Results with Random Fuzzy Metric Spaces

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Abstract
In this paper we obtain some fixed point results in random fuzzy metric space of two mappings.
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Introduction and Preliminaries

The concept of fuzzy metric space or a fuzzy set is introduced by Zadeh in 1965, Some times for the measurement of an ordinary length, it proves the concept of a fuzzy metric space. The author divides the results in two groups, in which a set X maps on fuzzy metric space defines the totality of all fuzzy points of a set and also the distance between objects is fuzzy and the objects together may or may not be fuzzy. By this the fuzzy objects has a numerical distances. Later then the concept of fuzzy metric space is introduced by Kramosil and Michalek it proves the the contraction principles.

Definition 1.1. An algebraic structure $(X, M,*)$ is called a fuzzy metric space if a non-empty set X, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$, satisfying the following conditions for each $x, y, z \in X \text{ and each } t \text{ and } s > 0$,

(1) $M(x, y, t) > 0$,
(2) $M(x, y, t) = 1 \text{ if and only if } x = y$,
(3) $M(x, y, t) = M(y, x, t)$,
(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$
(5) $M(x, y, .): (0, \infty) \to [0,1] \text{is continuous}$.

Let (X, M, $*$) be a fuzzy metric space. for$t > 0$ *and* the open ball $B(x, r, t)$ with center $x \in X$ *radius* $0 < r < 1$is defined as

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}$$

A subset $A \subset X$ is called open If for each$x \in A$, there exist$t > 0$ and $0 < r < 1$such that $B(x, r, t) \subset A$. Let τ denotes the family of all open subsets of X. Then is called the topology on X induced by the fuzzy metric M. This topology is Hausdorff and first countable. A subset A of X is said to be F-bounded if there exist $t > 0$ and $0 < r < 1$ such that $M(x, y, t) > 1 - r$ for all $x, y \in A$.

Lemma 1.2: Let (X, M, $*$) be a fuzzy metric space. Then for all $x, y \in A$. we have a non decreasing function $M(x, y, t) \text{ with respect } t$.

Definition 1.3: Abinary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t- norm if it satisfies the following conditions

(1) $*$ is associative and commutative ,
(2) $*$ is continuous,
(3) $a * 1 = a$ for all $a \in [0,1]$,
(4) $a * b \leq c * d \text{ whenever } a \leq c \text{ and } b \leq d, \text{for each } a, b, c, d \in [0,1]$,

Two typical examples of continuous t-norm are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 1.4: Let (X, M, $*$) be a fuzzy metric space. M is said to be continuous on $X^2 \times (0, \infty)$ If

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t)$$

Whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0, \infty)$ converges to a point $(x, y, t) \in X^2 \times (0, \infty)$, i.e.

$$\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1 \text{ and } \lim_{n\to\infty} M(x, y, t_n) = M(x, y, t)$$

Lemma 1.5: Let (X, M, $*$) be a fuzzy metric space. Then M is continuous function on $X^2 \times (0, \infty)$.

Definition 1.6: Let f and g be self –mappings on a fuzzy metric space (X, d). Then the mappings are said to be weakly compatible if they commute at their coincidence point, that is, $fx = gx$ implies that $fgx = gfx.$

Definition 1.7: A sequence$\{x_n\}$ in a fuzzy metric space (X, M, $*$) is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1.$ The space (X, M, $*$) is said to be complete If every Cauchy sequence in X is convergent in X.

Definition 1.8: A fuzzy metric space (X, M, $*$) is said to be precompact if for each $0 < r < 1$ and each $t > 0$there is a finite subset A$\in X$ such that $X = \bigcup_{a\in A} B(a, r, t).$ A fuzzy metric space (X, M, $*$) is called compact if (X, τ) is a compact topological space. Also it is clear that every compact set is closed F-bounded.

