where the increment, Δy_{n-1} is defined as;

$$\Delta y_{n-1} = \frac{1}{6}[k_1 + 2k_2 + 2K_3 + k_4]$$

So, from the dynamic equations (13) and (14) of multiple pole pairs of an induction motor; if the relative position of the rotor to the stator is set at $\theta = 0^0$ (i.e Maximum. magnetic coupling) and the number of pole pairs P=3; we have:

$$V_{\alpha s} = R_s i_{\alpha s} + L_s \frac{di_{\alpha s}}{dt} + M \frac{di_{\alpha r}}{dt} \quad \text{(a)}$$

$$V_{\beta s} = R_s i_{\beta s} + L_s \frac{di_{\beta s}}{dt} + M \frac{di_{\beta r}}{dt} \quad \text{(b)}$$

$$0 = R_r i_{\alpha r} + L_r \frac{di_{\alpha r}}{dt} + M \frac{di_{\alpha s}}{dt} \quad \text{(c)}$$

$$0 = R_r i_{\beta r} + L_r \frac{di_{\beta r}}{dt} + M \frac{di_{\beta s}}{dt} \quad \text{(d)}$$

and

$$J\frac{d\theta}{dt} = PM\,(i_{\beta s} i_{\alpha r} - i_{\alpha s} i_{\beta r}) - T_L$$

If the retarding torque produced by the friction in the ball bearing of the motor is modeled by $f\omega$, where f=Coefficient of Viscous Friction.

Therefore, the equivalent Newton's differential equation is;

$$J\frac{d\theta}{dt} = PM(i_{\beta s} i_{\alpha r} - i_{\alpha s} i_{\beta r}) - f\omega - T_L$$

Multiply equations (a) and (c) by L_r and M respectively;

$$L_r V_{\alpha s} = R_s L_r i_{\alpha s} + L_s L_r \frac{di_{\alpha s}}{dt} + ML_r \frac{di_{\alpha r}}{dt} \quad \text{(e)}$$

$$0 = MR_r i_{\alpha r} + ML_r \frac{di_{\alpha r}}{dt} + M^2 \frac{di_{\alpha s}}{dt} \quad \text{(f)}$$

Subtracting equations (f) from (e) to simplify;

$$L_r V_{\alpha s} = R_s L_r i_{\alpha s} + MR_r i_{\alpha r} + (L_s L_r - M^2)\frac{di_{\alpha s}}{dt} \quad \text{(g)}$$

Likewise, multiply equations (b) and (d) by L_r and M respectively we have;

$$L_r V_{\beta s} = R_s L_r i_{\beta s} + L_s L_r \frac{di_{\beta s}}{dt} + ML_r \frac{di_{\beta r}}{dt} \quad \text{(h)}$$

$$0 = MR_r i_{\beta r} + ML_r \frac{di_{\beta r}}{dt} + M^2 \frac{di_{\beta s}}{dt} \quad \text{(i)}$$

Subtracting equation (i) form (h), we have;

$$L_r V_{\beta s} = R_s L_r i_{\beta s} - MR_r i_{\beta r} + (L_s L_r - M^2) \frac{di_{\beta s}}{dt} \quad \text{(j)}$$

Also, if equations (a) and (c) are multiplied by M and Ls respectively we have;

$$MV_{\alpha s} = MR_s i_{\alpha s} + ML_s \frac{di_{\alpha s}}{dt} + M^2 \frac{di_{\alpha r}}{dt} \quad \text{(k)}$$

$$0 = R_r L_s i_{\alpha r} + L_r L_s \frac{di_{\alpha r}}{dt} + ML_s \frac{di_{\alpha s}}{dt} \quad \text{(l)}$$

Subtracting equation (l) from (k);

$$MV_{\alpha s} = MR_s i_{\alpha s} - R_r L_s i_{\alpha r} - (L_s L_r - M^2) \frac{di_{\alpha r}}{dt} \quad \text{(m)}$$

Also, if equations (b) and (d) are multiplied by M and L_s respectively, the resulting equations are (o) and (p);

$$MV_{\beta s} = MR_s i_{\beta s} + ML_s \frac{di_{\beta s}}{dt} + M^2 \frac{di_{\beta r}}{dt} \quad \text{(o)}$$

$$0 = R_r L_s i_{\beta r} + L_r L_s \frac{di_{\beta r}}{dt} + ML_s \frac{di_{\beta s}}{dt} \quad \text{(p)}$$

Subtracting (p) from (o), we have (q);

$$MV_{\beta s} = MR_s i_{\beta s} - R_r L_s i_{\beta r} - (L_r L_s - M^2) \frac{di_{\beta r}}{dt} \quad \text{(q)}$$

Arranging equations (g), (m), (j) and (q);

$$L_r V_{\alpha s} = R_s L_r i_{\alpha s} + MR_r i_{\alpha r} + (L_s L_r - M^2) \frac{di_{\alpha s}}{dt} \quad \text{(g)}$$

$$MV_{\alpha s} = MR_s i_{\alpha s} - R_r L_s i_{\alpha r} - (L_s L_r - M^2) \frac{di_{\alpha r}}{dt} \quad \text{(m)}$$

$$L_r V_{\beta s} = R_s L_r i_{\beta s} - MR_r i_{\beta r} + (L_s L_r - M^2) \frac{di_{\beta s}}{dt} \quad \text{(j)}$$

$$MV_{\beta s} = MR_s i_{\beta s} - R_r L_s i_{\beta r} - (L_r L_s - M^2) \frac{di_{\beta r}}{dt} \quad \text{(q)}$$

