Optimization and Analysis of Underactuated Linkage Robotic Finger

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Abstract
In this study it is required to maximize the transmission performance, which is leading to increase the transmitted torque from the actuated joints to the underactuated joints through transmission mechanism. Accordingly grasping forces in finger phalanges will increase. Studying the four bar mechanism parameters of a specific configuration within defined limits led to the linkage transmission defect parameter, which play a major role in deciding the linkage performance, used as optimization objective function to be minimized. This study presents an optimization procedure carried out using matlab **fminunc** function, formulated by using Freudenstein's equations to be applied on a (Cassino-Underactuated-Multifinger-Hand) design , using one finger and a thumb .A mathematical model of grasping forces of the finger were introduced taking into account the solid links in the(Ca.U.M.Ha) robotic finger .
Keywords: Linkage , underactuated, optimization

1. Nomenclature

δ	transmission defect
g, H	length of link EH, link HI ,respectively
d_1	force arm of f_1 with respect to joint E.
d_2	force arm of f_2 with respect to joint B.
d_3	force arm of f_3 with respect to joint A.
J	jacobian matrix of the finger
θ°	joint velocities vector of the finger
θ_a°	actuating velocity of the driving bar of the finger
k_1, k_2, k_3	Frudenstein's equation constants.
l1,l2,l3	Proximal phalanx length, middle phalanx length, distal phalanx length.
$\theta_1, \theta_2, \theta_3$	Rotation angle of the proximal phalanx, middle phalanx, and distal phalanx,
$\theta_{1M}, \theta_{2M}, \theta_{3M}$	Maximum angle of the proximal , middle phalanx ,distal phalanx
$\gamma_1, \gamma_2, \gamma_3$	Angle of solid links H,E,F ,G,B,C ,D,A,P ,respectively.
f_1, f_2, f_3	Grasping force on the proximal ,middle,and distal phalanx .
k_{s1}	Stiffness of the finger second joint torsional spring.
k_{s2}	Stiffness of the finger third joint torsional spring.

2. Introduction.

In underactuated robotic manipulators, the need arises to differential mechanisms that transmit the actuation torque to the underactated joints .Then many ideas for mechanisms are used to serve this purpose .In general pulleys and tendons for small and medium grasping forces and linkages for large grasping forces are the main types used in this field. Using these mechanisms, usually have a large size and not able to fully control the underactuated robotic finger .there is also some requirements for the design in linkage based robotic finger ,that introduced linkage geometrical parameters which result in specified mechanical configuration to ensure the desired motion envelop .To make use, as much as possible, from these transmission mechanisms ,different optimization algorithms are adopted, to obtain better results under desired circumstances following one or multiple objective functions such as reduced power consumption, reduced structure errors for different mechanisms, transmit as large as possible, of the actuating torque, reducing weight and size …etc.

3. Literature review

The behavior of an under-actuated bar linkage finger such as its grasping kinematics and force depends greatly on its geometry and stiffness parameters(Birglen, Lionel, Laliberte’, Thierry,Gosselin, 2008).A design for an anthropomorphic finger should fulfill some features that depend on the aim that hand used for Lionel Birglen et al. (2008)presented an optimal design of under-actuated fingers considering several issues among which the force isotropy of the grasp, such characteristic is only local i.e. it depends on the finger configuration defined by θ_2 and θ_3 ,and the contact locations k_1, k_2, and k_3 .Changing the actuation torque modified the isotropic configuration of the finger, the researchers introduced the force isotropy equations, Their final aim was to guarantee stability for all grasps if possible, they introduced Kronecker-like symbol for characterizing the stability of the contact situation. The same researcher pointed that ,for the two -phalanx fingers the behavior of the underactuated linkage-driven finger was mainly dictated by the ratios $R_i = \frac{c_i}{a_i}$ with i=1,2 .In order to, minimize the size of the finger ,the length c_i should be chosen as small as possible but it was limited by consideration on mechanical interference ,so the R_i are used as design parameters , while the lengths $b_i$and angle (ψ) should be selected in order to obtain these parameters. E.Ngale Haulin et al. (2001) introduced a crossed four-link mechanism used in prosthesis hand of four fingers and a thumb using two performance criteria, the optimal dimension are obtained with the smallest value of the energy consumed , in addition to the smallest value of the mean quadratic error with minimum acceptable transmission angle. Effect of number of positions in the optimal dimensions of the mechanisms was also introduced, the optimal variables of the mechanisms are presented for 3,4,5,6,7,8 and 9 positions of the output and the coupling links. Their study permitted to confirm that the three positions synthesis led to a null mean quadratic error and the energy consumed obtained was very high. E.Ngale Haulin and R.Vinet (2003) expressed a constrained multiobjective optimization to the previous four –bar mechanism prosthesis hand. This was with reference to seven positions and with respect to five design criteria, considering the driving system of the finger. It was also to ensure grasping, holding and pinching operations efficiency of the hand prosthesis. Performance criteria are simultaneously considered in optimization of the mechanism. The five design goals were: minimum of the maximum driving torque, minimum energy consumed for a cycle of the mechanism's closure, minimum motion posture error, minimum deviation of the transmission angle from its ideal value 90º, and finally the minimum value of the mechanical error. It was concluded that maximum driving torque, energy consumed and angular accelerations can be greatly reduced, while the mechanical advantage and transmission angle can be increased with the inclusion of drive system during the mechanism optimization. Jorge Eduardo Parada Puig et al. (2008) proposed a multi-objective optimization for LARM hand ,where the driving mechanism was series of crossed –four bar linkages. The design of an anthropomorphic finger must fulfill basically the requirements of human-like motion and grasp, compact size, actuation lightweight, efficiency and position and force control. The researchers could obtain the numerical evaluation by noticing and relating the joint motion as compared to a finger design scheme then, interpolating the phalanx human hand experimental data to a cubic line. Accordingly, criterion for achieving an optimum solution can be formulated as the difference of those functions for a finger motion. J.A.Cabrera et al. (2008)introduced an optimal design of planar 1DoF mechanisms used for a robotic hand or a gripper. The researchers used genetic algorithm based on evolutionary approach based on differential evolution technique .They proposed (POEMA) method. They subdivided the problem into two parts, First part: To optimize the mechanism to grasp specific object, where the goal functions were: 1- maximizing the grasping index (GI) that is similar to the mechanical advantage, 2-minimizing the acceleration in the contact point to avoid a big impact on the object, 3-reducing the weight of the mechanism , 4-standarize the link length in the mechanism to avoid great difference between the lengths of the different links. For the second part, the researchers used the same hand robot mechanism, the mechanism was able to grasp different objects with different sizes within a determined range, where the goal functions in this case: 1- the contact point follow several precision points to determine the range of the size of the object and made the contact point follow a predefined trajectory. 2-The second goal function was also minimizes the grasping index (GI) as the first part, one of the features of the used method is that there was not a unique solution to the problem, as the method found several solutions which were called non-dominated solutions and every non –dominated solution was a good solution to the proposed problem so, it was concluded to determine which characteristic or goal function was a priority and which was not.

