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Two-stage Interval Time Minimization Transportation Problem with Capacity Constraints

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Abstract

This paper discusses a two-stage interval time minimization transportation problem with capacity constraints on each source-destination link. In the current problem, the exact total demand of all the destinations lies in an interval whose end points are equal to the minimum and the maximum total available amount of the homogeneous product (which is being transported), at the sources. The minimum amount available at the sources is transported in the first stage of transportation, while enough amount of the product is shipped in the second stage so as to meet the exact total demand of the destinations. The objective of the current paper is to obtain a transportation schedule so that the transportation in both the stages is carried out in the minimum possible time. A polynomial time iterative algorithm is proposed which generates a sequence of pairs of Stage-I and Stage-II times. Firstly, a related standard cost minimization transportation problem (CMTP) is solved and then at each iteration, a restricted version of this CMTP is solved in which the time of Stage-I decreases strictly and the time of Stage-II increases. Out of these generated pairs, a pair with the minimum sum of transportation times of Stage-I and Stage-II is considered as the global optimal solution. Numerical illustration is included in the support of the theory.

Keywords: Time transportation problem, Combinatorial optimization, Non-convex programming.

1. Introduction

The most commonly studied transportation problem in the literature is the transportation problem with a single objective of minimization of the total transportation cost. But, in a variety of real world transportation problems, like the transportation of perishable goods from the stores to the market and in emergency situations like war, where army troops are to be sent to the battle field etc., the time of transportation problem is relevant too. This gives rise to time minimization transportation problem (TMTP) which was first discussed by Hammer. (Hammer 1969).

Later on, many authors studied this problem and proposed different solution methodologies (Garfinkel and Rao 1971), (Szwarc 1971), (Bhatia, Swarup and Puri 1977), (Sharma and Swarup 1977), (Arora and Puri 1997, 2001), (Parkash 1982). The mathematical structure of the standard TMTP which was proposed by Hammer is as follow:

$$Min_{\{X=\{x_{ij}\}\in S\}} Max_{I\times J}\, t_{ij}(x_{ij})$$

where the set S is given by

$$S = \left\{X = x_{ij} \middle| \sum_{j\in J} x_{ij} = a_i \quad \forall\, i \in I,\ \sum_{i\in I} x_{ij} = b_j \ \ \forall\, j \in J, x_{ij} \geq 0\, \forall\, (i,j) \in I \times J\right\}$$

where $I = \{1,2,\ldots,m\}$ and $J = \{1,2,\ldots,n\}$ denote the index set of m sources and n destinations respectively. The availability and the demand of the homogeneous product at the i^{th} source and the j^{th}destination is denoted by a_i and b_j respectively. Also, x_{ij} denotes the number of units of the homogeneous product to be transported from the i^{th} source to the j^{th} destination. This problem is said to be a balanced transportation problem if the total availability of the homogeneous product at the various sources is equal to the total demand of all the destinations. The time of transportation associated to the link joining i^{th} source and the j^{th} destination is denoted as t_{ij} where

$$t_{ij} = \{t_{ij}(\geq 0) \quad if\ x_{ij} > 0$$
$$\qquad 0 \qquad if\ x_{ij} = 0\,\}$$

A variant of the standard TMTP is a two-stage time minimization transportation problem (TSTMTP) where due to the storage constraints or some kinds of duties or may be due to some political reasons, it is not possible to transport the amount equal to the total demand in one go, so the transportation has to take place in two stages.

