 

# Effect of Thermogeometric Parameters on Heat Transfer Rate in Straight Fin with Variable Thermal Conductivity Having a Convective Tip

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**Abstract**

Temperature distribution in a straight fin with variable thermal conductivity and a convecting tip was analyzed using regular perturbation method. Approximate analytical solution was obtained and used to investigate the effects of Biot number, varying thermal conductivity, convective and insulated tips on the temperature distribution and heat transfer in a straight fin.In the limit of small parameter, the approximate solution was obtained for a straight fin with variable thermal conductivity having a convecting tip. The results of Aziz and Huq were fully recovered when the thermal conductivity at the tip was set to zero. From the results, for a high value of fin parameter, it is shown that the fin with convecting tip convecting tip has an enhanced 20 % heat conducting capacity than the fin with an assumed insulated.

**Keywords:** Straight Fin, Regular perturbation, Variable Thermal Conductivity,Biot Number and Convective Tip

**Nomenclature**

A Cross sectional area of the fins, $m^2$
$Bi_e$ Biot number
h Heat transfer coefficient, $Wm^{-2}k^{-1}$
K Thermal conductivity of the fin $Wm^{-1}k^{-1}$
L Length of the fin, m
M Dimensionless fin parameter
$m^2$ fin parameter $m^{-1}$
P perimeter of the fin, m
T Temperature, K
$T_\infty$ Ambient temperature, K
x axial distance, m
X dimensionless fin length
Q dimensionless heat transfer
η Efficiency of the fin

**Greek Symbols**

δ Thickness of the fin, m
ε Small parameter that depends on the thermal conductivity
θ Dimensionless temperature
σ Stefan-Boltzmann constant
$\delta_b$ Fin thickness at its base.

## 1.0 Introduction

The expanding demand for high-performance heat transfer components with progressively smaller weights, volume, costs or accommodating shapes have greatly increased the use of extended surfaces to enhance heat dissipation from hot primary surfaces. In the design and construction of various types of heat-transfer equipment, such extended surfaces are used to implement the flow of heat between a source and a sink due to the wide

 

varieties of applications of the extended surfaces in engineering devices.
In the literature, there are numerous studies of fins with constant thermal conductivity, but fewer studies exist variable thermal conductivity with temperature. Some of these studies include the work of Aziz and Enamul-Huq (1973) who considered a pure convection fin with temperature dependent thermal conductivity and developed a three term regular perturbation expansion in terms of the thermal conductivity parameter $\varepsilon$. He (1976) extended the previous analysis to include a uniform internal heat generation in the fin. In some years later, Arslanturk (2005) adopted the Adomian Decomposition Method (ADM) to obtain the temperature distribution in a pure convection fin with thermal conductivity varying linearly with temperature. The same problem was solved by Ganji (2006) with the aid of the homotopy perturbation method originally proposed by He (1996, 1999, 2002). In the following year, Coskun *et al*. (2008) utilised variational iteration method for the analysis of convective straight and radial fins with temperature-dependent thermal conductivity. Chowdhury *et al.*( 2009) investigated a rectangular fin with power law surface heat flux and made a comparative assessment of HAM, HPM, and ADM while Khani and Aziz (2010) considered a trapezoidal fin with both the thermal conductivity and the convection heat transfer coefficient varying as functions of temperature and reported an analytic solution generated using the homotopy analysis method (HAM). To the best of our knowledge, previous studies were based on the assumption of insulated tip and therefore, the effects convecting tip and the Biot number on the temperature distribution and the performance of the extended surfaces were not carried out. Therefore, this paper presents a regular perturbation solution for a straight fin with temperature-dependent thermal conductivity having a convecting tip, where the small parameter is a measure of the ratio of variation of thermal conductivity to the tip thermal conductivity value. In the limit of small parameter, the approximate solution was obtained for a straight fin with variable thermal conductivity having a convecting tip. The results of Aziz and Enamul-Huq were fully recovered when the thermal conductivity at the tip was set to zero. The solution was used to investigate the effects Biot number, the variable thermal conductivity, convective and insulated tips on the temperature distribution in the straight fin. From the results, for a high value of fin parameter, it is shown that the fin with convecting tip convecting tip has an enhanced 20 % heat conducting capacity than the assumed insulated fin of Aziz and Enamul-Huq.