4. The Design of Underactuated Robotic Finger

An optimization algorithm will be applied on Ca.U.M.Ha (Cassino-Underactuated-Multifinger-Hand) .This hand is composed of three links which composed of proximal, median , and distal phalanges . A simplified sketch of the finger is shown in figure (1), θ_{1M} ,θ_{2M} ,θ_{3M}, represent the maximal angels of rotation of the first, second ,and third phalanges respectively, l1 ,l2,and l3 denoted the first, second, third phalanx lengths .The overall characteristics of the finger are listed in table 1.The finger mechanism shown in figure (2) is composed of one slider-crank mechanism I,H,F,E with a solid crank link of an angle γ_1 =43º and two four-bar linkages A,B,C,D and B,E,F,G connected in series by the rigid body B,C,G of angle $\gamma_2 = 55°$,this is used to transmit the motion between the first four bar linkage which represents the first phalanx and the second four bar linkage which stand for the median phalanx .The distal phalanx represented by the rigid body, A,D, P with an angle $\gamma_3 = 50°$.In order to prevent the hyper-flexion of the finger mechanism ,the finger mechanism is provided with a suitable mechanical stoppers .At the second and third joints A,B respectively torsional springs are added to constrain the motion(Rea, 2011) .

5. Optimal synthesis of planar four-link mechanism used in under- actuated robotic manipulator.

The dimensioning and selection of these mechanisms is a crucial issue in the design of the overall transmission mechanism. The optimal synthesis of two planar four-bar linkages and the offset slider-crank mechanism are carried out with reference to three positions of the input and output links .These are formulated using Freudensteins equations. Force transmission optimization was achieved using the transmission defect as an objective function to be minimized. Where the four –bar linkages are used as transmission mechanism for the motion from the linear actuator to the three phalanxes of the finger mechanism .

5.1 Synthesis of the four bar linkage A, B, C, D

Referring to figure (3), the Freudenstein's equations for the four bar linkage ABCD, can be expressed as (Hartenberg & Denavit, 1964):

$$k_1 \cos \varepsilon_i - k_2 \cos \rho_i + k_3 = \cos(\varepsilon_i - \rho_i) \qquad i = 1,2,3. \tag{1}$$

$$k_1 = \frac{l_2}{a}, \qquad k_2 = \frac{l_2}{c}, \qquad k_3 = \frac{a_2 - b^2 + c^2 + l_2^2}{2ac} \tag{2}$$

The four-bar linkage shown is in equilibrium ,let a, b, c and l2 be the magnitudes of the links AD,DC,BC and AB, respectively ,where l2 represent the second phalanx length of the suggested underactuatd linkage robotic finger. The angles ε_i and ρ_i for i=1,2,3 are the input and output angles of the four bar linkage of both BC and AD through given pairs of (ε_i, ρ_i) for i=1,2,3 .Angle $\alpha = 50°$ for link AD, while angles $\gamma = 40°$ and $\beta_1 = 25°$ for link BC are assumed for the specific proposed design. Equations (1) and (2) can be solved when three positions 1, 2 and 3 of both of the output and the input angles are given through the pairs of angle (ε_i, ρ_i) .The pairs of angles $(\varepsilon_1 = 115°, \rho_1 = 130°)$ and $(\varepsilon_3 = 140°, \rho_3 = 208°)$ are obtained for the starting and final configuration, respectively. Since that the above synthesis technique is used to synthesize a mechanism where three finitely separated positions of the input and output links are known, an extra position point is needed in addition to these two points. An optimization procedure in terms of force transmission has been developed by assuming (ε_2, ρ_2) as starting values. This corresponds to both middle position between 1 and 3 of links BC and AD , respectively. The transmission defect for the linkage is expressed in the form:

$$\text{fun}_{1=}\delta_1 = \sqrt{\frac{1}{\Delta\varepsilon}\int_{\varepsilon 1}^{\varepsilon 2} \cos^2 \mu_1 \, d\varepsilon} \qquad \Delta\varepsilon = \varepsilon_2 - \varepsilon_1 \tag{3}$$

Where:-

$$\mu_2 = \cos^{-1} \frac{l_2^2 + c^2 - a^2 - b^2 - 2l_2 c \cos(\pi - \varepsilon)}{2ab} \tag{4}$$

Substituting the initial values of (ε_2, ρ_2),in addition to the assumed values of (ε_1, ρ_1) and (ε_3, ρ_3), into equations (1) and (2), result in the links lengths values, substitute these values in equation (4) , getting (μ_2), substituting the later value in transmission defect equation (δ) if the later value is the minimum, then the values of a,b,c,d ,will be the optimum values, if it is not, another value of (ε_2, ρ_2)will be taken until getting the minimum value of the transmission defect within the given specified range of the input and output angles of the input and output links. The same initial values as the previous researcher's values were used for (ε_2, ρ_2) which they are (70º, 169º), using the new optimization algorithm. The new values of transmission defect were less than in the previous one. Figure (4) shows the optimization iteration by matlab optimization toolbox, while table (2) shows comparison between the new and the previous linkage ABCD parameters.

5.2 Synthesis of the four bar B, E, F, G

The same method has been applied to the synthesis of the function –generating four-bar linkage B, E, F, G, shown in figure (5) the Freudenstein's equation can be expressed in the form:

$$k_1 \cos \nu_i - k_2 \cos \emptyset_i + k_3 = \cos(\nu_i - \emptyset_i) \qquad i = 1,2,3. \tag{5}$$

$$k_1 = \frac{l_1}{d}, \qquad k_2 = \frac{l_1}{f}, \qquad k_3 = \frac{d^2 - e^2 + f^2 + l_1^2}{2df} \tag{6}$$