Definition 1.9: Throughout this paper $(\mathbf{\Omega}, \mathbf{\Sigma})$ denotes a measurable space. $\boldsymbol{\xi} : \mathbf{\Omega} \to \mathbf{X}$ is a measurable selector. X is any non empty set. $\star$ is continuous t-norm, **M** is a fuzzy set in $X^2 \times [0,\infty)$ .A binary operation $*$:[0,1]x[0,1]$\to$[0,1] is called a continuous t-norm if ([0,1],$*$) is an abelian Topological monodies with unit 1 such that $a * b \geq c * d$ whenever
$a \geq c$ and $b \geq d$, For all a, b, c, d, $\in [0, 1]$
Example of t-norm are $a * b = a\,b$ and $a * b = \min \{a, b\}$

Definition1.9 (a): The 3-tuple (X, M, Ω $*$) is called a **Random fuzzy metric space**, if X is an arbitrary set,* is a continuous t-norm and M is a fuzzy set in X^2 x $[0,\infty)$ satisfying the following conditions: for all
ξx, ξy, ξz $\in$ X and s, t > 0,

$$(RFM-1): M(\xi x, \xi y, 0) = 0$$

$$(RFM-2): M(\xi x, \xi y, t) = 1, \forall t \succ 0, \Leftrightarrow x = y$$

$$(RFM-3): M(\xi x, \xi y, t) = M(\xi y, \xi x, t)$$

$$(RFM-4): M(\xi x, \xi z, t+s) \geq M(\xi x, \xi y, t) * M(\xi z, \xi y, s)$$

$$(RFM-5): M(\xi x, \xi y, \xi a) : [0,1) \to [0,1] \, is\ left\ continuous$$

In what follows, (X, M, Ω ,*) will denote a random fuzzy metric space. Note that M (ξx, ξy, t) can be thought of as the degree of nearness between ξx and ξy with respect to t. We identify ξx = ξy with M (ξx, ξ y, t) = 1 for all t > 0 and M (ξx, ξy, t) = 0 with ∞.In the following example, we know that every metric induces a fuzzy metric.

Example Let (X, d) be a metric space.
Define $a * b = a\,b$, or ab =min {a, b}) and for all x, y, $\in$ X and t > 0,

$$M(\xi x, \xi y, t) = \frac{t}{t + d(\xi x, \xi y)}$$

Then (X, M, Ω , *) is a fuzzy metric space. We call this random fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition1.9 (b): Let (X, M, Ω , *) is a random fuzzy metric space.

(i)A sequence {ξx_n} in X is said to be convergent to a point ξx $\in$ X,

$$\lim_{n\to\infty} M(\xi x_n, \xi x, t) = 1$$

(ii) A sequence {ξx_n} in X is called a Cauchy sequence if

$$\lim_{n\to\infty} M(\xi x_{n+p}, \xi x_n, t) = 1, \forall t \succ 0 \, and \, p \succ 0$$

(iii) A random fuzzy metric space in which every Cauchy sequence is convergent is said to be Complete.

Let (X.M,*) is a fuzzy metric space with the following condition.

(RFM-6) $\lim_{t\to\infty} M(\xi x,\xi y,t)=1, \forall \xi x,\xi y \; \varepsilon \, X$

. **Definition1.9 (c)**: A function M is continuous in fuzzy metric space iff whenever

$\xi x_n \to \xi x, \xi y_n \to \xi y \Rightarrow \lim_{n\to\infty} M(\xi x_n,\xi y_n,t) \to M(\xi x,\xi y,t)$

Definition1.9 (d): Two mappings A and S on fuzzy metric space X are weakly commuting iff

M (AS ξ u, SA ξ u, t) ≥ M (A ξ u, S ξ u, t)

Main Results

Theorem 2.1: Let R and S be self-maps of on a F-bounded Random fuzzy metric space (X, , N,*) satisfying

(i) R(X) ⊆ S(X), S(X) is complete. If (R,S) is a weakly compatible pair.

(ii) N$(R\xi x, R\xi y, u)$

$$\geq \varphi\left[min\left\{\begin{array}{c} N(S\xi x,S\xi y,u), N(S\xi x,R\xi x,u), N(S\xi y,R\xi y,u), N(S\xi x,R\xi y,u) \\ N(S\xi y,R\xi x,u), \quad \dfrac{N(S\xi x,R\xi x,u)+N(S\xi y,R\xi y,u)}{1+N(S\xi x,S\xi y,u)}, \\ \dfrac{N(S\xi x,R\xi x,u)+N(S\xi x,R\xi y,u),\; N(S\xi y,R\xi x,u)+N(S\xi y,R\xi y,u)}{1+N(S\xi x,R\xi y,u),\; N(S\xi y,R\xi x,u)+N(S\xi x,S\xi y,u)N(S\xi y,R\xi x,u)} \end{array}\right\}\right]$$

$\forall\, \xi x,\xi y \in X$ *and* $\forall\, u>0$, *where* $\varphi : [0,1] \to [0,1]$ *is continuous and monotonically increasing such that* $\varphi(t) > t, \forall\, t \in [0,1)$.