Two-stage transportation problem is studied by many authors. Pandian and Natrajan proposed a zero point method to study the two-stage transportation problem (Pandian and Natrajan 2011). Gani and Sudhakar studied the two-stage transportation problem under fuzzy envoirnment (Gani et al. 2006, Sudhakar et al. 2011). Gen proposed a genetic algorithm for the two-stage transportation problem (Gen et al. 2006). The Variants of TSTMTP are also studied thoroughly in literature. Sharma et al. discussed TSTMTP with the capacity constraints (Sharma et al. 2010) in which due to some additional restrictions, a capacity was introduced to each of the source-destination links. Further TSTMTP with restricted flow was studied by Kaur and Dahiya (Kaur and Dahiya 2014). In 2010, Sharma et al studied a two-stage interval TMTP in which the total demand of all the destinations lies in an interval with the end points equal to the total minimum and the total maximum available amount of the product at the sources (Sharma et al. 2008). In the first stage, the sources ship all of their on-hand material to the demand points whereas in the second stage, shipment covers the demand that is not fulfilled in the first stage.

The current paper is an extension of the problem discussed by Sharma (Sharma et al. 2010) in a way that a capacity u_{ij} is introduced on each $(i,j)^{th}$ source-destination link. It is assumed that vehicles are available for the transportation of the product from various sources to the various destinations. Due to some additional constraints like, fuel budget, capacity of vehicles, some kinds of duties or may be due to some political reasons, it is not possible to transport more than a particular number of units from a source to a particular destination. Thus, the current problem is formulated as the two-stage interval TMTP with the capacity constraints.

3. Mathematical formulation

Let the minimum and the maximum availability of the homogeneous product at the source $i,\ \forall\, i \in I$ be denoted by a_i and a_i' respectively and $b_j,\ \forall\, j \in J$ be the demand of the same at the destination j, where

$$\sum a_i < \sum b_j < \sum a_i'.$$

In the first stage of the two-stage interval TMTP, the quantity $a_i < a_i'$ is shipped from each source i to the various destinations and then, enough quantity of the product is dispatched in the second stage so as to satisfy the demand b_j at the destination j exactly. Thus the Stage-I problem can be formulated as:

$$Min_{\{X=\{x_{ij}\}\in S'\}}[T_1(X)] = Min_{\{X=\{x_{ij}\}\in S'\}}[Max_{I\times J}\, t_{ij}(x_{ij})]$$

where the set S' is given by

$$S' = \left\{X = x_{ij} \,\middle|\, \sum_{j\in J} x_{ij} = a_i \ \ \forall\, i \in I,\ \sum_{i\in I} x_{ij} \le b_j \ \ \forall\, j \in J,\ 0 \le x_{ij} \le u_{ij},\ \forall\,(i,j) \in I \times J\right\}$$

Corresponding to a feasible solution $X = \{x_{ij}\}$ of Stage-I problem, let $S'(X)$ be the set of feasible solutions of Stage-II problem which is stated below:

$$Min_{\{Y=\{y_{ij}\}\in S'(X)\}}[T_2(Y)] = Min_{\{Y=\{y_{ij}\}\in S'(X)\}}[Max_{I\times J}\, t_{ij}(y_{ij})]$$

where the set $S'(X)$ is given by

$$S'(X) = \left\{Y = y_{ij} \,\middle|\, \sum_{j\in J} y_{ij} \le a_i' - a_i \ \ \forall\, i \in I,\ \sum_{i\in I} y_{ij} = b_j - b_j' \ \ \forall\, j \in J,\ 0 \le y_{ij} \le u'_{ij},\ \forall\,(i,j) \in I \times J\right\}$$

Here $b_j' = \sum_{i\in I} x_{ij},\ j \in J$ and $u'_{ij} = u_{ij} - x_{ij}$.

Thus a capacitated two-stage time minimization transportation problem can be defined as:

$$Min_{\{X=\{x_{ij}\}\in S'\}}[T_1(X) + Min_{\{Y=\{y_{ij}\}\in S'(X)\}}[T_2(Y)]] \qquad (P)$$

A capacitated interval time minimizing transportation problem (P_α) defined below is closely related to the problem (P), where (P_α) is defined as:

$$Min_{\{Z=\{z_{ij}\}\in S\}}[T(Z)] = Min_{\{Z=\{z_{ij}\}\in S\}}[Max_{I\times J}\, t_{ij}(z_{ij})] \qquad (P_\alpha)$$

where the set S is given by

$$S = \left\{Z = z_{ij} \,\middle|\, a_i \le \sum_{j\in J} z_{ij} \le a_i' \ \ \forall\, i \in I,\ \sum_{i\in I} z_{ij} = b_j \ \ \forall\, j \in J,\ 0 \le z_{ij} \le u_{ij},\ \forall\,(i,j) \in I \times J\right\}$$