Where l1 is the length of the first phalanx, and, d, e and f are the lengths of BG, GF, FE. ν_i and Φ_i for i=1, 2, 3 are the input and output angels of the links EF and BG ,respectively. Equations (5) and (6) can be solved for three positions of the input link EF and output link BG, with pairs of angles (ν_i, ϕ_i) for i=1, 2 , 3 .According to the proposed mechanical design of the finger, the design parameters $\gamma = 40°$, $\beta_1 = 30º$,and ,$\chi = 10º$ are assumed empirically. Then the pairs of angles $(\nu_1 = 80°, \phi_1 = 60°)$ and $(\nu_3 = 140°, \phi_3 = 190°)$ are obtained for the starting and final position respectively of both links EF and BG. As described previously, the initial value of (ν_2, ϕ_2) is (110º, 125º) which correspond to the middle position between the starting and final positions of the links EF and BG, consequently. The transmission defect (δ) can be formulated as,

$$fun_{2=}\delta_2 = \sqrt{\frac{1}{\Delta\nu}\int_{\phi 1}^{\phi_2} \cos^2\mu_2 \, d\nu} \qquad \Delta\nu = \nu_2 - \nu_1 \tag{7}$$

Where:-

$$\mu_2 = \cos^{-1}\frac{l_1^2 + f^2 - d^2 - e^2 - 2l_1 f\cos(\pi - \phi)}{2de} \tag{8}$$

The middle position angle pair value in addition to the values of (ν_1, ϕ_1) and (ν_3, ϕ_3) used earlier in the previous research, are substituted three in Freudenstein's equations. Solving these equations, will give links lengths. Substitute these lengths in transmission angle equation (8) then in equation (7), if (δ) value is the minimum value, then the lengths will represent the optimal values if not, changing (ν_2, ϕ_2) values continues within specific range until getting the minimum value of (δ_2).The resulting links lengths will give the maximum force transmitted through this linkage. The obtained value of (δ_2) in this method is less than the previous value. Figure (6) shows the optimization iteration, while table (3) shows comparison between the new and the previous linkage BEFG parameters.

5.3 Synthesis of the slider-crank mechanism EHL

In the same way, as for both the four-bar linkage ABCD and BEFG, the offset slider –crank mechanism EHL, shown in figure (7) in which the motion of the slider is the input of the system and the output is the motion of the crank that represent the input for the next stage which represent the first phalanx, is synthesized by using Freudenstein's equations, which take the form:

$$k_1(s_1 - x_i)\cos\lambda_i + k_2\sin\lambda_i - k_3 = (s_1 - x_i)^2 \tag{9}$$

Where:-

$$k_1 = 2g \tag{10}$$
$$k_2 = 2go_f \tag{11}$$
$$k_3 = g^2 + o_f^2 - h^2 \tag{12}$$

o_f is the offset, h and g are the lengths of the links EH and HI, respectively

(x_i, λ_i) for i=1, 2, 3 represent the input displacement, and the output rotation angle of the link EH, respectively. Equations (9) through (12) can be solved when three precision points of the linear slider and rotation angle of the crank are known through the pairs of (x_i, λ_i) for i=1, 2, 3. According to the suitable design for the proposed design $(x_1 = 0\text{mm}, \lambda_1 = 37°)$and $(x_3 = 75\text{mm}, \lambda_3 = 180°)$ are assumed empirically for the starting position 1 and final position 3 of both slider of the linear actuator and link EH .The optimization procedure in terms of force transmission has been done by assuming the middle position between 1) and 3) for both the slider position and link EH, respectively as starting values of the optimization procedure. The transmission defect (δ_3) for this mechanism could be written in the form:

$$fun_3 = \delta_3 = \frac{1}{x_3 - x_1}\int_{x1}^{x3} cos^2\mu_3 dx \tag{13}$$

Where the transmission angle μ_3 is expressed as:

$$\mu_3 = \cos^{-1}\left(\frac{(s_1 - x)^2 + o_f^2 - g^2 - h^2}{2gh}\right) \tag{14}$$

The optimization procedure followed was by using the same previous initial values for (x_2, λ_2) in addition to $(x_1, \lambda_1)$and (x_3, λ_3) , substituting these values in Freudenstein's equations will give links lengths, substituting these lengths in equations (13) and (14), to get μ and (δ_3) until getting the minimum value of (δ_3), the later value was close to Rea's value which give the same link lengths as shown in table (4)(Rea, 2006).

6. Formulation of optimum design

The optimization is a way of finding a set of parameters that can in some way be defines as optimal .These parameters are obtained by minimizing or maximizing an objective function subject to equality or inequality constrains and /or parameter bounds. Optimization techniques are used to find a set of design parameters, $x = \{x_1, x_2, \dots x_n\}$ that can in some way be defined as the optimal. In the synthesis of function –generators, the linkage should generate a set of input and output pairs that verify certain functional relations. It cannot find that linkage which produce set of input-output exactly, but the optimum linkage, which produces the specified pairs, could be found approximately. An optimum design procedure for 1DOF finger driving mechanism can be formulated by using the above mentioned evaluation criteria for finger design using multi-objective optimization problem by using (Fminunc) function in matlab software("Optimization Toolbox TM User ' s Guide R 2015 a," 2015).The problem can be specified as:-

$$\min_{x} f(x) \tag{15}$$

Where: - x is a vector and f(x) is a function that returns scalar.
The syntax of this function is:

$$[\mathrm{x}, \mathrm{f_{val}}] = \mathrm{fminunc}(\mathrm{fun}, \mathrm{x_o}, \mathrm{options}), \tag{16}$$

fminunc function returns in $\mathbf{f_{VAL}}$ the value of the objective function fun at solution **x** .Starts at the point $\mathbf{x_o}$ and attempts to find a local minimum x of the function described in fun. , $\mathbf{X_o}$ **c**an be a scalar, vector, or matrix.

7. Finger static equilibrium model

A model of the finger under study is presented in figure (8) .The general form of contact forces expressions for linkage underactuated finger have been established in(Birglen, Lionel, Laliberte', Thierry,Gosselin, 2008) .A modification to these equations which describe the forces had been made, taking into account, the effect of the rigid bodies E,F,H and B,C,G used in finger mechanism. These equations form the relationship between the input actuator torque and the grasping forces exerted on the object. According to the principle of virtual work, by equating the input and output virtual powers, one obtains:

$$t^T \omega_a = f^T \mathrm{v} \tag{17}$$

Where:-

$$V = J\dot{\theta} \tag{18}$$

$$\dot{\theta} = T\omega_a \tag{19}$$

Where f is the grasping force vector on the three phalanges, t is the input torque vector exerted by the motor and the underactuation springs, and ω_a is the corresponding joint velocity vector. V is the linear velocity vector of contact points and T is the transmission matrix. Contact forces are assumed to be normal to the phalanges and without friction (ZHANG et al., 2013) .Substitution of equations (18) and (19) into (17) ,gives:

$$f = J^{-T} T^{-T} t \tag{20}$$

Where:

$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, t = \begin{bmatrix} t_a \\ t_2 = k_{s2}\Delta\theta_2 \\ t_3 = k_{s3}\Delta\theta_3 \end{bmatrix},\ \omega_a = \begin{bmatrix} \dot{\theta}_a \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}, \theta^{\circ} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}, \tag{21}$$