Then R and S have a unique common random fixed point in X.

Proof: Let $\xi f_0 \,\epsilon\, X$ from R(X) ⊆ S(X), there exist a sequence $\{\xi f_n\}$ in X such that

$R\xi f_n = S\xi f_{n+1} = \xi E_n$ for some n

Case (i) Suppose $\xi E_{n+1} = \xi E_n$ for some n, Then $R\xi z = S\xi z$, *where* $\xi z = \xi f_{n+1}$

Denotes $K = R\xi z = S\xi z$

Since (R,S) is a weakly compatible pair, we have $R_k = S_k$

Therefore from (ii) we have

$N(R_k, \xi k, \xi u) = N(\xi R_k, \xi R_z, u)$

$$\geq \varphi\left[min\left\{\begin{array}{c} N(\xi S_k,\xi S_z,u), N(\xi S_k,\xi R_k,u), N(\xi S_z,\xi R_z,u), N(\xi S_k,\xi R_z,u) \\ N(\xi S_z,\xi R_k,u), \quad \dfrac{N(\xi S_k,\xi R_k,u)+N(\xi S_z,\xi R_z,u)}{1+N(\xi S_k,\xi S_z,u)}, \\ \dfrac{N(\xi S_k,\xi R_k,u)+N(\xi S_k,\xi R_z,u),\; N(\xi S_z,\xi R_k,u)+N(\xi S_z,\xi R_z,u)}{1+N(\xi S_k,\xi R_z,u),\; N(\xi S_z,\xi R_k,u)+N(\xi S_k,\xi S_z,u)N(\xi S_z,\xi R_k,u)} \end{array}\right\}\right]$$

$= \varphi[min\{N(\xi R_k, \xi k, u)\}]$

$> \{N(\xi R_k, \xi k, u)\}, iF \; \{N(\xi R_k, \xi k, u)\} < 1$

Hence $\xi R_k = \xi k$ *Thus* $\xi S_k = \xi R_k = \xi k$

If v is another common fixed point of Rand S, Then $N(\xi k, \xi v, u) = N(\xi R_k, \xi R_v, u)$

$= \varphi[min\{N(\xi k,\xi v,u), 1, 1, \}N(\xi k,\xi v,u), N(\xi k,\xi v,u), 1, 1\}]$

$= \varphi[N(\xi k, \xi v, u)]$

$> [N(\xi k, \xi v, u)]$ if $N(k, v, u) < 1$

Hence $\xi\, k = \xi v$. *Thus* ξk *is the unique common fixed point of S and R.*

Case (ii) Assume that $\xi\, E_{n+1} \neq \xi E_n \;\forall\, n \in N$,

$$let\; \xi\beta_n(u) = inf\{N(\xi E_i, \xi E_j, u); i > n, j > n\}$$

$\forall\, u > 0$. *Then* $\{\xi\beta_n(u)\}$ *is a monotonically increasing sequence of real number betweeen* 0 *and* 1 *for all* $u > 0$.

Hence $\lim_{n\to\infty} \xi\beta_n(u) = \xi\beta_n(u)$ for some $0 \leq \xi\beta_n(u) \leq 1$ for any n∈ N *and integer* $i \geq n$,