## 2.0 Problem Formulation

Consider a straight fin of length L and thickness $\delta$ expressed on both faces to a convective environment at temperature $T_\infty$ and with heat transfer co-efficient shown in Fig.1., assuming that the heat flow in the fin and its temperatures remain constant with time, the convective heat transfer coefficient on the faces of the fin is constant and uniform over the entire surface of the fin, the temperature of the medium surrounding the fin is uniform, the fin base temperature is uniform., there is no contact resistance where the base of the fin joins the prime surface, there are no heat sources within the fin itself, the fin thickness is small compared with its height and length, so that temperature gradients acrossthe fin thickness and heat transfer from the edges of the fin may be neglected, the governing differential equation of the fin is given by equation (1) below.

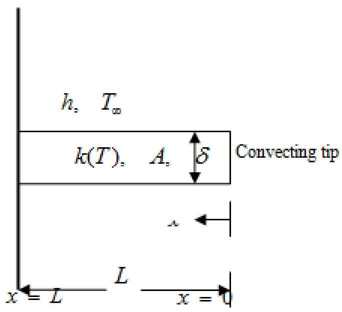


Figure 1

$$\frac{d}{dx}\left(k(T)\frac{dT}{dx}\right)-\frac{2h}{\delta}(T-T_\infty)=0 \qquad (1)$$

The boundary conditions are given as:

$$x=0, \quad \frac{\partial T}{\partial x}=\frac{hL}{k_a}(T-T_\infty), \qquad x=L, \quad T=T_b, \qquad (2)$$

Where $k$ is assumed to vary linearly with temperature and is given by

$$k(T) = k_a\left(1+\beta(T-T_a)\right) \tag{3}$$

The developed equation and the boundary conditions were rendered dimensionless by defining the following parameters

$$X = \frac{x}{L} \quad \theta = \frac{T-T_\infty}{T_b - T_\infty}, \quad M^2 = m^2L^2 \Rightarrow M = mL, \quad m^2 = \frac{hP}{kA} = \frac{2h}{k\delta} \tag{4}$$

The governing differential equation now becomes

$$(1+\varepsilon\theta)\frac{d^2\theta}{dX^2} + \varepsilon\left(\frac{d\theta}{dX}\right)^2 - M^2\theta = 0 \tag{5}$$

Where $$\varepsilon = \frac{k_b - k_\infty}{k_\infty} = \beta(T_b - T_\infty) \tag{6}$$

$k_b$ and $k_\infty$ are the thermal conductivity at the fin base and the tip respectively.

**3.0 Solution method**

In solving equation (5), we expand the temperature as

$$\theta = \theta_0 + \varepsilon\theta_1 + \varepsilon^2\theta_2 + \varepsilon^3\theta_3 + \ldots \tag{7}$$

Substituting equation (7) into equation (5), up to first order approximate, we have

$$\frac{d^2\theta_0}{dX^2} - M^2\theta_0 + \varepsilon\left[\frac{d^2\theta_1}{dX^2} + \theta_0\frac{d^2\theta_0}{dX^2} + \left(\frac{d\theta_0}{dX}\right)^2 - M^2\theta_1\right] = 0 \tag{8}$$

Leading order and first order equations with the appropriate boundary conditions are given as:

Leading equation:

$$\frac{d^2\theta_o}{dx^2} - M^2\theta_o = 0 \tag{9}$$

Subject to:

$$x = 0, \quad \frac{d^2\theta_o}{dx} = \frac{hL\theta_o}{k} = Bi_e\theta_o$$
$$x = 1, \quad \theta_o = 1 \tag{10}$$