Also;

$$\mathrm{J} \frac{\mathrm{d}\omega}{\mathrm{dt}} = \left[PM(i_{\beta s} i_{\alpha r} - i_{\alpha s} i_{\beta r}) - f\omega - T_L \right] \quad \text{(r)}$$

From equations (g), (m),, (j), (q) and (r), equations (20), (21), (22), (23) and (24) are obtained respectively;

$$\frac{di_{\alpha s}}{dt} = \left[L_r V_{\alpha s} - R_s L_r i_{\alpha s} + MR_r i_{\alpha r} \right] / \left[L_s L_r - M^2 \right] \quad (20)$$

$$\frac{di_{\alpha r}}{dt} = \left[MV_{\alpha s} - MR_s i_{\alpha s} + R_r L_s i_{\alpha r} \right] / \left[-(L_s L_r - M^2) \right] \quad (21)$$

$$\frac{di_{\beta s}}{dt} = \left[L_r V_{\beta s} - R_s L_r i_{\beta s} + M R_r i_{\beta r}\right] / \left[L_s L_r - M^2\right] \quad (22)$$

$$\frac{di_{\beta r}}{dt} = \left[M V_{\beta s} - M R_s i_{\beta s} + R_r L_s i_{\beta r}\right] / \left[-(L_r L_s - M^2)\right] \quad (23)$$

$$\frac{\mathrm{d}\omega}{\mathrm{dt}} = \left[PM(i_{\beta s} i_{\alpha r} - i_{\alpha s} i_{\beta r}) - f\omega - T_L\right] / J \quad (24)$$

And;

$$\frac{\mathrm{d}\theta}{\mathrm{dt}} = \omega \quad (25)$$

In order to simulate the two-phase equivalent of the three phase dynamic model of induction motor, the differential equations (20-25) will be solved using the 4th Orrder Runge-Kutta described above with the aid of MATBLAB code program (M-file).

4. Computer simulation and results

The 3hp, 415volts, 50H_Z, 1440rpm, 6poles, three phase induction motor chosen for this dynamic modeling and simulation has the following operating characteristics parameters:
Per phase stator resistance R_s = 0.0135Ω
Per phase rotor resistance referred to stator side = R_r = 0.9i6Ω
Per phase stator inductance L_s = 0.5H
Per phase rotor referred to stator side L_r = 1.7H
Per phase mutual inductance M = 0.0117H
Moment of inertial J = 0.001kg-m^2
Load Torque T_L = 0.8 Nm

The MATLAB code program developed to solve the dynamic differential equations of the three-phase induction motor was achieved using the algorithm stated in the flow-chat of fig.3 below.

