Estimation of Electrical Characteristics in Equivalent Circuit Model of Non-ideal Potential Transformer

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Abstract – In this paper equivalent circuit model of a non-ideal potential transformer (PT) with the model parameters determined practically using open-circuit and short-circuit tests. The experimental setup for determining the equivalent circuit parameters is presented. As an example of the usefulness of the non-ideal equivalent circuit of the PT, the parameters evaluated in the lab are used to calculate two important transformer characteristic; percentage regulation and maximum efficiency. The equivalent circuit of the tested transformer is also simulated using the LTspice/SwCAD III simulator to study some electrical characteristics of the PT. For instance, the frequency behavior of some parameters in the equivalent circuit is investigated accordingly. The parameters behaviors at frequencies up to 100 Hz, such as copper loss, iron loss, series impedance and shunt impedance, has been indicated in this paper.

Keywords: Potential transformers, equivalent circuit model, parametric identification, Electrical simulation, transformer characteristics.

I. Introduction

A transformer is an apparatus for converting electrical power in an AC system at one voltage or current into electrical power at some other voltage or current without the use of rotating parts. It is critical and costly component for the utility industry. Therefore, it is useful to use an equivalent circuit model to characterize the non-ideal operation of the transformer. While an ideal model may be well suited for rough approximations, the non-ideal parameters are needed for careful transformer circuit designs. Knowing the non-ideal parameters and characteristics allows the manufactures to optimize a design using equations rather than inefficiently spending time testing physical implementations in the manufacturing workshops.

If the material properties of a transformer are determined, the non-ideal parameters can be directly evaluated, hence the importance. In this paper, a practical method for determining the parameters of the equivalent circuit model of a potential transformer (PT) using two specific tests (open circuit and short circuit) is firstly described. The obtained parameters then have been employed for simulating the characteristics of the voltage transformer by means of the proposed model and by using the LTspice/SwCAD III simulator. It is a high performance simulator, schematic capture and waveform viewer with enhancements and models for easing the simulation of switching regulators. It allows the user to view waveforms for all components of the schematic in just a few fractions of a second [1]. Results of this process are also presented and described in this paper.

II. Equivalent Circuit Model of PT

Modeling of transformers is necessary for many reasons, depending on the application of the transformer. Consequently, many different equivalent circuit models have been set up to model the transformer in different frequency ranges. Many of the equivalent circuit models are set up on the basis of the open-circuit and short-circuit impedance of the transformers obtained from tests or from calculations For potential transformers, the models are mostly set up for measuring purposes, as it is of interest to know the transfer of different signals, when the potential transformer is used outside its normal frequency rating, 50-60 Hz [2].

A basic equivalent circuit model for the non-ideal potential transformer is shown in Figure 1. It is known as "high side equivalent circuit model" because all parameters have been moved to the primary side of the transformer [3]. Essentially when reflecting/referring impedance to the primary side of a transformer, you are just seeing what the secondary impedance "looks like" to the primary side. Since the secondary impedance will determine the load on the primary, it is helpful to know how to relate it in terms of the primary so as to calculate the current flow in the primary due to the load on the secondary.

In Figure 1, the series resistance (R_{eq}) is the resistance of the copper winding. The series inductance (X_{eq}) represents the flux leakage where a small amount of flux passes through the air outside the magnetic core path. The parallel resistance (R_m) represents the core loss of the magnetic core material due to hysteresis. The parallel inductance (X_m) is the magnetizing inductance and represents the finite permeability of the magnetic core.

Generally speaking, equivalent circuits are used to simplify a complex circuit into terms that are solvable with known relations. For example, in the transformer equivalent circuit, you can account for winding losses and flux leakage with a series resistance and reactance on the primary side. Core losses can be also modeled similarly with a parallel resistance and reactance on the primary.