First order equations:

$$\frac{d^2\theta_1}{dX^2} - M^2\theta_1 = -\left(\frac{d\theta_o}{dX}\right)^2 - \theta_0\frac{d^2\theta_o}{dX^2} \tag{11}$$

Subject

$$x = 0, \quad \frac{d^2\theta_1}{dx} = 0$$
$$x = 1, \quad \theta_1 = 0 \tag{12}$$

It can be easily shown from (9) and (11) with the corresponding boundary conditions of equations (10) and (11) that the:

Leading order solution for $\theta_o$ is

$$\theta_o = \frac{M\cosh Mx + Bi_e \sinh Mx}{M\cosh M + Bi_e\sinh M} \tag{13}$$

While the first order solution $\theta_1$ is

$$\theta_1 = \frac{1}{3}\{\left[\frac{(M^2 + Bi_e^2)\cosh 2M + 2Bi_eM\sinh 2M - 4Bi_eM\sinh M}{\cosh M(M\cosh M + Bi_e\sinh M)^2}\right]\cosh Mx$$
$$+ \frac{4Bi_eM\sinh Mx}{(M\cosh M + Bi_e\sinh M)^2} - \left[\left[\frac{(M^2 + Bi_e^2)\cosh 2Mx + 2Bi_eM\sinh 2Mx}{(M\cosh M + Bi_e\sinh M)^2}\right]\}\right] \tag{14}$$

Substituting equation 13 and 14 into equation 7 up to the first order (i.e. neglecting the higher orders), we arrived at

$$\theta = \frac{M\cosh Mx + Bi_e \sinh Mx}{M\cosh M + Bi_e \sinh M}$$
$$+\frac{\varepsilon}{3}\left\{\left[\frac{\left(M^2 + Bi_e^{\,2}\right)\cosh 2M + 2Bi_e M \sinh 2M - 4Bi_e M \sinh M}{\cosh M\left(M\cosh M + Bi_e \sinh M\right)^2}\right]\cosh Mx\right.$$
$$\left. + \frac{4Bi_e M \sinh Mx}{\left(M\cosh M + Bi_e \sinh M\right)^2} - \left[\left[\frac{\left(M^2 + Bi_e^{\,2}\right)\cosh 2Mx + 2Bi_e M \sinh 2Mx}{\left(M\cosh M + Bi_e \sinh M\right)^2}\right]\right\}\right]$$

(15)

It is easy to see that the solution of Aziz and Huq is easily recovered from equation (15) by setting $Bi_e = 0$ (Fin with assumed insulated tip), to give

$$\theta = \mathrm{sec}hM \cosh Mx + \frac{\varepsilon \,\mathrm{sec}hM}{3}\left[\left(1 + \tanh^2 M\right)\cosh Mx - \mathrm{sec}hM \cosh 2Mx\right] \qquad (16)$$

## 4.0 Results and Discussion

Fig.2 shows the effect of Biot number on a straight fin with convective tip. From the results, it is found that the temperature of the straight fin is not significantly affected by the variation in the Biot number at a comparably high value of the fin parameter, M. However, a significant variation in the results was obtained when the fin tip was assumed insulated as depicted in Fig. 3. From the analysis of the results, it was found that the fin with convecting tip has an enhanced 20 % of heat conducting capacity than the fin with an assumed insulated tip. Actually, the enhanced performance by the fin with convecting tip is expected but the quantitative analysis was carried out in this work.