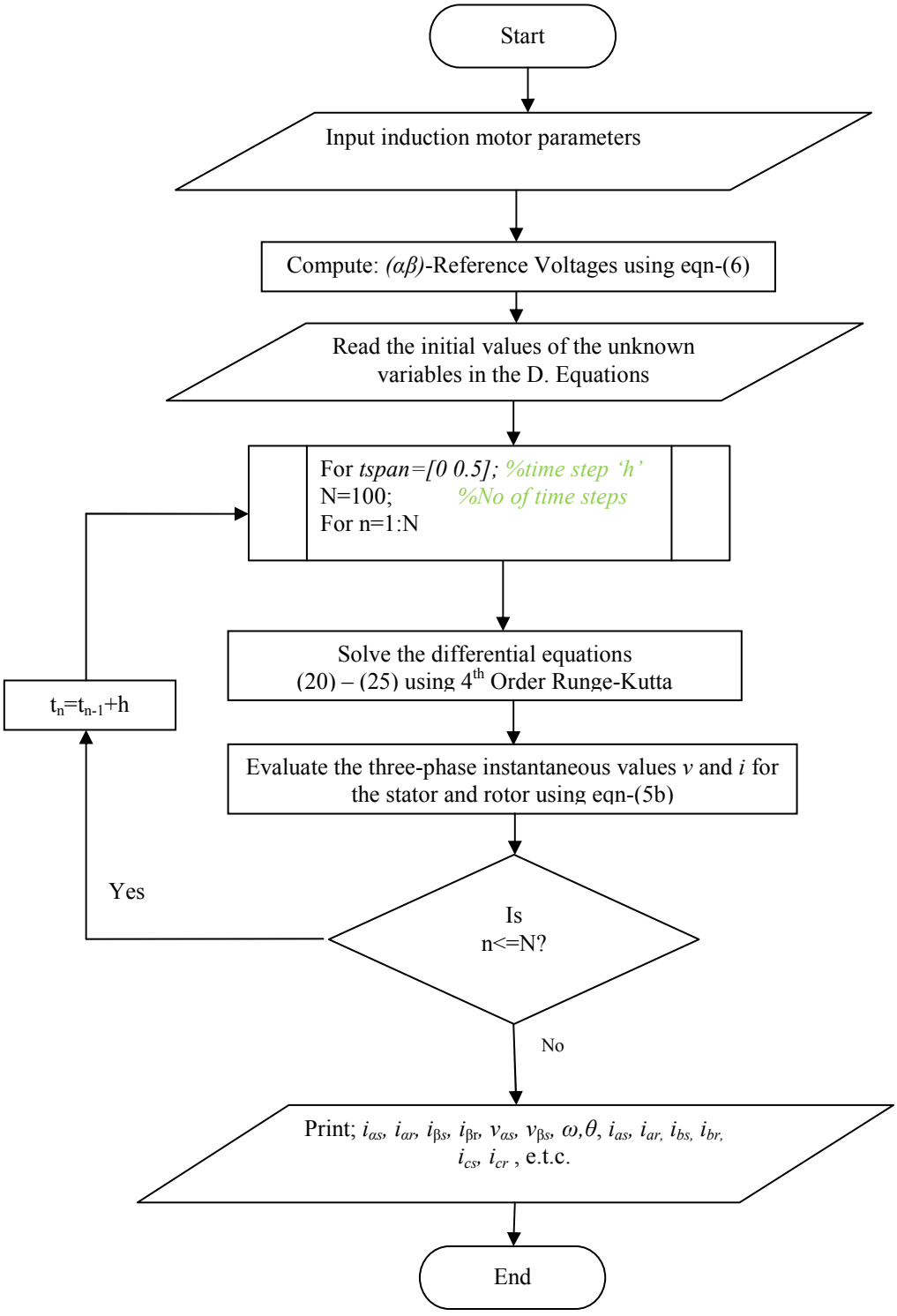

Fig. 3: The Flow-Chart for the Dynamic Model of the Three-Phase Induction Motor

4.1 Results and Discussions

The MATLAB computer simulation of the model was carried out under the following basic assumptions:- (i) Lumped Parameter Assumptions (LPA) and (ii) Effect of magnetic saturation is neglected.

The graphical results obtained from the computer simulation of the induction motor for dynamic analysis in a stationary reference frame are shown in the figures below:

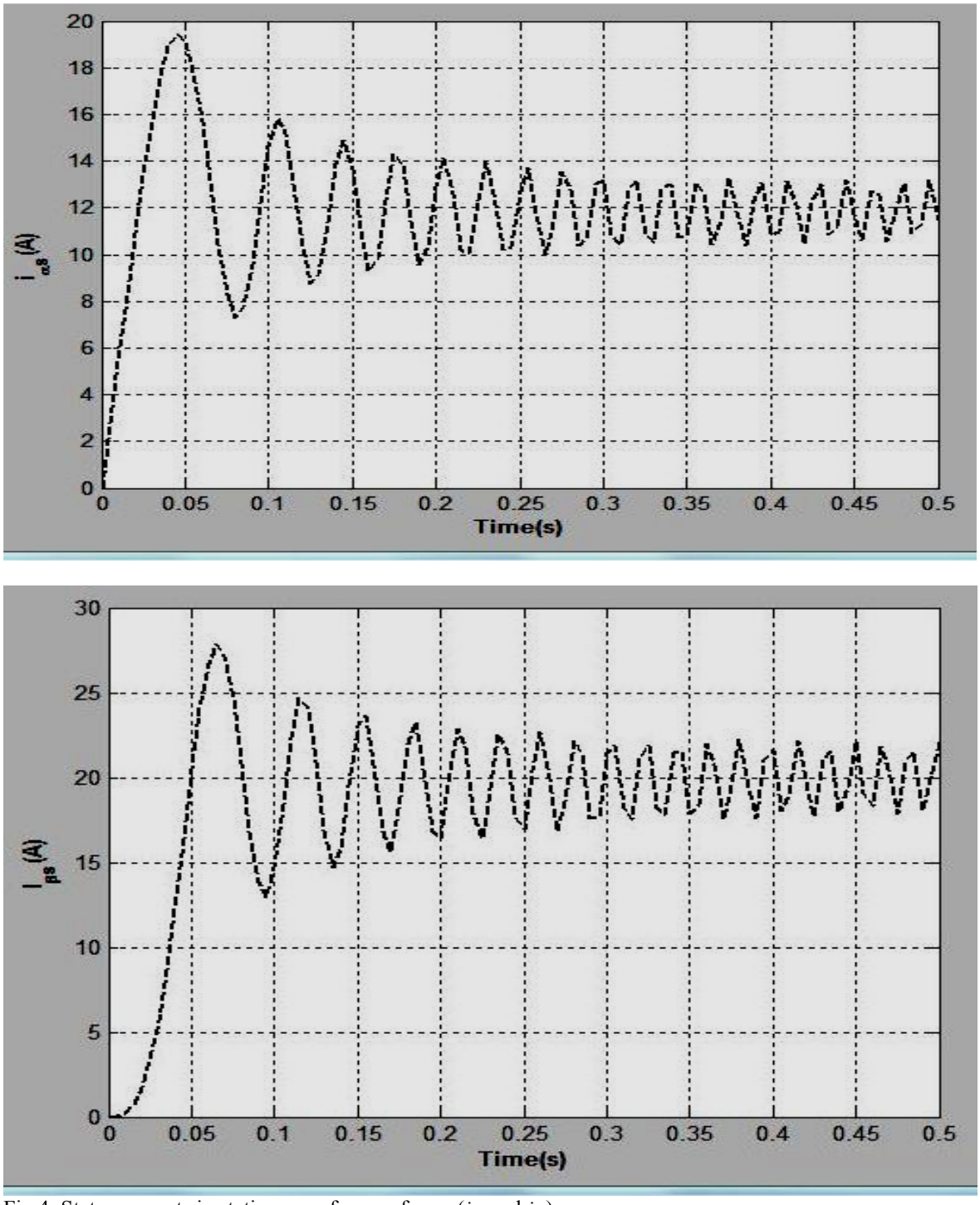

Fig.4: Stator currents in stationary reference frame ($i_{\alpha s}$ and $i_{\beta s}$)