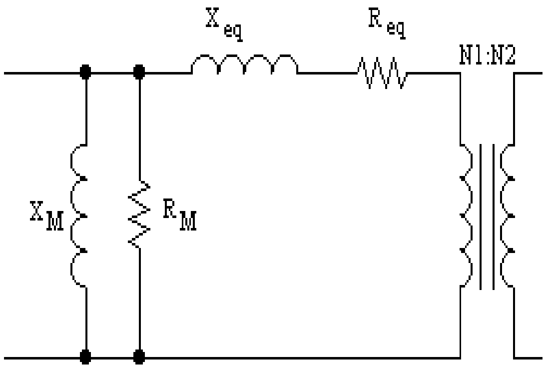

Fig. 1. Basic equivalent circuit model of the PT

In addition, each parameter of the equivalent circuit model could be adjusted by changing the transformer design. For example, increasing the diameter of the wire in the windings decreases the series resistance. Therefore, the equivalent circuit model parameters can be used as a reliable method to evaluate a transformer, or compare transformers.

The parameters can be found in the same way that Thevenin equivalent circuit parameters are found: open circuit and short circuit tests. The parallel parameter values are found with no load connected to the secondary (open circuit) and the series parameter values are found with the secondary terminals shorted (short circuit). Figure 2 gives the equivalents circuits for the two tests. For the open circuit test, the series parameters are neglected for convenience. This is reasonable since the voltage drops across R_{eq} and X_{eq} are normally small [4].

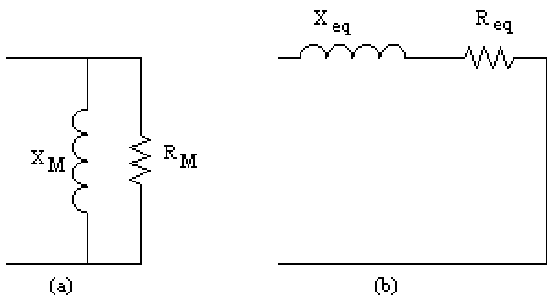

Fig. 2. Equivalent circuits for the transformer tests.
(a) Open circuit (b) Short circuit.

Relations (1), (2), (3), and (4) for the non-ideal transformer parameters are derived from the equivalent circuits shown in Figure 2. All parameters are expressed in terms of quantities measured in the open circuit and short circuit tests.

$$R_M = \frac{V_{oc}^2}{P_{oc}} \tag{1}$$

$$X_M = \frac{1}{\sqrt{\left(\frac{i_{oc}}{V_{oc}}\right)^2 - \frac{1}{R_M^2}}} \tag{2}$$

$$R_{eq} = \frac{P_{sc}}{i_{sc}^2} \tag{3}$$

$$X_{eq} = \sqrt{\left(\frac{V_{sc}}{i_{sc}}\right)^2 - R_{eq}^2} \tag{4}$$

III. Experimental Setup and Results

The textbook method [5, 6], for the determination of equivalent-circuit parameters, involves open-circuit and short-circuit tests of the transformer. This method yields satisfactory result since the two series-branch impedances in the equivalent-circuit for most medium and large size transformers are negligible, compared to the shunt-branch impedance. In this work, (1:1) of 60 Hz potential transformer was practically tested to determine its non-ideal parameter values. Figure 3 illustrates the circuit diagram used to perform the open circuit test. With the secondary open, the primary voltage was increased from zero to the rated voltage (110 V). A high sensitive digital multimeter (DMM) was used as an ammeter to measure the open circuit current (I_{oc}). A wattmeter was used to measure the open circuit power (P_{oc}).

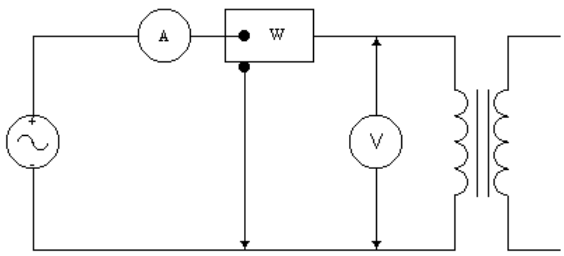

Fig. 3. Circuit diagram for open circuit test.

The short circuit diagram is shown in Figure 4. With the secondary terminals shorted, the primary voltage was increased from zero until the rated current of the primary.

At this point the primary voltage was measured. It was much less than rated voltage. Again, the power and current were measured.

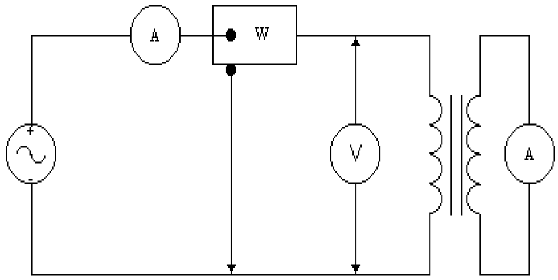

Fig. 4. Circuit diagram for short circuit test.