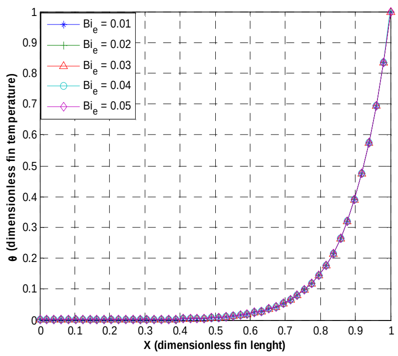


Fig.2.Temperature distribution in a straight fin with convecting tip, M=10

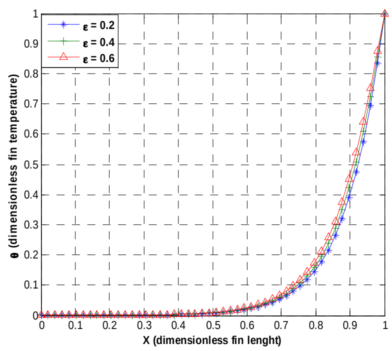


Fig. 3. Temperature distribution in a straight fin with insulated tip, M=10

Fig. 4 and Fig. 5 show the effects of Biot number on a straight fin with convective tip at fin parameters of 0.5 and 1.0 respectively while. Fig.6 and Fig. 7 depicts the effects of the small parameter, ε (as defined in this work), on a straight fin with an assumed insulated tip at fin parameters of 0.5 and 1.0 respectively. From the results, it was established that at a comparably low value of the fin parameter, the temperature of at the straight fin is significantly affected by the variation in the Biot number and the small parameter, ε. However, as the fin parameter increases, the effects of these parameters seem on the heat conducting capacity of the fin become insignificant.

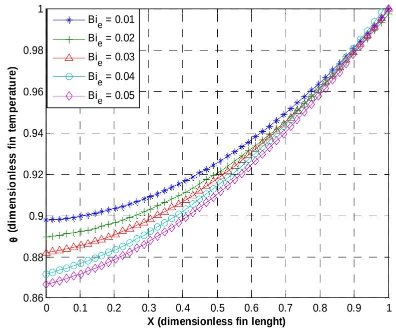


Fig 4.Temperature distribution in a straight fin with convecting tip, M=0.5

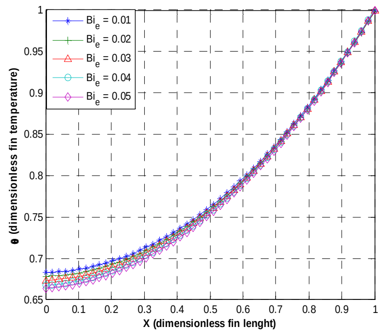


Fig..5 Temperature distribution in a straight fin with convecting tip, M=1.0

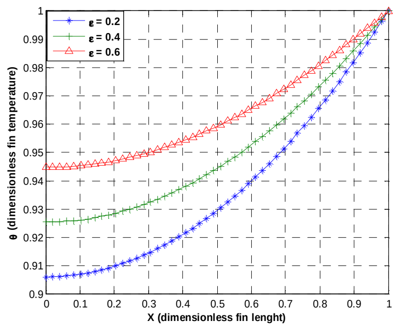


Fig. 6.Temperature distribution in a straight fin with insulated tip, M=0.5

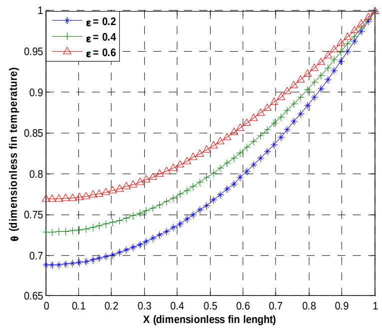


Fig.7. Temperature distribution in a straight fin with insulated tip, M=1.0

The effect of fin tip end condition on the rate of heat loss from the fin tip is shown Fig. 8 and 9. While Fig. 10-13 show the effects Biot number and the small parameter, ε, on the efficiency of the fin. Since, the fin temperature drops along the fin length, the fin efficiency decreases with increase in fin parameter. Therefore, in practice required fin length should be properly determined because the fin length that causes the fin efficiency to drop below 60% cannot be justified economically and should be avoided.