The velocity vector is related to the angles and corresponding arms, therefore they are expressed as:-

$$V = J\dot{\theta} = \begin{bmatrix} d_1 & 0 & 0 \\ l_1 \cos\theta_2 + d_2 & d_2 & 0 \\ l_1 \cos(\theta_2 + \theta_3) + l_2 \cos\theta_3 + d_3 & l_2 \cos\theta_3 + d_3 & d_3 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{22}$$

In which J is expressed as:-

$$J = \begin{bmatrix} d_1 & 0 & 0 \\ l_1 \cos\theta_2 + d_2 & d_2 & 0 \\ l_1 \cos(\theta_2 + \theta_3) + l_2 \cos\theta_3 + d_3 & l_2 \cos\theta_3 + d_3 & d_3 \end{bmatrix} \tag{23}$$

The transmission matrix (T), represents the underactuation characteristic, for the linkage mechanism(Birglen, Lionel, Laliberte', Thierry,Gosselin, 2008).

$$T = \begin{bmatrix} 1 & -\dfrac{h_2}{h_2 + l_1} & -\dfrac{h_2 h_3}{(h_2 + l_1)(h_3 + l_2)} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{24}$$

Each coefficient of matrix (T) is a function of the transmission mechanism used to propagate the actuator torque to i^{th} phalanx. Matrix T relates vector ω_a to the time derivatives of the phalanx joint coordinates ($\dot{\theta}_i$). Referring to figure (8) for the four-bar linkage ,and according to the transmission principle , which known as Kennedy's Theorem, which gives the angular velocity ratio of this linkage, and by considering the two mechanisms in figure (9) which shows the rigid links added to the general four bar linkage mechanism. The modified equations taking into account the rigid links will be discussed below in details. By superposition, one obtains,

$$\dot{\theta}_{ai-1} = \dot{\theta}_{i-1} + \frac{h_i}{h_i + l_{i-1}} \dot{\theta}_i \tag{25}$$

Where, the velocity output of each stage is the input of the next stage. That the transmission stages are assumed to be connected in series, referring to figure (9) b), (h_2) is the distance between point o_2 and the intersection of lines $\overline{o_1 o_2}$ and $\overline{p_1 p_2}$.

$$h_2 = c_1(\cos(\theta_2 - \varphi_2 - \gamma_2) - \sin(\theta_2 - \varphi_2 - \gamma_2)\, cot\beta_1) \tag{26}$$

The new expressions of angles β_i and φ_i which include the angle γ_2 of solid link as follows:

$$\cot\beta_1 = \frac{c_1 \sin(\theta_2 - \varphi_2 - \gamma_2)\sqrt{4a_1^2 b_1^2 - N_2^2} + M_2(l_1 + c_1\cos(\theta_2 - \varphi_2 - \gamma_2))}{-(l_1 + c_1\cos(\theta_2 - \varphi_2 - \gamma_2))\sqrt{4a_1^2 b_1^2 - N_2^2} + M_2 c_1 \sin(\theta_2 - \varphi_2 - \gamma_2)} \tag{27}$$

$$M_2 = -l_1(l_1 + 2c_1\cos(\theta_2 - \varphi_2 - \gamma_2)) + a_1^2 - b_1^2 - c_1^2 \tag{28}$$

$$N_2 = -l_1(l_1 + 2c_1\cos(\theta_2 - \varphi_2 - \gamma_2)) - a_1^2 - b_1^2 - c_1^2 \tag{29}$$

$$\varphi_1 = \tan^{-1}\left[\frac{-c_1\sin(\theta_2 - \varphi_2 - \gamma_2)}{l_1 + c_1\cos(\theta_2 - \varphi_2 - \gamma_2)}\right] + \cos^{-1}\left[\frac{l_1^2 + a_1^2 + c_1^2 - b_1^2 + 2c_1 l_1\cos(\theta_2 - \varphi_2 - \gamma_2)}{2a_1\sqrt{c_1^2 + 2c_1 l_1\cos(\theta_2 - \varphi_2 - \gamma_2) + l_1^2}}\right] \tag{30}$$

While, φ_3 and β_2 will not change, and h_3 is the distance between o_3 and the intersection of lines $\overline{o_2 o_3}$ and $\overline{p_3 p_4}$.

$$h_3 = (c_2(\cos(\theta_3 - \varphi_3) - \sin(\theta_3 - \varphi_3)\cot(\beta_2)) \tag{31}$$

$$\cot\beta_2 = \frac{c_2\sin(\theta_3 - \varphi_3)\sqrt{4a_2^2 b_2^2 - N_3^2} + M_3(l_2 + c_2\cos(\theta_3 - \varphi_3)}{-(l_2 + c_2\cos(\theta_3 - \varphi_3))\sqrt{4a_2^2 b_2^2 - N_3^2} + M_3 c_2\sin(\theta_3 - \varphi_3)} \tag{32}$$

Where:-

$$M_3 = -l_2(l_2 + 2c_2\cos(\theta_3 - \varphi_3)) + a_2^2 - b_2^2 - c_2^2 \tag{33}$$

$$N_3 = -l_2(l_2 + 2c_2\cos(\theta_3 - \varphi_3)) - a_2^2 - b_2^2 - c_2^2 \tag{34}$$

$$\varphi_2 = \tan^{-1}\left[\frac{-c_2\sin(\theta_3 - \varphi_3)}{l_2 + c_2\cos(\theta_3 - \varphi_3)}\right] + \cos^{-1}\left[\frac{l_2^2 + a_2^2 + c_2^2 - b_2^2 + 2c_2 l_2\cos(\theta_3 - \varphi_3)}{2a_2\sqrt{c_2^2 + 2c_2 l_2\cos(\theta_3 - \varphi_3) + l_2^2}}\right] \tag{35}$$

And finally, substitute equations (21) through (35) into equation (20) give equation (36), which states the final expression to the contact forces using the solid links.

$$f = \begin{bmatrix} \frac{l_1 U T_a}{d_1 d_2 d_3 (h_2 + l_1)(h_3 + l_2)} - \frac{(k_2 + l_1 \cos\theta_2) T_2}{d_1 d_2} + \frac{l_1 V T_3}{d_1 d_2 d_3} \\ \frac{h_2 l_2 (d_3 - h_3 \cos\theta_3) T_a}{d_2 d_3 (h_2 + l_1)(h_3 + l_2)} + \frac{T_2}{d_2} - \frac{(d_3 + l_2 \cos\theta_3) T_3}{d_2 d_3} \\ \frac{h_2 h_3 T_a}{d_3 (h_2 + l_1)(h_3 + l_2)} + \frac{T_3}{d_3} \end{bmatrix} \quad (36)$$

