Innovative Systems Design and Engineering www.iiste.org
ISSN 2222-1727 (Paper) ISSN 2222-2871 (Online)
Vol 2, No 3

Vibration of Non-Homogeneous Visco-Elastic Square Plate of Variable Thickness In both directions With Thermal Effect

Anupam Khanna (Corresponding author)
Dept. Of Mathematics, MMEC, MMU (MULLANA), Ambala, Haryana, INDIA
3/1413, Janakpuri, Saharanpur, UP, INDIA
Tel: - +91-903409199, +91-8439448048, E-mail: anupam_rajie@yahoo.co.in

Ashish Kumar Sharma
Dept. Of Mathematics, MMEC, MMU (MULLANA), Ambala, Haryana, INDIA
PO box Sunder Bani, Jammu & Kashmir, Pin code 185153, INDIA
Tel: - +91-8529525643, +91-9858687507, E-mail: ashishk482@gmail.com

The research is financed by Asian Development Bank. No. 2006-A171(Sponsoring information)

Abstract

Plates are being increasingly used in the aeronautical and aerospace industry as well as in other fields of modern technology. Plates with variable thickness are of great importance in a wide variety of engineering applications i.e. nuclear reactor, aeronautical field, naval structure, submarine, earth-quake resistors etc. A mathematical model is presented for the use of engineers and research workers in space technology; have to operate under elevated temperatures. It is considered that the temperatures vary linearly in one direction and thickness of square plate varies linearly in two directions. An approximate but quite convenient frequency equation is derived for a square plate by using Rayleigh-Ritz technique. Both the modes of the frequency are calculated by the latest computational technique, MATLAB, for the various values of taper parameters and temperature gradient. All the results are presented in the form of table.

Keywords: Visco-elastic, Thermal Effect, Square plate, Vibration, Taper constant.

1. Introduction

With the advancement of technology, the requirement to know the effect of temperature on visco-elastic plates of variable thickness has become vital due to their applications in various engineering branches such as nuclear, power plants, engineering, industries etc. Plates are inherently associated with many mechanical structures which are designed to perform under extreme dynamic loading conditions. There has been a constant need for the light weight and high strength materials for various applications like aerospace and automobiles. Therefore visco-elastic materials are utilized in making structural parts of equipment used in modern technological industries. Further in mechanical system where certain parts of machine have to operate under elevated temperature, its effect is far from negligible and obviously cause non-homogeneity in the plate material of the materials becomes functions of space variables.

Many researchers have analyzed the free vibration of visco-elastic plates with variable thickness for many years. During the past four decades, vibration of plates has become an important subject in engineering applications. There are several papers about plate vibrations in open technical literature. Square plates have many engineering applications. These are commonly found in spacecrafts, missiles, land base vehicles, off-shore platforms, and underwater acoustic transducers.

The aim of present investigation is to study the free vibration of visco-elastic square plate whose thickness varies linearly in both directions. It is assumed that the plate is clamped on all the four edges and its temperature varies linearly in one direction. For various numerical values of thermal gradient and taper constants; frequency at different points for the first two modes of vibration are evaluated with the help of MATLAB. All the numerical calculations will be carried out using the material constants of alloy 'Duralium'. All results are shown in the form table.

2. Equation of Motion and Analysis

Differential equation of motion for visco-elastic square plate of variable thickness in Cartesian coordinate is in equation (2.1):

$$\begin{aligned}&[D_1\left(W,_{xxxx}+2W,_{xxyy}+W,_{yyyy}\right)+2D_1,_x\left(W,_{xxx}+W,_{xyy}\right)+2D_{1,y}\left(W,_{yyy}+W,_{yxx}\right)+\\&D_{1,xx}(W,_{xx}+\nu W,_{yy})+D_{1,yy}(W,_{yy}+\nu W,_{xx})+2(1-\nu)D_{1,xy}W,_{xy}]-\rho hp^2W=0\end{aligned} \tag{2.1}$$

which is a differential equation of transverse motion for non-homogeneous plate of variable thickness.

Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = Eh^3/12(1-\nu^2) \tag{2.2}$$

and corresponding two-term deflection function is taken as

$$W=[(x/a)(y/a)(1-x/a)(1-y/a)]^2[A_1+A_2(x/a)(y/a)(1-x/a)(1-y/a)] \tag{2.3}$$

Assuming that the square plate has a steady one dimensional temperature distribution i.e.

$$\tau=\tau_0(1-x/a) \tag{2.3}$$

where τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "*a*" is the length of a side of square plate.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this form,

$$E=E_0(1-\gamma\tau) \tag{2.4}$$

where, E_0 is the value of the Young's modulus at reference temperature i.e. $\tau=0$ and γ is the slope of the variation of E with τ. The modulus variation (2.4) become

$$E=E_0[1-\alpha(1-x/a)] \tag{2.5}$$

where, $\alpha=\gamma\tau_0(0\le\alpha<1)$, a parameter.