The measurements taken for the open and short circuit tests are listed in Table 1.

TABLE 1
MEASUREMENTS OF THE OPEN AND SHORT CIRCUIT TESTS

Open circuit measurements	
V_{oc}	109.445 V
i_{oc}	0.229 A
P_{oc}	6.1 W
Short circuit measurements	
V_{sc}	10.619 V
i_{sc}	3.698 A
P_{sc}	5.8 W

IV.

Referring to the open circuit measurements, the parallel parameters of the transformer (R_m and X_m) are evaluated using Equations (1) and (2). While the short circuit measurements are used to evaluate the series parameters of the transformer (R_{eq} and X_{eq}) by using Equations (3) and (4). The calculated parameters of this transformer are listed in Table 2.

TABLE 2
EVALUATED PARAMETERS OF THE TRANSFORMER

Parameter	value
R_m	1963.342 Ω
X_m	493.24 Ω
R_{eq}	0.425 Ω
X_{eq}	2.839 Ω

Using these parameters, it would be possible to determine the Percentage Regulation (*P.R*) of the tested PT. The regulation of a transformer is the change in secondary voltage from no load to full load. It is generally as a percentage of the full-load secondary voltage [6]:

$$P.R = \frac{V_{oc} - V_{FL}}{V_{FL}} x100$$

$$P.R = \frac{109.445 - 110}{110} x100$$

$$P.R = -0.5\%$$

The *P.R* depends upon the design of the transformer and the power factor of the load. The *P.R* increases to possible about 5 % in the inductive load. If the motor load is large and fluctuating, it is recommendable to use separate transformer for that motor.

It would be also possible practically to determine the maximum efficiency of the transformer by setting the load so that the transformer is operating at maximum efficiency. While the actual efficiency of the transformer could be found by dividing the power out by the power in. To simplify the procedures during this work, the parameters evaluated for the transformer tested (R_{eq} and R_m) can be used to find the minimum current (I_{min}) and the maximum current (I_{max}):

$$I_{\min} = \frac{V_{oc}}{R_m} = \frac{109.445}{1963.342} = 55.744mA \quad (5)$$

$$I_{\max} = \frac{V_{oc}}{R_{eq}} = \frac{109.445}{0.42506} = 257.4816A \quad (6)$$

In this case, the maximum efficiency is given by [7]:

$$\eta_{\max} = \frac{\left(1 - \sqrt{\frac{I_{\min}}{I_{\max}}}\right)^2}{1 - \frac{I_{\min}}{I_{\max}}} = 97.1\% \quad (7)$$

IV. Electrical Simulation and Frequency Behavior

The evaluated lumped-parameter model of the non-ideal PT equivalent circuit, shown in Figure 5, was designed and simulated by LTspice/SwCAD III simulator to study and expect the electrical characteristics of the PT at 50 Hz and 60 Hz.

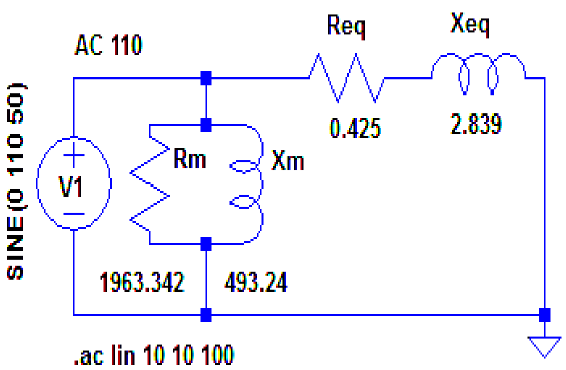

Fig. 5. Electrical Simulation of the PT

To verify and confirm the simulation circuit, a comparison between the practical and the simulated results was compared as in Table 3. The comparison exhibits very good agreement between the two methods. The simulation error in estimation of the maximum efficiency of the tested PT is about 3 ppm (parts per million).