The torques exerted by the springs are ignored ,since their magnitude are relatively small compared to the motor torque .It has been verified by practice that, this assumption will not influence the effect of static analysis (Zhao & Zhang, 2010)Equation(36) is reformed without spring torques as follows:

$$f = \begin{bmatrix} \frac{l_1 U T_a}{d_1 d_2 d_3 (h_2 + l_1)(h_3 + l_2)} \\ \frac{h_2 l_2 (d_3 - h_3 \cos\theta_3) T_a}{d_2 d_3 (h_2 + l_1)(h_3 + l_2)} \\ \frac{h_2 h_3 T_a}{d_3 (h_2 + l_1)(h_3 + l_2)} \end{bmatrix} \quad (37)$$

8. Contact force representation

As discussed in the last section that the new lengths for the under actuated linkage robotic hand with less transmission defect and greater transmitted force through each four bar linkage so the grasping force that result in each phalanx should be larger than the old mechanism .An matlab program based on [3]equations for the Kinostatic analysis of linkage finger have been adopted after made some changes on these equations by taking into account the angle of the solid links that connect the first phalanx with the second phalanx and the other solid link which connect the slider-crank mechanism to the first phalanx. The grasping force obtained from the program effected mainly by two factors .one the configuration of the finger described by θ_2 andθ_1, the other factor was the contact locations on the three phalanges, denoted byd_1, $d_2$and d_3, the effect of the solid link in the first phalanx which is described by angle $\gamma_2$will influence to the second phalanx equation and that was clarified in the static analysis of the under actuated finger, for space to be bounded $0° < \theta_3 < 78°$ and $0° < \theta_2 < 105°$,it has expressed a comparison between the grasping force for the two mechanisms as shown in figures (10)and(11).

9. Conclusions

The new re –optimization method gives transmission defects values less than the ordinary programming old method for the same design specifications. Consequently the adaptability of the finger is good and the grasping forces maximized in addition to the increasing in stable area with 31% within the working area limits.

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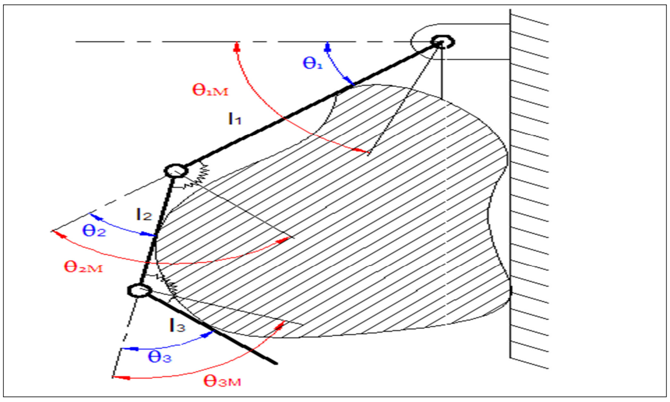

Figure (1) Simplified sketch of under actuated finger mechanism.

Table (1) Characteristics of the robotic finger

Phalanx	Lengths	Max. angle
l1	86mm	$\theta_{1M} = 83°$
l2	50mm	$\theta_{2M} = 105°$
l3	46mm	$\theta_{3M} = 78°$

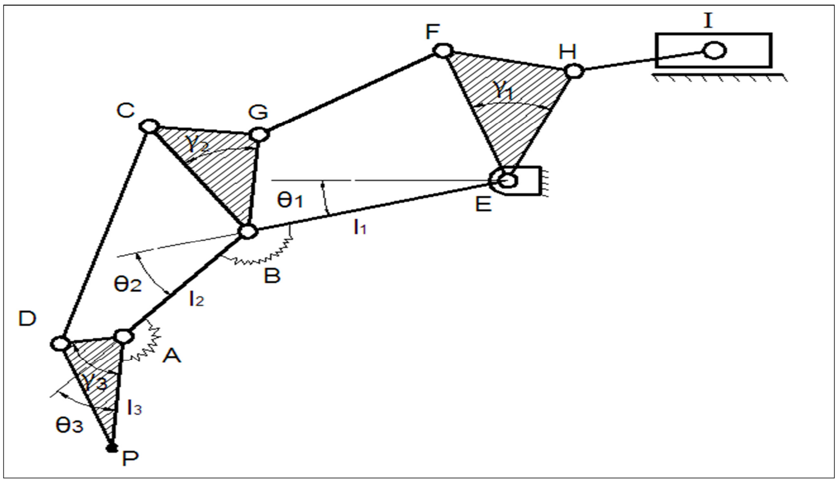

Figure (2) Sketch of underactuated linkage finger mechanism.

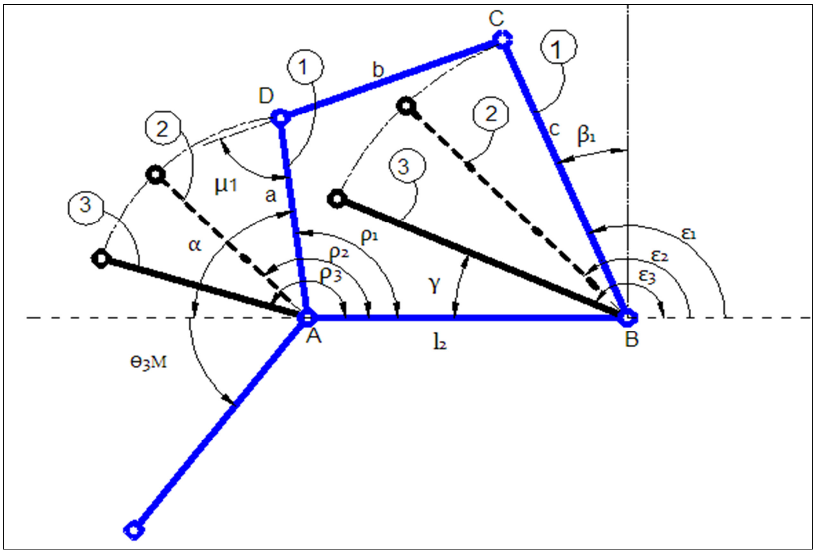

Figure (3) Sketch for the kinematic synthesis of the four bar linkage ABCD.