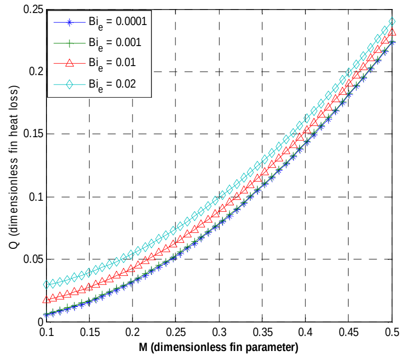


Fig.8. Rate of heat loss from a straight fin with convecting tip, ε=0.2

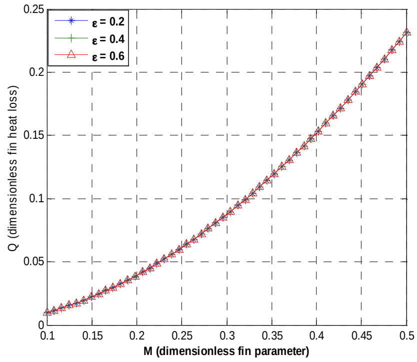


Fig. 9. Rate of heat from a straight fin with insulated tip, ε=0.2

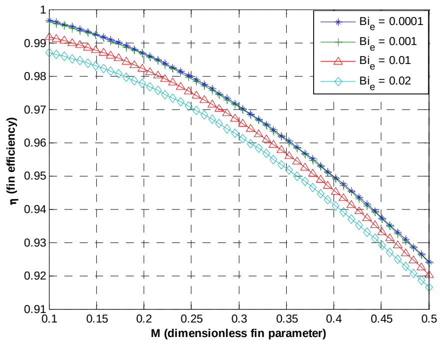


Fig.10. Efficiency of a straight fin with convecting tip, ε=0.2

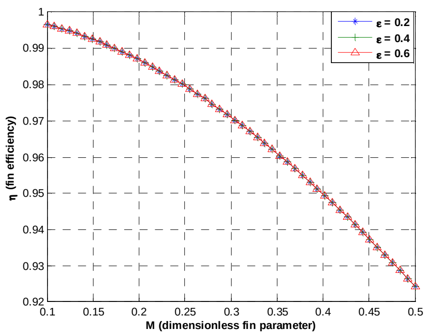


Fig. 11. Efficiency of a straight fin with insulated tip, ε=0.2

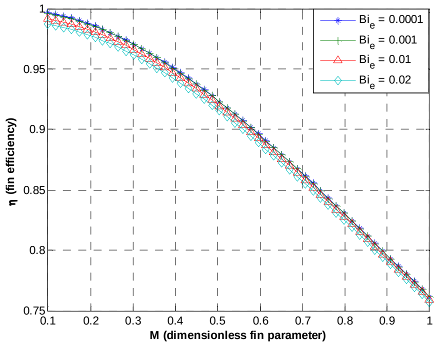


Fig.12. Rate of heat loss from a straight fin with convecting tip, ε=0.6

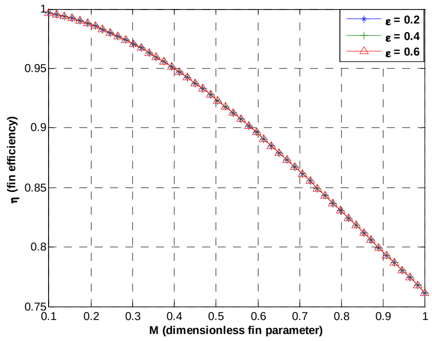


Fig. 13 Rate of heat from a straight fin with insulated tip, ε=0.6

## 4.0 Conclusion and Future Work

In this work, the approximate analytical solution for Temperature distribution in a straight fin having variable thermal conductivity with convecting tip was obtained using regular perturbation solution. The approximate solution methods used obtain the effects Biot number, varying thermal conductivity, convective and insulated tips on the temperature distribution in the straight. From the results, it shown that the variation in the thermal conductivity of the fin has pronounced effects on the temperature distribution especially when a large temperature difference exists between the prime surface and the environment. From the results, it is found that the Biot number has great influence on the temperature distribution and consequently on the performance of the straight fin. Also, for a high value of fin parameter, the fin with convecting tip is enhanced by 20 % heat conducting capacity than the assumed insulated fin of Aziz and Huq.

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