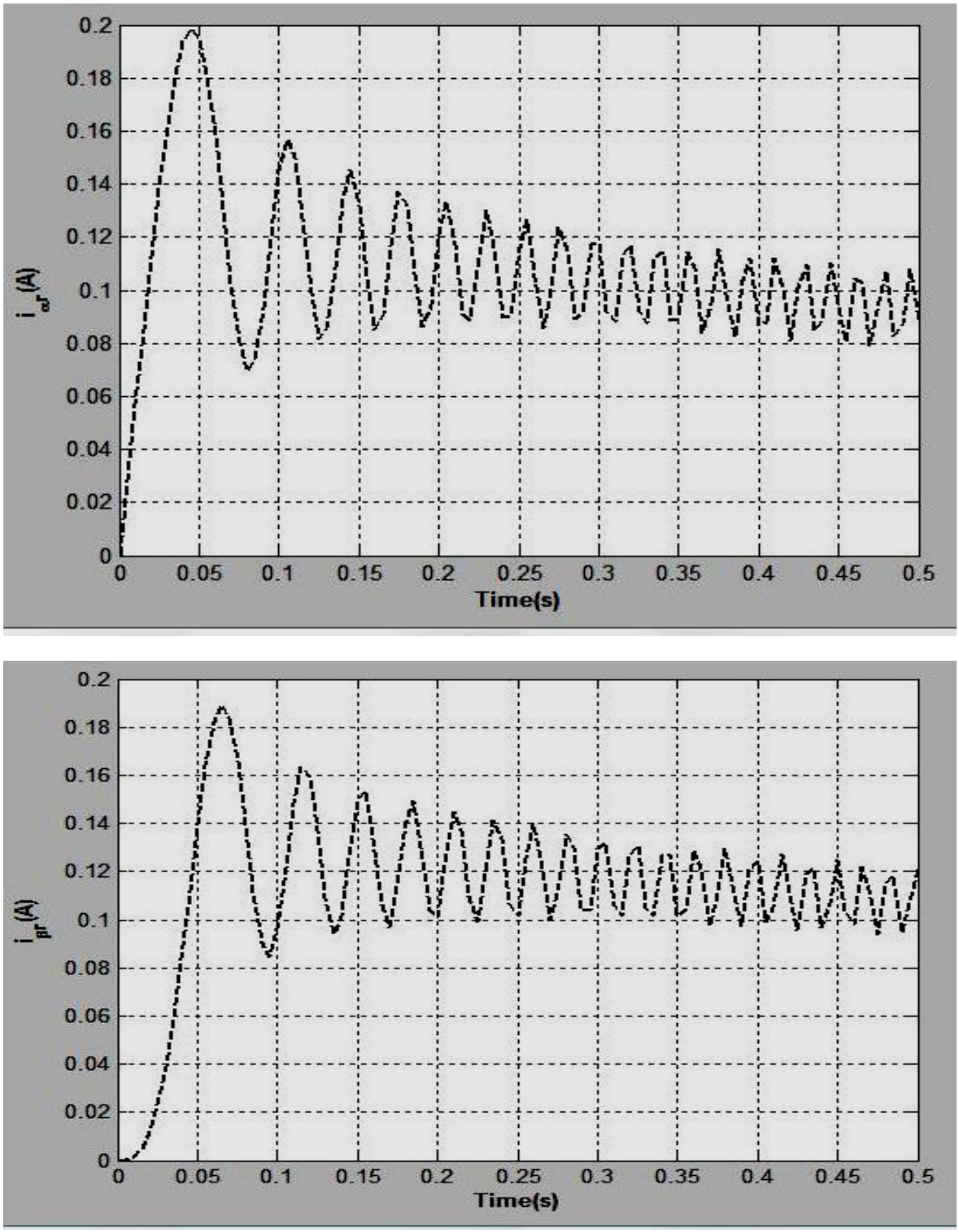

Fig. 5: Rotor currents in stationary reference frame ($i_{\alpha r}$ and $i_{\beta r}$)