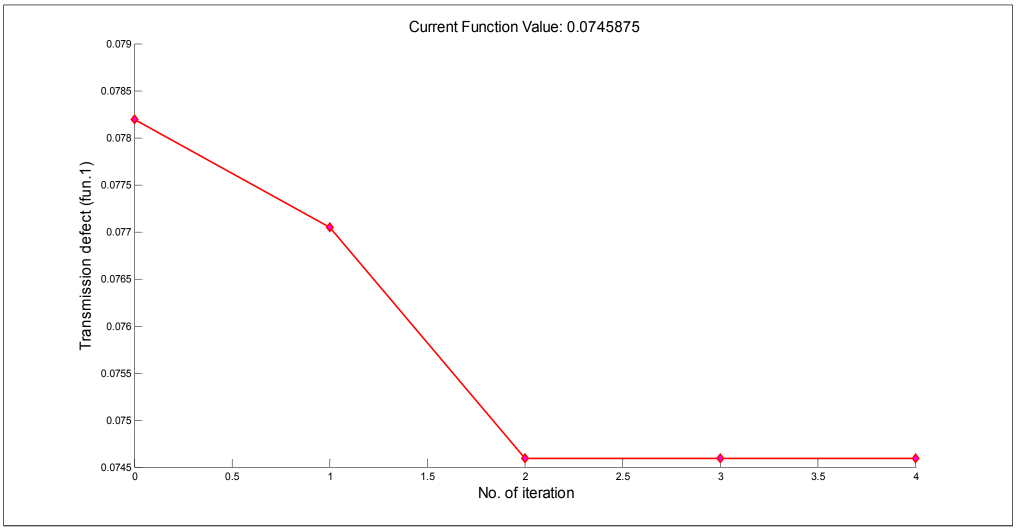

Figure (4) Evaluation of the objective function fun.1

Table (2) Comparison between the new and the previous linkage ABCD parameters.

Parameter	New finger	Rea finger
Transmission defect(δ)	0.07458	0.13469
(ε_2, ρ_2) (degree)	(127.6048,168.9655)	(128.56,169.08)
a(mm)	23	22.584
b(mm)	73.275	58.374
c(mm)	96.4016	71.0558
l2(mm)	50	50

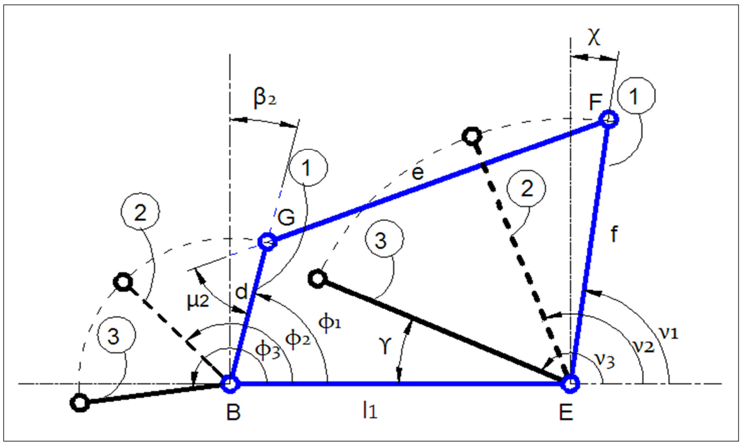

Figure (5) Sketch for the kinematic synthesis of the four bars - linkage BEFG.

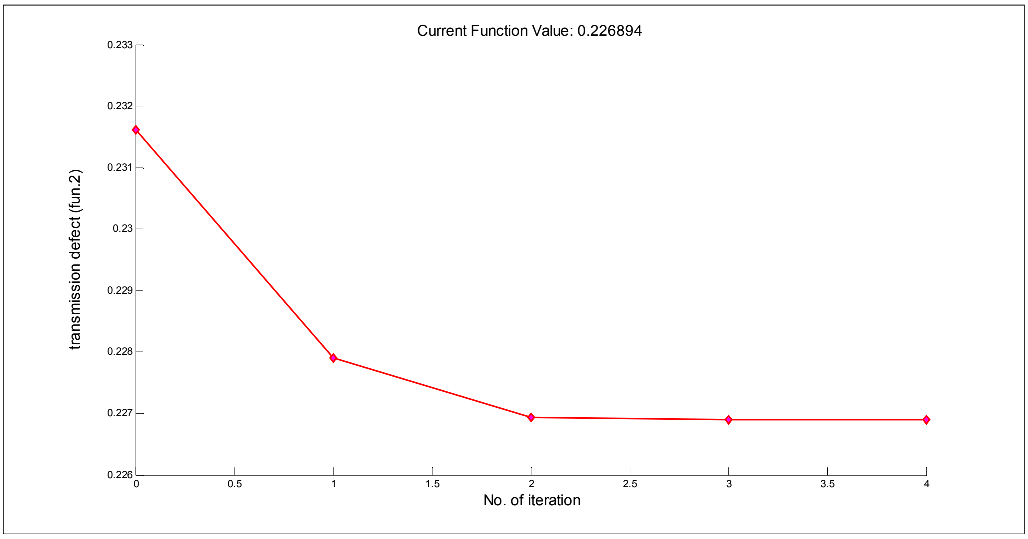

Figure (6) Evaluation of the objective function fun.2

Table (3) Comparison between the new and the previous linkage BEFG parameters.

Parameter	New finger	Rea finger
Transmission defect(δ)	0.22689	0.23723
(υ_2, ϕ_2) (degree)	108.924,125.56)(	(110.58,124.69)
e(mm)	112.997	96
f(mm)	144.205	104.97
d(mm)	68.3468	53.40118
l1	86	86

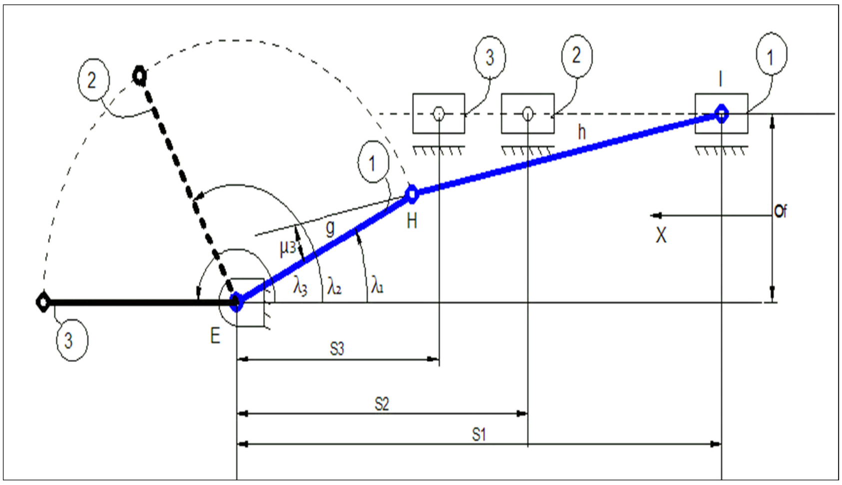

Figure (7) Kinematic scheme of the offset slider-crank mechanism EHI.