It is assumed that thickness also varies linearly in two directions as shown below

ISSN 2222-1727 (Paper) ISSN 2222-2871 (Online)
Vol 2, No 3

$$h = h_0(1+\beta_1 x/a)(1+\beta_2 y/ \quad (2.8)$$

where a is length of a side of square plate and β_1 & β_2 are taper parameters in x- & y- directions respectively and $h=h_0$ at $x=y=0$.

Assuming that density varies linearly in x-direction as

$$\rho = \rho_0(1+\alpha_1 \frac{x}{a}) \quad (2.9)$$

Put the value of E & h from equation (2.5) & (2.8) in the equation (2.2), one obtain

$$D_1 = [E_0[1-\alpha(1-x/a)]h_0^3(1+\beta_1 x/a)^3(1+\beta_2 y/a)^3]/12(1-v^2) \quad (2.10)$$

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(V^* - T^*) = 0 \quad (2.11)$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$\left.\begin{aligned} W = W,_x = 0 &, \quad x = 0, a \\ W = W,_y = 0 &, \quad y = 0, a \end{aligned}\right\} \quad (2.12)$$

Now assuming the non-dimensional variables as

$$X = x/a, Y = y/a, \overline{W} = W/a, \overline{h} = h/a \quad (2.13)$$

The kinetic energy T* and strain energy V* are

$$T^* = \int_0^1\int_0^1 \rho h \overline{W^2}] dYdX \quad (2.14)$$

and

$$V^* = \int_0^1\int_0^1 D_1(1+\beta_2 Y)^3\{(\overline{W},_{XX})^2 + (\overline{W},_{YY})^2 + 2v\overline{W},_{XX}\overline{W},_{YY} + 2(1-v)(\overline{W},_{XY})^2\}dYdX \quad (2.15)$$

Using equations (2.14) & (2.15) in equation (2.11), one get

$$(V^{**} - \lambda^2 p^2 T^{**}) = 0 \quad (2.16)$$

where

$$V^{**} = \int_0^1\int_0^1 [1-\alpha(1-X)](1+\beta_1 X)^3(1+\beta_2 Y)^3\{(\overline{W},_{XX})^2 + (\overline{W},_{YY})^2 + 2v\overline{W},_{XX}\overline{W},_{YY} + 2(1-v)(\overline{W},_{XY})^2\}dYdX \quad (2.17)$$

and

$$T^{**} = \int_0^1 \int_0^1 [1+\alpha_1 X][(1+\beta_1 X)(1+\beta_2 Y)\overline{W^2}]dYdX \qquad (2.18)$$

Here, $\lambda^2 = 12\rho(1-v^2)a^2 / E_0 h_0^2$ is a frequency parameter.

Equation (2.16) consists two unknown constants i.e. A_1 & A_2 arising due to the substitution of W. These two constants are to be determined as follows

$$\partial(V^{**} - \lambda^2 T^{**}) / \partial A_n \quad , n = 1, 2 \qquad (2.19)$$

On simplifying (2.19), one gets

$$bn_1 A_1 + bn_2 A_2 = 0 \quad , n = 1, 2 \qquad (2.20)$$

where, bn_1, bn_2 (n=1,2) involve parametric constant and the frequency parameter.
For a non-trivial solution, the determinant of the coefficient of equation (2.20) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} b_{12} \\ b_{21} b_{22} \end{vmatrix} = 0 \qquad (2.21)$$

With the help of equation (2.21), one can obtains a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1(Mode1) & λ_2(Mode2) for different values of taper constant and thermal gradient for a clamped plate.

3. Result and Discussion

Frequency equation (2.21) is quadratic in λ^2, so it will give two roots. A two term deflection function is used to give the solution. The frequency is derived for the first two modes of vibration for non-homogenous square plate having linearly varying thickness in both the directions, for various values of non-homogeneity constant, taper constant and thermal gradient. The value of Poisson ratio v has been taken 0.345.
These results are tabulated in tables (1-3) for first two modes of vibration for clamped square plate.
In Table 1:- It is clearly seen that value of frequency decreases as value of thermal gradient increases from 0.0 to 1.0. These results have been displayed for the following three cases:
α_1=0.0, β_1=0.0, β_2=0.0
α_1=0.4, β_1=0.4, β_2=0.4
α_1=0.8, β_1=0.8, β_2=0.8
In Table 2:- Also, it is observed that for both modes of vibration, the frequency decreases with increases in non-homogeneity constant α_1 from 0.0 to 1.0. These results have been displayed for the following three cases:
α =0.0, β_1=0.2, β_2= 0.4
α =0.2, β_1=0.4, β_2= 0.4
α =0.4, β_1=0.6, β_2= 0.8
In Table 3:- It is observed that for both modes of vibration, frequency parameter increases with increases in taper constant β_1 from 0.0 to 1.0. These results have been displayed for the following three cases:

α =0.0, α_1=0.2, β_2= 0.4

α =0.2, α_1=0.4, β_2= 0.4

α =0.4, α_1=0.6, β_2= 0.8

Conclusion

The main aim for our research is to develop a theoretical mathematical model for scientists and design engineers so that they can make a use of it with a practical approach, for the welfare of the human beings as well as for the advancement of technology. Therefore engineers are advised to analyze and develop the plates in the manner so that they can fulfill the requirements.