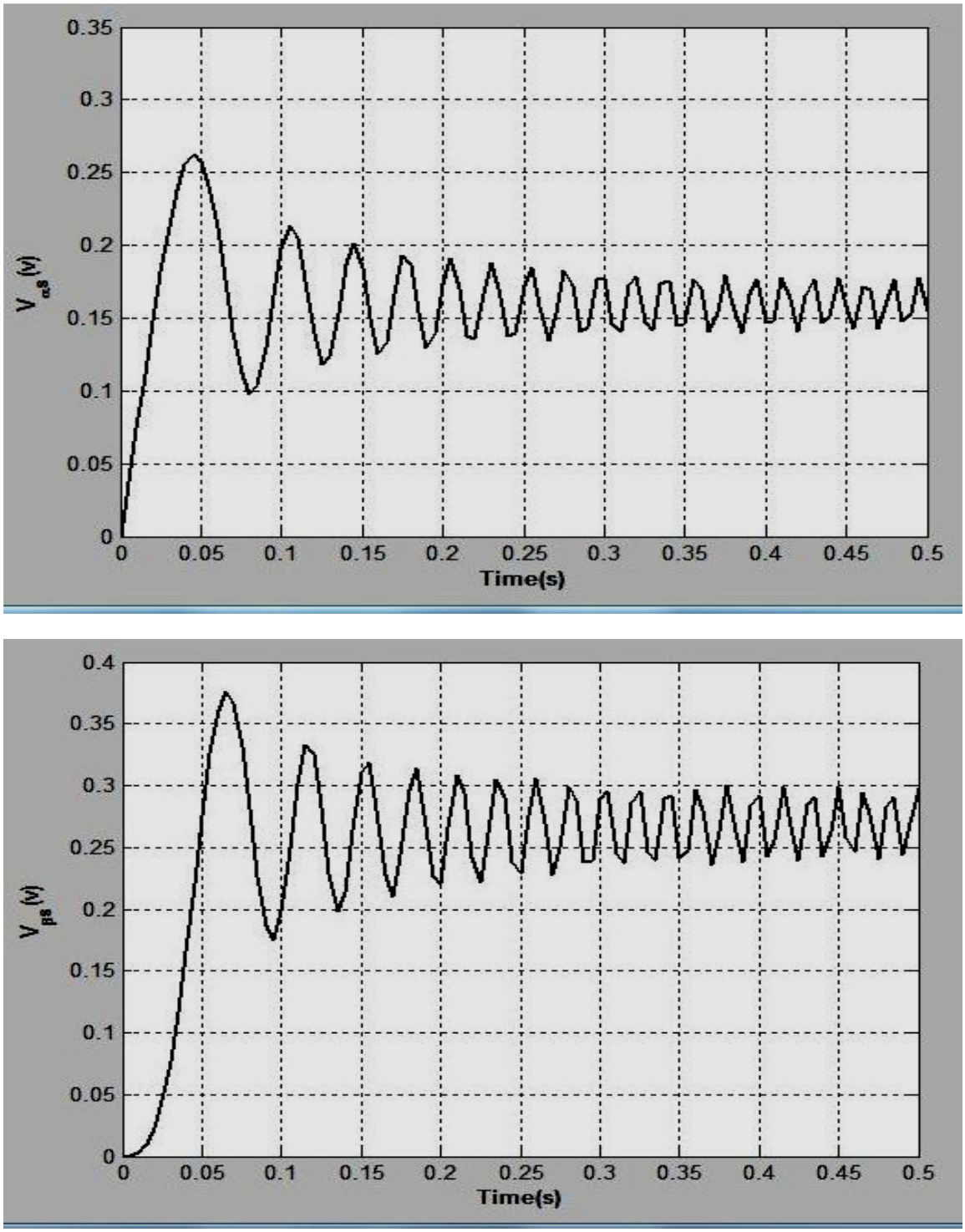

Fig. 6: Stator voltages in stationary reference frame ($V_{\alpha s}$ and $V_{\beta s}$)