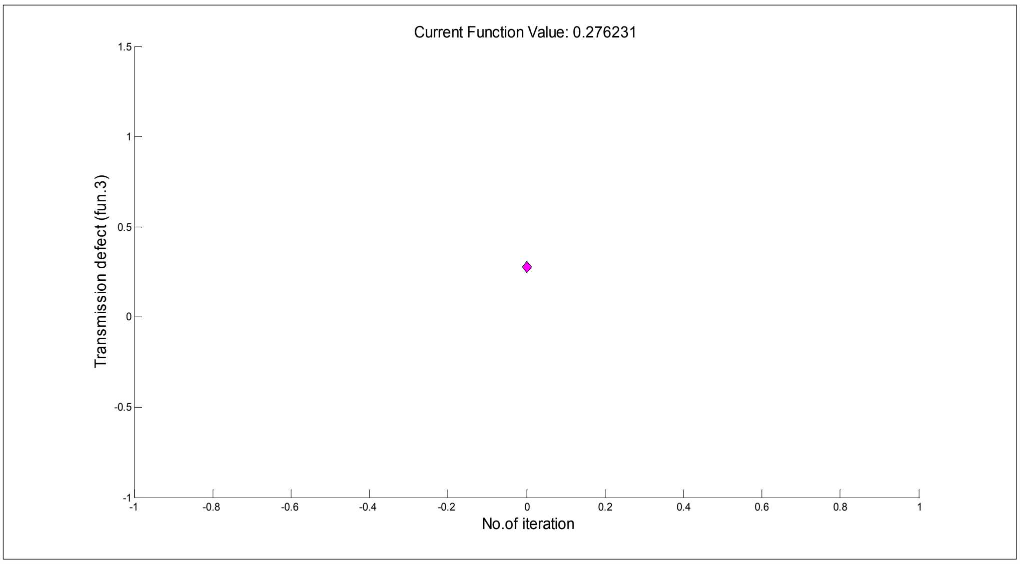

Figure (3-10) Evaluation of the objective function fun.3

Table (4) Comparison between the new and the previous linkage HIG parameters.

Parameter	New finger	Rea finger
Transmission defect(δ)	0.27643	0.274
(λ_2, x_2) (degree)	(37.5,108.5)	(37.4,108.9)
h(mm)	74.5	74.72
o(mm)	42.746	43.48
g(mm)	35.73	35.773
S1 (mm)	100	100

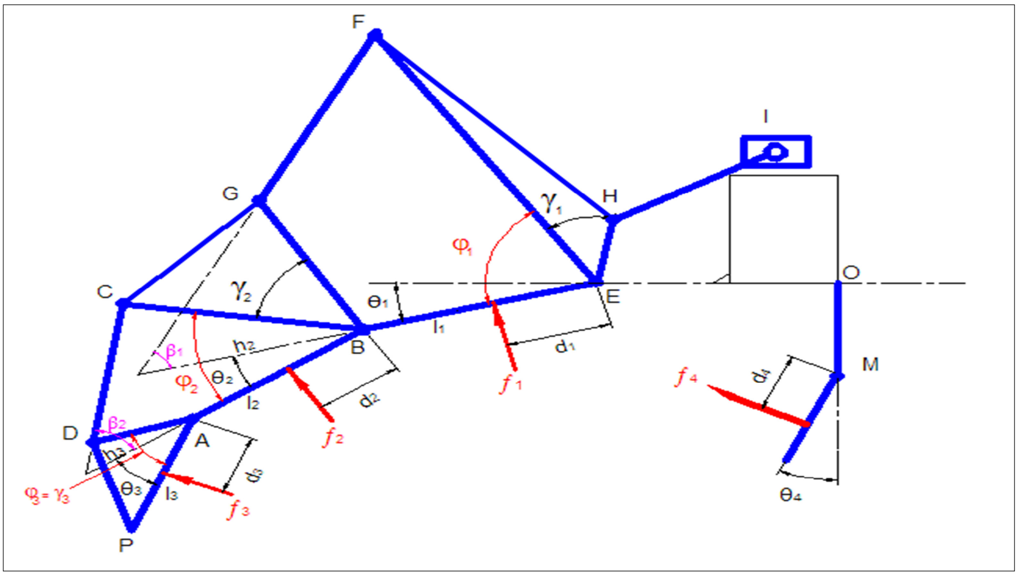

Figure (8) Static model of three –phalanx finger with the thumb.