A feasible solution of the problem (P) provides a feasible solution to the problem P_α and conversely (Sharma et al. 2010). A balanced capacitated transportation problem (P_β) associated with the problem P_α is defined as:

$$Min_{\{Z=\{z_{ij}\}\in \hat{S}\}}[\hat{T}(Z)] = Min_{\{Z=\{z_{ij}\}\in \hat{S}\}}[Max_{\hat{I}\times \hat{J}}\, \hat{t}_{ij}(z_{ij})]$$

where,

$$\hat{S} = \{\ \sum_{j\in \hat{J}} z_{ij} = \hat{a}_i \ \ \forall\, i \in \hat{I},\ \ \sum_{i\in \hat{I}} z_{ij} = \hat{b}_j \ \ \forall\, j \in \hat{J},\ \ z_{ij} + z_{m+i,j} \le \hat{u}_{ij},\ \forall\,(i,j) \in I \times J,$$
$$0 \le z_{i,n+1} \le \hat{u}_{i,n+1},\ \forall\, i \in I,\ 0 \le z_{m+i,n+1} \le \hat{u}_{m+i,n+1},\ \forall\, i \in I\} \qquad (P_\beta)$$

where,

$$\hat{I} = \{1,2, \ldots \ldots . m, m+1, \ldots \ldots . . n\} \qquad \hat{J} = J \cup \{n+1\}$$

$$\hat{a}_i = a_i, \ \forall\, i \in I \qquad \hat{a}_{m+i} = a_i' - a_i, \quad \forall\, i \in I \ .$$

$$\hat{b}_j = b_j, \ \forall\, j \in J \qquad \hat{b}_{n+1} = \textstyle\sum_{i \in I} a_i{}' - \sum_{j \in J} b_j$$

$$\hat{t}_{ij} = t_{ij}, \forall\, (i,j) \in I \times J \qquad \hat{t}_{m+i,j} = t_{ij} \quad \forall\, i \in I$$

$$\hat{t}_{i,n+1} = M \quad \forall\, i \in I \quad \text{where M is a very large positive number.}$$

$$\hat{u}_{ij} = u_{ij}, \quad \forall\, (i,j) \in I \times J \qquad \hat{u}_{i,n+1} = \infty \quad \forall\, i \in I$$

$$\hat{u}_{m+i,n+1} = \infty \quad \forall\, i \in I$$

Definition (**M-feasible solution**): A feasible solution $Z = \{z_{ij}\}_{\hat{I} \times \hat{J}}$ of the problem (P_β) is called an M-feasible solution (MFS) if $z_{ij} = 0 \ \ \forall\, (i,j) \in \ \hat{I} \times \hat{J}$ for which $t_{ij} = M$.

An optimal basic feasible solution (OBFS) of (P_α) can be obtained from an optimal MFS solution of (P_β) as these problems are equivalent. Equivalence between (P_α) and (P_β) can be established by proving various results discussed by Sharma et al. in their paper (Sharma et al. 2010) . All these results can be proved for a TMTP on the same lines as prove for CMTP (Sharma et al. 2010).

3. Theoretical Development

In 1980, Bansal and Puri proved that the objective function in a TMTP is a concave function. So, the objective function in Stage-I and the Stage-II problem and hence in the problem (P) is concave and it belongs to the class of the concave minimization problem (CMP). As the global minimum of a CMP over a polytope is attainable at an extreme point of the polytope, it is desirable to investigate only its extreme points. Let the set of transportation times on the various routes of (P_β) be partitioned into a number of disjoint sets B_k, $k = 1,2, \ldots \ldots , s$.

where $B_k = \{(i,j) \in I \times J : t_{ij} = t^k\}$ and $t^j > t^{j+1} \ \forall\, j = 1,2 \ldots \ldots . . s-1$.