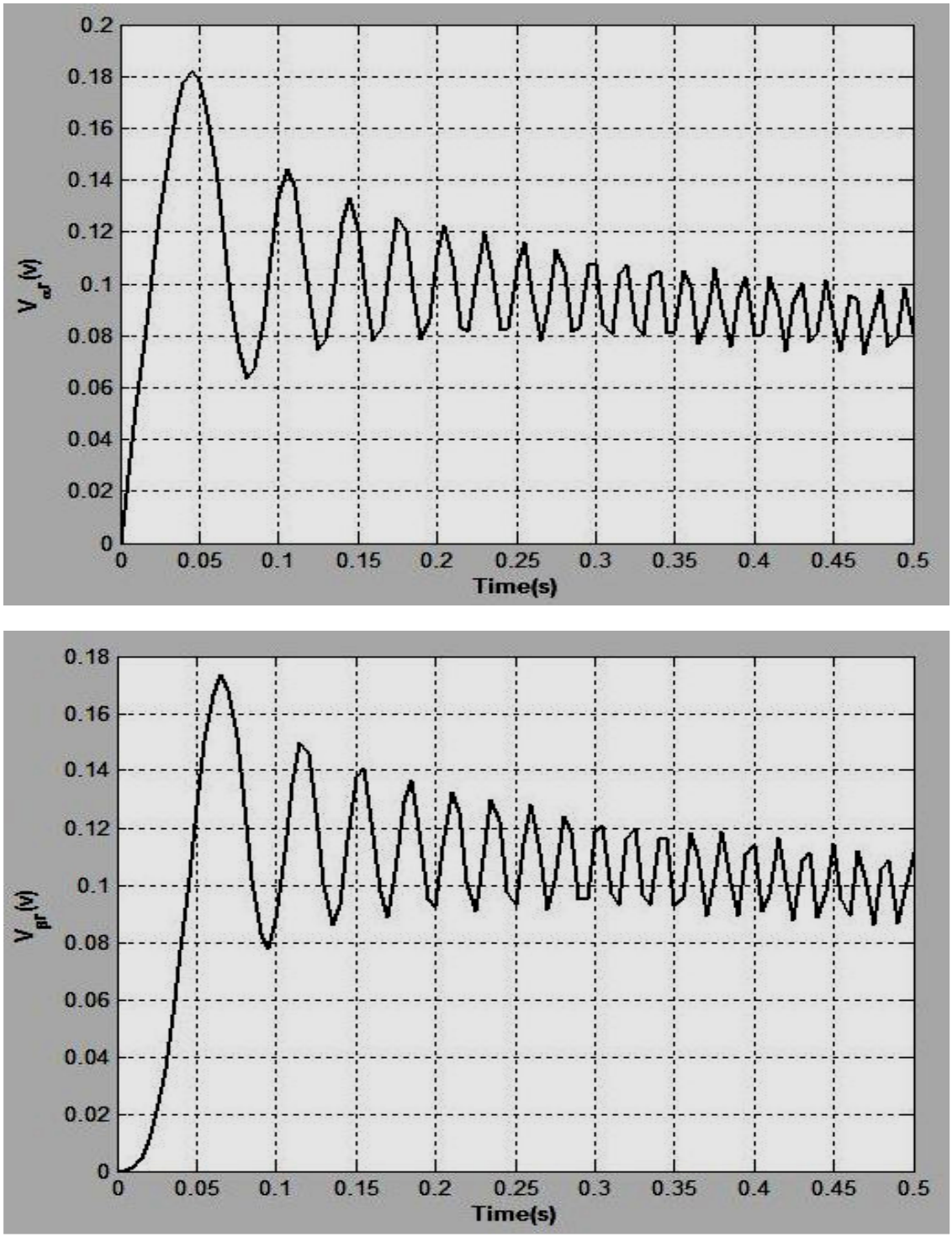

Fig. 7: Rotor voltages in stationary reference frame ($V_{\alpha r}$ and $V_{\beta r}$)