References

A.K. Gupta and Lalit Kumar (2008), "Thermal effects on vibration of non-homogeneous visco-elastic rectangular plate of linearly varying thickness in two directions", *Meccanica,* Vol.43, 47-54.

A. W. Leissa (1969), Vibration of plates, *NASA SP* 160.

A.K. Gupta and Anupam Khanna (2010), "Thermal Effect On Vibrations Of Parallelogram Plate Of Linearly Varying Thickness", *Advanced Studies Of Theoretical Physics*, (Vol.4, No.17, pp: 817-826).

D, Young (1950), "Vibration of rectangular plates by the Ritz method", *J. App. Mech., Trans. ASME*, Vol. 17(4), 448-453.

A.K.Gupta and Anupam Khanna (2008), "Vibration of clamped visco-elastic rectangular plate with parabolic thickness variations", *Journal shock and vibration*, Vol.15 (6), 713-723.

B. Singh and V, Saxena (1996), "Transverse vibration of rectangular plate with bi-directional thickness variation", Journal sound and vibration, Vol.198 (1), 51-65.

A.K.Gupta and Harvinder Kaur (2008), "Study of the effect of thermal gradient on free vibration of clamped visco-elastic rectangular plates with linearly thickness variations in both directions", *Meccanica*, Vol. 43(4), 449-458.

M, Sasajima, J, Kakudate and Y, Narita (2002), "Vibration behavior and simplified design of thick rectangular plates with variable Thickness", *Journal sound and vibration*, Vol.124 (2), 302-309.

A.K. Gupta and Anupam Khanna (2009), "Free vibration of clamped visco-elastic rectangular plate having bi-direction exponentially thickness variations", *Journal of Theoretical and Applied Mechanics*, Vol. 47 (2), pp. 457-471.

A. Khanna, A. Kumar and M. Bhatia (2011), "A Computational Prediction on Two Dimensional Thermal Effect on Vibration of Visco-elastic Square Plate of Variable Thickness", *Presented and Published in Proceeding of CONIAPS XIII*, held in UPES, Deharadun.

A.K.Gupta and A. Khanna (2007), "Vibration of visco-elastic rectangular plate with linearly thickness variations in both directions", *J. Sound and Vibration*, Vol.301, 450-457.

Table 1:- Frequency Vs Thermal gradient.

α	**$\alpha_1=\beta_1=\beta_2=0.0$**		**$\alpha_1=\beta_1=\beta_2=0.4$**		**$\alpha_1=\beta_1=\beta_2=0.8$**	
	Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0	140.88	35.99	188.87	48.30	246.74	63.45
0.2	133.65	34.15	180.53	46.25	236.94	60.92
0.4	126.11	32.19	171.82	44.12	226.71	58.27
0.6	117.87	30.11	162.61	41.64	216.11	55.48
0.8	109.12	27.88	152.84	39.13	204.75	52.53
1	99.62	25.45	142.43	36.43	192.83	49.38

Table 2:- Frequency Vs Non-homogeneity constant.

α_1	**$\alpha=0.0, \beta_1=0.2, \beta_2=0.4$**		**$\alpha=0.2, \beta_1=0.4, \beta_2=0.4$**		**$\alpha=0.4, \beta_1=0.6, \beta_2=0.8$**	
	Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0	188.67	48.25	216.83	55.51	247.11	63.28
0.2	179.75	45.99	206.44	52.92	235.15	60.28
0.4	171.99	44.12	197.42	50.62	224.68	57.67

0.6	165.15	42.28	189.48	48.62	215.56	55.37
0.8	159.17	40.73	182.48	46.82	207.47	53.33
1	153.62	39.34	176.13	45.22	200.23	51.49

Table 3:- Frequency Vs Taper constant.

β_1	**α =0.0, α_1=0.2, β_2= 0.4**		**α =0.2, α_1=0.4, β_2= 0.4**		**α =0.4, α_1=0.6, β_2= 0.8**	
	Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0	162.88	41.65	163.36	41.80	163.71	41.91
0.2	179.72	45.97	180.21	46.14	180.53	46.24
0.4	197.52	50.56	197.96	50.73	198.29	50.83
0.6	216.11	55.34	216.42	55.51	216.68	55.61
0.8	235.11	60.27	235.43	60.44	235.62	60.51
1	254.64	65.31	254.91	65.42	255.11	65.53