Positive weights say λ_{s-k+1}, $k = 1,2 \ldots . . , s$ are attached to these sets where $\lambda_{j+1} >> \lambda_j \ \ \forall\, j = 1,2 \ldots . , s-1$.

By doing so, the given TMTP (P_β) changes to a standard CMTP:

$$Min \ \textstyle\sum_{k=1,2\ldots s} \lambda_k \left(\sum_{(i,j) \in B_k} z_{ij}\right)$$

where $Z = \{z_{ij}\}$ belongs to the transportation polytope over which (P_β) is being studied. The OBFS of this standard CMTP yields an optimal solution of the problem (P_β) in which the transportation cost is minimum not only on the routes of the longest duration, but also on the routes of the second-longest duration if the transportation cost on the routes of the longest duration is minimum and on the routes of the third-longest duration if the transportation cost on the routes of the longest and the second-longest duration is minimum and so on. The proposed algorithm generates a sequence of the pairs of Stage-I and Stage-II times, where at each iteration, Stage-I time decreases strictly and Stage-II time increases. Therefore, firstly we concentrate on finding the minimum possible time for the Stage-II by finding the optimal feasible solution (OFS) of the following CMTP defined as:

$$(CP) \qquad Min_{\xi} \textstyle\sum_{\hat{I} \times \hat{J}} c_{ij} z_{ij} \, ,$$

where,

$$c_{i,n+1} = M \qquad \forall\, i \in I$$

$$c_{m+i,n+1} = 0 \qquad \forall\, i \in I$$

$$c_{ij} = 0 \qquad \forall\, (i,j) \in I \times J.$$

$$c_{m+i,j} = \lambda_{s-k+1} \quad \text{if} \quad t_{m+i,j} = t^k \ \forall\, (i,j) \in B_k \text{ and } k = 1,2 \ldots \ldots s.$$

Denote the Stage-II time obtained by finding the (OFS), Z^0 of the above (CP) by T_2^0 and the corresponding Stage-I time by T_1^0 yielding a pair of Stage-I and Stage-II times (T_1^0, T_2^0). Let at any iteration k, the pair of times of the Stage-I and Stage-II be (T_1^{k-1}, T_2^{k-1}) where $T_1^{k-1}, T_2^{k-1} \in \{t_1, t_2, \ldots . . t_s\}$. The restricted version of the problem (CP) denoted by $(CP_k), k \geq 1$, which concentrates on decreasing the Stage-I time is defined as:

$$(CP_k) \qquad Min_{\xi} \textstyle\sum_{\hat{I} \times \hat{J}} c_{ij} z_{ij} \, ,$$

where,

$$c_{ij} = M \quad \text{if} \quad t_{ij} \geq T_1^{k-1}, \quad \forall (i,j) \in I \times J.$$
$$c_{ij} = 0 \quad \text{if} \quad t_{ij} < T_1^{k-1}, \quad \forall (i,j) \in I \times J$$
$$c_{i,n+1} = M \quad \forall i \in I$$
$$c_{m+i,n+1} = 0 \quad \forall i \in I$$
$$c_{m+i,j} = \lambda_{s-k+1} \quad \text{if} \quad t_{m+i,j} = t^k \ \forall \ (i,j) \in B_k \text{ and } k = 1,2 \dots \dots s.$$

Let Z^k be an optimal MFS of the problem (CP_k) yielding the corresponding times of Stage-I and Stage-II as T_1^k and T_2^k respectively.

Remark 1: The problems (CP) and (CP_k) are the capacitated cost minimizing transportation problems but the capacity constraint viz. $z_{ij} + z_{m+i,j} \leq \hat{u}_{ij}, \forall (i,j) \in I \times$ J where, $\hat{u}_{ij} = u_{ij}, \ \forall (i,j) \in I \times$ J can be considered as the partial sum constraints. Therefore, these problems can be solved by treating them as the uncapacitated CMTPs with the partial sum constraints. The method to solve the transportation problems with partial sum constraints is discussed by Dantzing and Thapa and this method is used to solve the current problem (Dantzing and Thapa 2003).