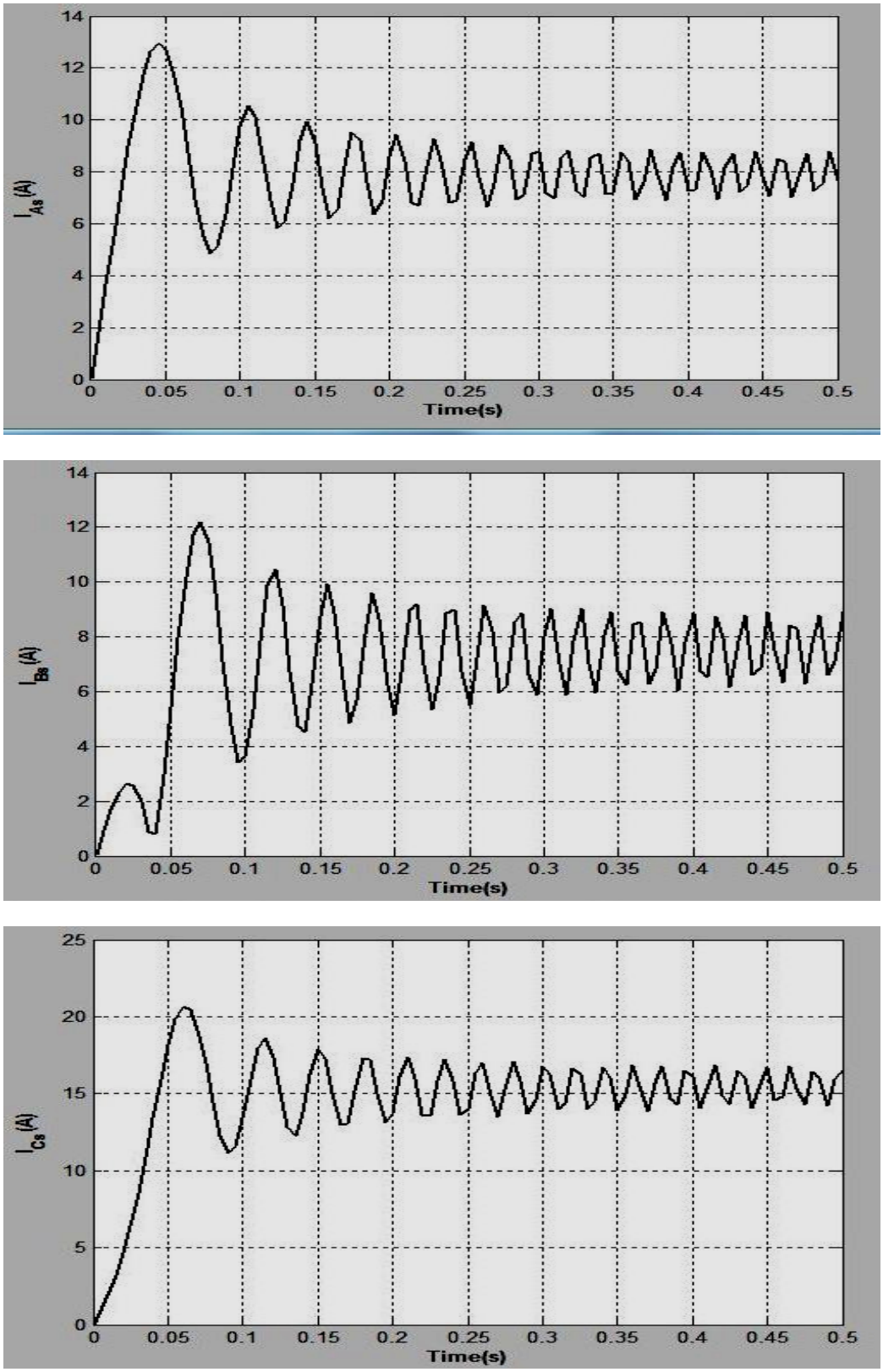

Fig. 8: Stator phase currents I_{As}, I_{Bs} and I_{Cs} respectively.

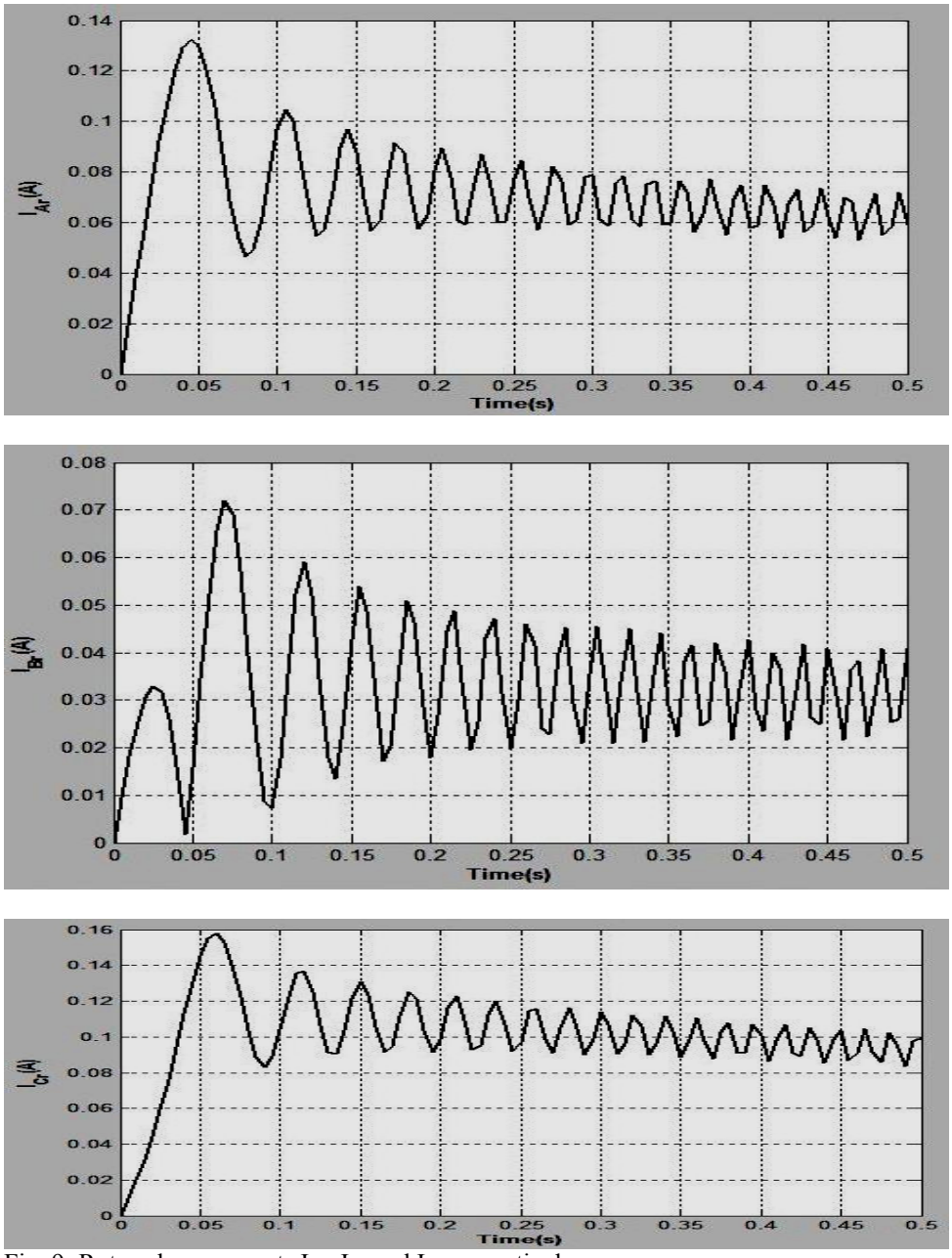

Fig. 9: Rotor phase currents I_{Ar}, I_{Br} and I_{Cr} respectively.