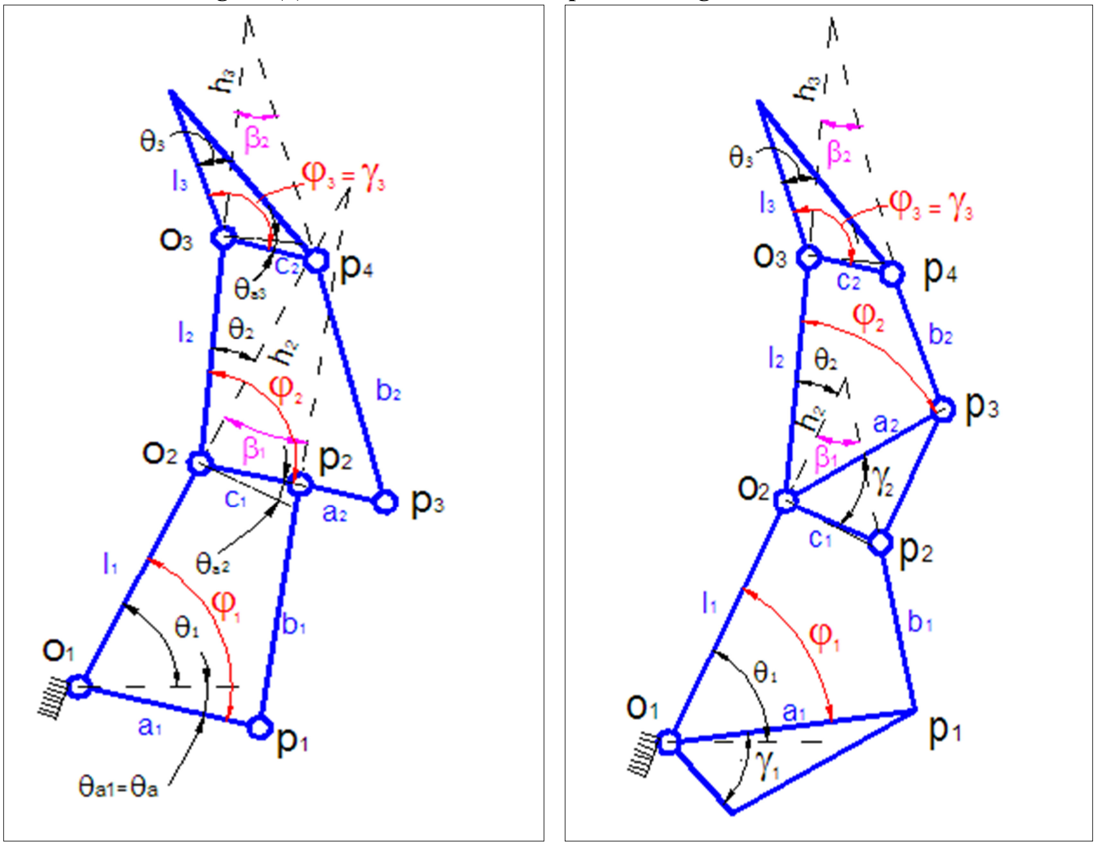

Figure (9) a) General modeling of a linkage –driven finger.
b) General modeling of linkage –driven finger with solid link.

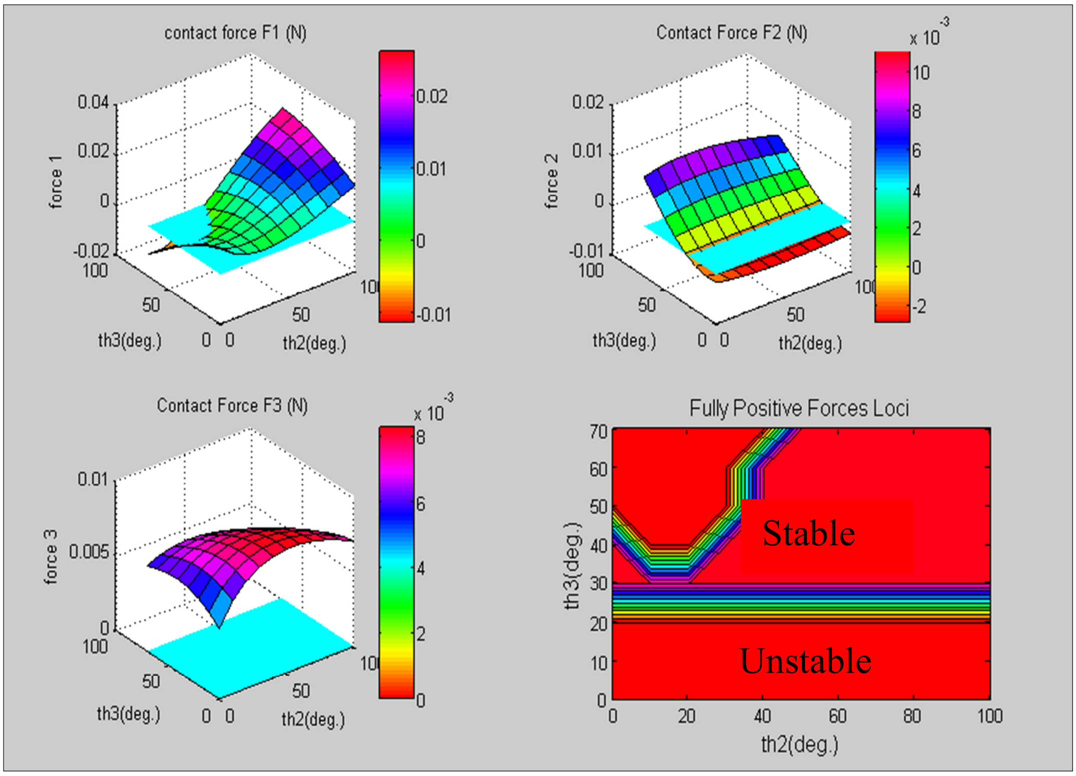

Figure (10) Contact forces and associated stability loci(d2=l2/2,d3=l3/2) Each force component is shown with the plane fi=0 in blue, for the Rea's parameters.

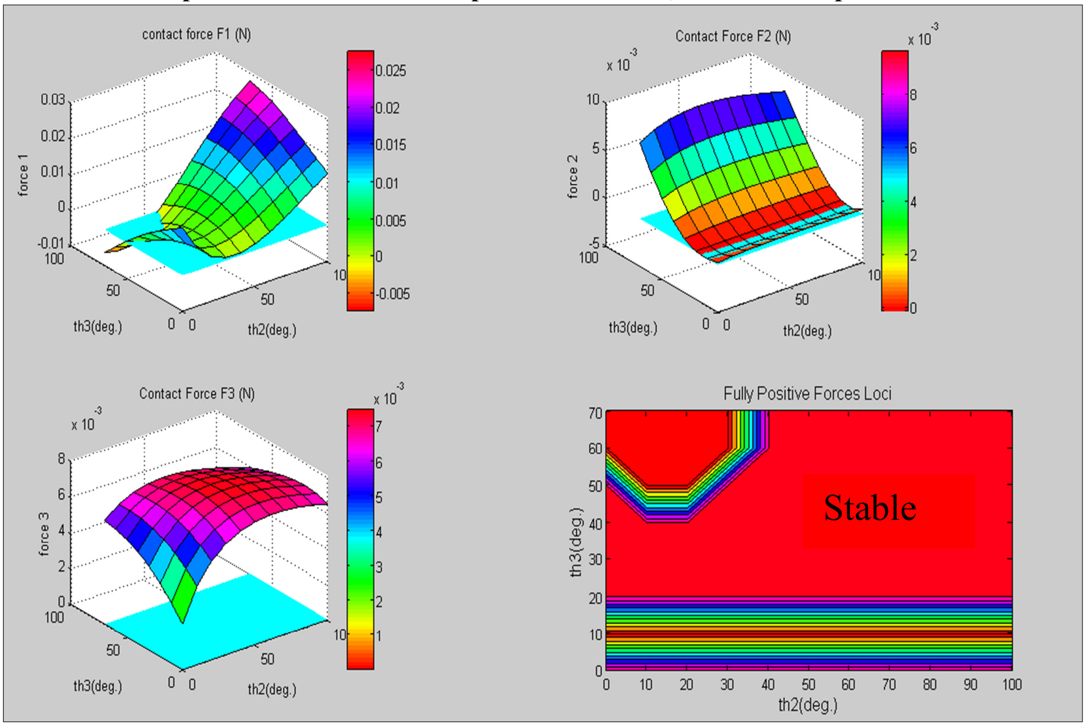

Figure (11) Contact forces and associated stability loci(d2=l2/2,d3=l3/2).Each force component is shown with the plane fi=0 in blue , for the new parameters.

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