Theorem 1. For an optimal MFS of the problem (CP_k), T_2^k is the minimum time of Stage-II corresponding to the Stage-I time $T_1^k < T_1^{k-1}$ 1 i:e: there does not exist a pair (T_1, T_2) such that $T_1 < T_1^{k-1}$ and $T_2 < T_2^k$.

Proof: Let, if possible, there exists a pair (T_1, T_2) yielded by some M-feasible solution $Z' = \{z'_{ij}\}$ of the problem (CP_k) such that $T_2 < T_2^k$ and $T_1 < T_1^{k-1}$ where $T_2 = t_p$ and $T_2^k = t_q$ for some $p, q \in \{1,2 \dots . s\}$. Since $T_2 < T_2^k$, therefore $p > q$; which implies $s - p + 1 < s - q + 1$.
Therefore,

$$C(Z') = \sum_{I \times J} c_{ij} z'_{ij} = \sum_{l=1,2..s} \lambda_{s-l+1} (\sum_{(i,j) \in B_l} z'_{ij} = \sum_{l=p,p+1..s} \lambda_{s-l+1} (\sum_{(i,j) \in B_l} z'_{ij})$$

Also,

$$C(Z^k) = \sum_{I \times J} c_{ij} z_{ij}^k = \sum_{l=1,2..s} \lambda_{s-l+1} (\sum_{(i,j) \in B_l} z_{ij}^k = \sum_{l=q,q+1..s} \lambda_{s-l+1} (\sum_{(i,j) \in B_l} z_{ij}^k)$$

Since $\lambda_{i+1} >> \lambda_i$, $i = 1,2 \dots . s - 1$.

$$\sum_{l=p,p+1..s} \lambda_{s-l+1} (\sum_{(i,j) \in B_l} z'_{ij} < \sum_{l=q,q+1..s} \lambda_{s-l+1} (\sum_{(i,j) \in B_l} z_{ij}^k))$$

This implies $\quad C(Z') < C(Z^k).$

But this contradicts the optimality of Z^k, therefore $T_2^k \leq T_2$.

Theorem 2: (CP) gives the optimal time of Stage-II.

Proof: It follows on the same lines as proof of Theorem 1.

Remark 2. By construction of (CP_k) it is observed that $T_1^0 > T_1^1 > T_1^2 \dots \dots . . > T_1^l$ and $T_2^0 \leq T_2^1 \leq T_2^2 \dots \dots \dots \leq T_2^l$ where T_1^1 is the optimal time of Stage-I.

Proof. The first sequence is clear from the construction of the problem (CP_k). Let if possible, $T_2^{k+1} < T_2^k$ for some k. Let $C_k = C(Z^k), C_{k+1} = C(Z^{k+1})$.

Since by reasoning used in the proof of Theorem 1, we see that $C_{k+1} < C_k$ for $T_2^{k+1} < T_2^k$.

This implies Z^{k+1} is a feasible solution of (CP_k) with $C_{k+1} < C_k$, a contradiction to the fact that Z^k is an OFS of (CP_k).

Remark 3. Since the optimal time of Stage-I problem is T_1^l, OFS of (CP_{l+1}) is not M-feasible.

Remark 4. If the overall time of transportation of the problem (CP) is $T(= t^r, r \in \{1,2 \dots \dots s\}$, then the maximum number of iterations required to solve the problem is $(s - r + 1)$.

Theorem 3: Let the generated pairs of the times of Stage-I and Stage-II be (T_1^k, T_2^k), $k \geq 0$ k. Then the optimal value of the problem (P) is given by $\min_{h=0,1,\ldots,l}[T_1^h + T_2^h]$

Proof: For proof, refer (Sharma et al. 2010).