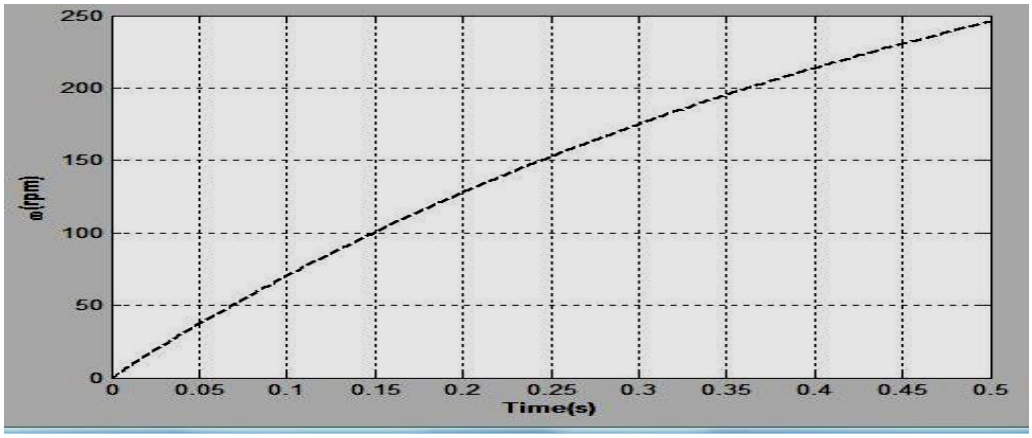

Fig.10: Rotor Angular Speed ω (rpm)-Time t (s) Profile

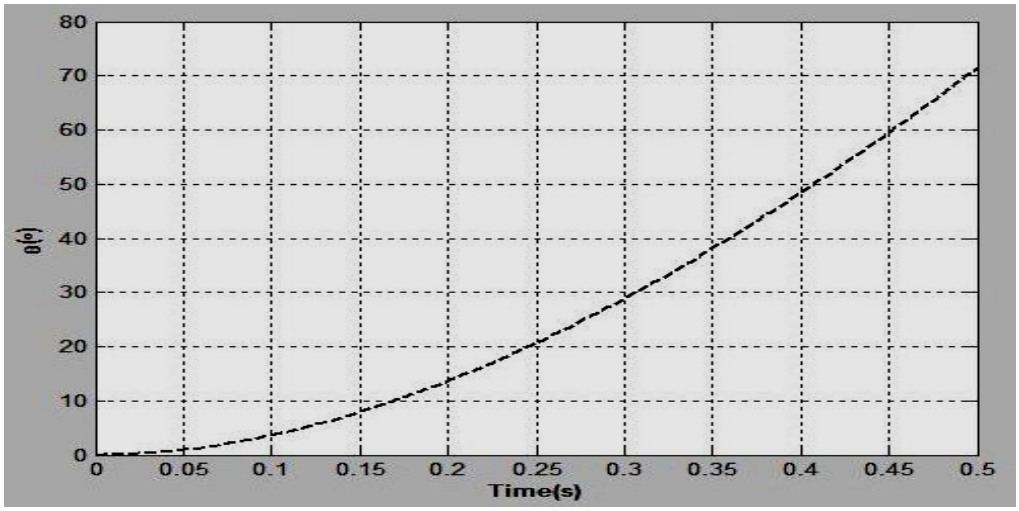

Fig.11: Rotor Angular Displacement θ (Deg.)-Time t (s) Profile

Figure 4 - 9 shows the electrical profiles of the three-phase induction motor model based on Faraday laws and Fig. 10&11 shows the mechanical behavior of the machine based of Newton's equation.

Conclusion

The paper gives a comprehensive mathematical dynamic model of a three-phase induction motor using a reference frame (α-β transformation) fixed in the stator. The stationary reference frame or α-β axes model presents a simpler and clear analysis of the machine such that all internal electrical variables is accessible for the study of dynamic and steady state behaviors of the induction motor. Also, it must be mentioned here that the 4th Order Runge-Kutta Method has proven suitable for providing a precise numerical solution to differential equations with initial conditions. Finally, the mathematical models and MATLAB program code developed gives a reliable dynamic behavior of induction motor using a reference frame theory.

References

E.S. Oluwasogo, and I.K. Okakwu, **(2014)**, "Performance Analysis of a Single-Phase AC Voltage Controller under Induction Motor Load" International Journal of Research in Engineering and Technology (IJRET). Vol.: 03, PP 184 – 191.

Adel Aktaibi, et al, "Dynamic Simulation of a Three-Phase Induction Motor Using Matlab Simulink"

Sifat Shah, A. Rashid, MKL Bhatti, **(2012)**, "Direct Quadrate (D-Q) Modeling of 3-Phase InductionMotor Using MatLab / Simulink" Canadian Journal on Electrical and Electronics Engineering Vol. 3, No. 5

Chiasson, J., **(2005),** "Modeling and High Performance Control of Electric Machine" IEEE Series on Power Engineering

Kriti, Jatinder Singh,Vivek Pahwa, **(2013)**, "Dynamic and Steady State Analysis of Induction Machine" International Journal of Emerging Science and Engineering (IJESE), Vol-1, Issue-12

Slobodan N. Vukosavic, **(2013)**, "Electrical Machines" Springer New York Heidelberg Dordrecht London

A.K. Babarinde, et al, **(2014)**, "Effects of Electrical Parameters Variation on the Dynamic Behaviour of Two-Phase Induction Motor" International Journal of Engineering Science Invention. Vol. 3, Issue 6, PP.15-25

Michael D. Greenberg **(1998)**, "Advanced Engineering Mathematics" 2nd Edition, Prentice Hall, Upper Saddle River, New Jersey.

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