$j \geq n$, we have

$N(\xi E_i, \xi E_j, u) = N(R\xi x_i, R\xi x_j, u)$

$$\geq \varphi\left[min\left\{\begin{array}{c} N(\xi E_{i-1},\xi E_{j-1},u), N(\xi E_{i-1},\xi E_i ,u), N(\xi E_{j-1},\xi E_j,u), N(\xi E_{i-1},\xi E_j,u) \\ N(\xi E_{j-1},\xi\, E_i,u), \quad \frac{N(\xi E_{i-1},\xi E_i ,u) + N(\xi E_{j-1},\xi E_j,u)}{1 + N(\xi E_{i-1},\xi E_{j-1},u)}, \\ \frac{N(\xi E_{i-1},\xi E_i,u) + N(\xi E_{i-1},\xi E_j,u),\ N(\xi E_{j-1},\xi E_i,u) + N(\xi E_{j-1},\xi E_j,u)}{1 + N(\xi E_{i-1},\xi E_j,u),\ N(\xi E_{j-1},\xi E_i,u) + N(\xi E_{i-1},\xi E_{j-1},u)N(\xi E_{j-1},\xi E_i,u)} \end{array}\right\}\right]$$

$\geq \varphi[\xi\beta_{n-1}(u)], since\ \varphi\ is\ monotonic\ increasing$

Hence$\xi\ \beta_n(u) \geq \varphi[\xi\beta_{n-1}(u)]$

Let $\xi\beta_n(u) \geq \varphi[\xi\beta_{n-1}(u)]$ $then$ at $n \to \infty$ we get

$\xi\beta_n(u) \geq \varphi\xi\beta_n(u) > \xi\beta_n(u)\ , if\ \xi\beta_n(u) < 1$

Hence $\xi\beta_n(u) = 1$ so that $\lim_{n\to\infty} \xi\beta_n(u) = 1$

Thus for given for given $\epsilon > 0, \exists\, n_0 \in N$ such that $\xi\beta_n(u) > 1 - \epsilon\ , \forall\ n > n_0$.

Therefore $n > n_0 . m \in N$ we have

M($\xi E_n, \xi E_{n+m}, u$)> $1 - \epsilon$

Hence $\{\xi E_n\}$ is a Cauchy sequence in X. Since S(X) is Complete, it follows that $\xi E_n \to \xi z$ for some $z \in S(X)$. Hence there exists $w \in X\ such\ that\ z = Sw$Now,

$$N(\xi R_w,\xi Rx_n,u) \geq \varphi\left[min\left\{\begin{array}{c} N(\xi S_w,\xi Sx_n,u), N(\xi S_w,\xi R_w ,u), N(\xi Sx_n,\xi Rx_n ,u), N(\xi S_w,\xi Rx_n,u) \\ N(\xi Sx_n,\xi R_w,u), \quad \frac{N(\xi S_w,\xi R_w ,u) + N(\xi Sx_n,\xi Rx_n ,u)}{1 + N(\xi S_w,\xi Sx_n,u)}, \\ \frac{N(\xi S_k,\xi R_k,u) + N(\xi S_k,\xi R_z,u),\ N(\xi S_z,\xi R_k,u) + N\xi(S_z,\xi R_z,u)}{1 + N(\xi S_w,\xi Rx_n,u), N(\xi Sx_n,\xi\, R_w,u)\ + N(\xi S_w,\xi Sx_n,u)N(\xi Sx_n,\xi R_w,u)} \end{array}\right\}\right]$$

Let $\lim_{n\to\infty}$ $we\ get$

$(\xi R_w,\xi z,u) \geq\ \varphi[min\{1, N(\xi z,\xi R_w ,u), 1,1, N(\xi z,\xi R_w ,u), 1,1\}]$

$= \varphi(N(\xi z,\xi R_w ,u))$

$> N(\xi z,\xi R_w ,u)$ if $N(\xi z, R_w ,u) < 1$

Hence $\xi\ R_w = \xi z$ ThusξS_w= ξR_w= ξz.