4. The Procedure

- Initial Step. Find an (OFS) of the problem (CP) and obtain the corresponding times of Stage-I and Stage-II as T_1^0 and T_2^0 respectively.
- General Step. If $k \geq 1$ at a given pair (T_1^{k-1}, T_2^{k-1}) of Stage-I and Stage-II times, solve the problem (CP_k). From the (OFS) of the problem (CP_k), construct the pairs (T_1^k, T_2^k).
- Terminal Step. If (OFS) of the problem (CP_k) is not M-feasible, then Stop. The optimal value of the objective function of the problem (P) is given by $\min_{h=0,1,\ldots,l}[T_1^h + T_2^h]$

5. Numerical Illustration

Consider the two-stage interval capacitated time minimization transportation problem given in Table 1. The capacity associated to each link is given in the Table 2.

Table 1. Cost associated to source-destination links with availabilities and demands.

a_i = Min availability at i^{th} source
a'_i = Max availability at i^{th} source
b_j = Demand at j^{th} destination

	D_1	D_2	D_3	a_i	a'_i
S_1	5	10	9	20	50
S_2	2	7	4	30	60
S_3	12	6	8	40	70
b_j	25	40	65		

Table 2. Capacity associated to various source-destination links.

	D_1	D_2	D_3
S_1	20	25	25
S_2	15	20	30
S_3	20	15	30

The partition of the various time routes is given by

$t_1(=12) > t_2(=10) > t_3(=9) > t_4(=8) > t_5(=7) > t_6(=6) > t_7(=5) > t_8(=4) > t_9(=2)$

as $t_s = t_9 = 2$, therefore $s = 9$.

The corresponding problem (P_8) is shown in Table 3.

Table 3. A balanced problem associated with the given problem (P)

	D_1	D_2	D_3	D_4	a_i
S_1	5	10	9	M	20
S_2	2	7	4	M	30
S_3	12	6	8	M	40
S_4	5	10	9	0	30
S_5	2	7	4	0	30
S_6	12	6	8	0	30
b_j	25	40	65	50	

An OBFS of the problem (CP) yields the Stage-I time as $T_1^0 = 10$ and Stage-II time as $T_2^0 = 5$ where 5 is the optimal time of Stage-II. Next pair is obtained by solving the time minimization transportation problem (CP_1), an OBFS of which yields Stage-I time as 9 and Stage-II time as 10 where, 10 is the minimum time for the Stage-II corresponding to the Stage-I time 9.

Table 4. An optimal feasible solution of the problem (P_β).

Entries in the lower right corner are costs associated with various times

Entries in the bold face are allocation of the product at various links.

$\lambda_i's$ = lexicographic costs associated with various times.

	D_1	D_1	D_1	D_2	D_2	D_2	D_3	D_3	D_3	D_4	a_i
S_1	0	*	*	**15** 0	*	*	**5** 0	*	*	M	20
S_2	*	0	*	*	**15** 0	*	*	**15** 0	*	M	30
S_3	*	*	**0** 0	*	*	**10** 0	*	*	**30** 0	M	40
S_4	**10** λ_3	*	*	λ_8	*	*	λ_7	*	*	**20** 0	30
S_5	*	**15** λ_1	*	*	λ_5	*	*	**15** λ_2	*	0	30
S_6	*	*	λ_9	*	*	λ_4	*	*	λ_6	**30** 0	30
b_j	10	15	0	15	15	10	5	30	30	50	

Similarly, on solving the problem (CP_2), the pair obtained is $(8, 10)$. Algorithm terminates here as (CP_3) is no more M-feasible. Thus $\min\{10 + 5, 9 + 10, 8 + 10\} = 15$. Hence the optimal value of the objective function of the problem (P) corresponds to the pair $(10; 5)$. The transportation schedule (an OBFS of the problem (CP)) which gives this optimal value is shown in the Table 4 where, the entries in the lower right corner represent the associated cost and the entries in boldface show the values of the basic variables and the links marked as (*) are considered as blocked.

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