Corollary 2.2: Let R be a self -map on a F- bounded Complete random fuzzy metric space (X, Ω ,N,*) satisfying

(i) N($R\xi x, R\xi y, u$)$\geq \varphi\left[min\left\{\begin{array}{c} N(\xi x,\xi y,u), N(\xi x,R\xi x,u), N(\xi y,R\xi y,u), N(\xi x,R\xi y,u) \\ N(\xi y,R\xi x,u),\ \frac{N(\xi x,R\xi x,u)+N(\xi y,R\xi y,u)}{1+N(x,y,u)}, \\ \frac{N(\xi x,R\xi x,u)+N(\xi x,R\xi y,u),N(\xi y,R\xi x,u)+N(\xi y,R\xi y,u)}{1+N(\xi x,R\xi y,u),N(\xi y,R\xi x,u)+N(\xi x,\xi y,u)N(\xi y,R\xi x,u)} \end{array}\right\}\right]$

$\forall\ \xi x,\xi y \in X\ and\ \forall\ u > 0, where\ \varphi : [0,1] \to [0,1]\ is\ continuous\ and\ monotonically\ increasing\ such\ that\ \varphi(t) > t, \forall\ t \in [0,1).$

Then R has a unique common fixed point in X.

Now we proves the following theorem in compact fuzzy metric spaces by using the methodology of Shih and Yeh

Theorem 2.3: Let (X, Ω, N,*) be a compact random fuzzy metric space $S, R : X \to X$ be satisfying:

(i) R is continuous, SR = RS and

(ii) N($R\xi x, R\xi y, u$) $> min\{N(\xi x_1,\xi y_1,u); \xi x_1,\xi y_1 \in Q(x) \cup Q(y)\}$
For all$\xi x,\xi y \in X\ with\xi\ x \neq \xi y, \forall u > 0\ where\ Q(\xi x) = \{g\xi x : g\xi \in \tau\}$, τ being the semi group of self maps on X generated by {S, R, I}, (I is the Identity map on X). Then S and R have a unique common fixed point z$\in X$.

Proof: We know that $R^n X$ is Compact and $R^{n+1}X \subseteq R^n X$ for n = 1,2,3,-----

Let $X_0 = \bigcap_{n=1}^{\infty} R^n X$, X_0 is a non empty compact subset of X, $RX_0 = X_0$ and $SX_0 \subseteq X_0$.

Since N is continuous on ${X_0}^2 \times (0,\infty)$ and X_0 is compact it follows that for each $u > 0, N(.,.,u)$ has a minimum value. Hence $\exists\xi\ v_1,\xi\ v_2 \in X_0$ such that

$N(\xi v_1,\xi v_2,u) = \inf\{N(\xi x,\xi y,u); \xi x,\xi y \in X_0\}$ For each $u > 0$.

since $TX_0 = X_0\ \exists\ \xi y_1,\xi y_2 \in X_0$ such that $R\xi y_1 = \xi v_1\ and\ R\xi y_2 = \xi v_2\ , suppose\ \xi y_1 \neq y_2\ Then\ from\ (ii)\ we\ have$

N($\xi v_1,\xi v_2,u$) = N(R$\xi y_1, R\xi y_2,u$)

$> min\{N(\xi x,\xi y,u); \xi x,\xi y \in Q(\xi y_1) \cup Q(\xi y_2)\}$

$\geq$ N($\xi v_1,\xi v_2,u$)

It is a contradiction. Hence$\xi y_1 =\ \xi y_2\ \ and\ \xi v_1 =\ \xi v_2$.

Hence $X_0\ is\ a\ singleton\ set, say$ {v} Thus v is a common fixed point of S and R.

Corollary 2.4: Let R be a continuous self map on a compact random fuzzy metric space (X, Ω N,*) satisfying

$$N(R\xi x, R\xi y, u) \geq \varphi\left[\min\left\{\begin{matrix} N(\xi x, \xi y, u), N(\xi x, R\xi x, u), N(\xi y, R\xi y, u), N(\xi x, R\xi y, u) \\ N(\xi y, R\xi x, u),\ \frac{N(\xi x, R\xi x, u)+N(\xi y, R\xi y, u)}{1+N(x,y,u)}, \\ \frac{N(\xi x, R\xi x, u)+N(\xi x, R\xi y, u), N(\xi y, R\xi x, u)+N(\xi y, R\xi y, u)}{1+N(\xi x, R\xi y, u), N(\xi y, R\xi x, u)+N(\xi x, \xi y, u) N(\xi y, R\xi x, u)} \end{matrix}\right\}\right]$$

$\forall \xi\ x, \xi y \in X$ $with$ $\xi x \neq \xi y$ and for all $u > 0.$
Then R has a unique common fixed point in X.

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