TABLE 3
COMPARISON BETWEEN PRACTICAL AND SIMULATION RESULTS

Parameter	Practical Results	Simulation Results	Simulation Error
I_{min}	55.744 mA	56.027 mA	0.51 %
I_{max}	257.482 A	258.824 A	0.52 %
η_{max}	97.09 %	97.11 %	2.7 ppm

Using the simulated circuit in Figure 5, some electrical characteristics of the tested PT can be investigated as discussed in the following sections.

The transformer cannot change the frequency of the supply. If the power supply is 60 Hz, the output signal will also be 60 Hz. In most parts of the Americas, it is typically 60 Hz, and in the rest of the world it is typically 50 Hz. Places that use the 50 Hz frequency tends to use 230 V RMS, and those that use 60 Hz tend to use 117 V RMS. The three common frequencies available are 50Hz, 60Hz and 400Hz. The 400 Hz is reserved for high-powered applications such as aerospace and some special-purpose computer power supplies and hand-held machine tools [8]. In the following sections, some studies of the frequency behaviors of the tested PT are discussed.

IV.1. Frequency Behavior of the Copper Loss

The copper loss of the PT is determined by the resistance of the high-tension and low-tension windings and of the leads. It is equal to the sum of the watts of (I^2R_{eq}) losses in these components at the load for which it is desired to compute the efficiency. The copper loss in the tested transformer in this paper is simulated as a function in the applied frequency as given in Table 4 and shown in Figure 6. It has been noticed that the copper loss of the PT decreases when the frequency increases.

TABLE 4
FREQUENCY BEHAVIOR OF THE COPPER LOSS

Freq. (Hz)	20	40	50	60	80	100
Copper Loss (mW)	40.4	10.1	6.46	4.49	2.53	1.62

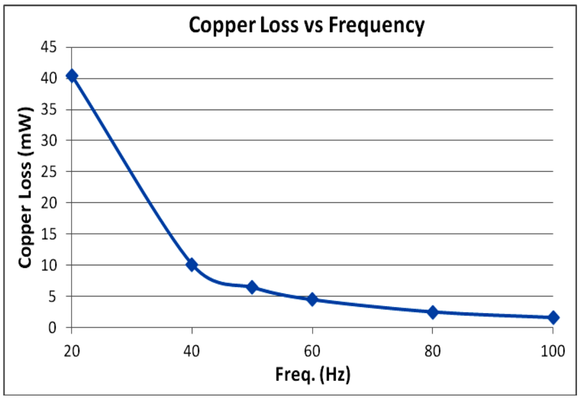

Fig. 6. Frequency Behavior of the Copper Loss

IV.2. Frequency Behavior of the Iron Loss

It is equal to the sum of the losses due to the parallel resistance (R_m). It represents the core loss of the magnetic core material due to hysteresis and equals to the sum of the watts of (I^2R_m) losses at the load for which it is desired to compute the efficiency. The iron loss in the tested transformer in this paper is simulated as a function in the applied frequency as given in Table 5 and shown in Figure 7. It has been noticed that the iron loss of the PT remains constant despite the change of frequency (frequency-independent).

TABLE 5
FREQUENCY BEHAVIOR OF THE IRON LOSS

Freq. (Hz)	20	40	50	60	80	100
Iron Loss (W)	6.16	6.16	6.16	6.16	6.16	6.16

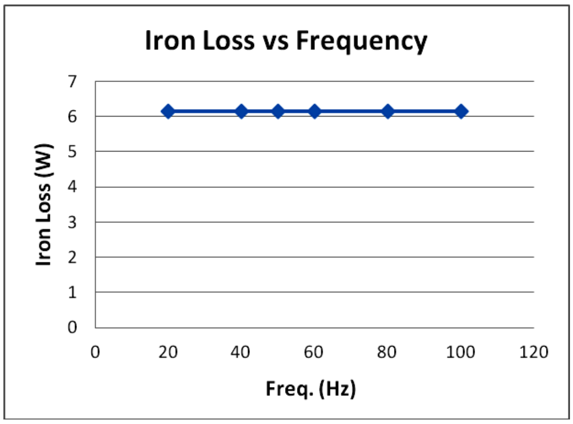

Fig. 7. Frequency Behavior of the Iron Loss

IV.3. Frequency Behavior of PT Impedance

Impedance is the current limiting characteristic of a transformer. In electrical power networks, it is usually used for determining the interrupting capacity of a circuit breaker or fuse employed to protect the primary of a transformer. The impedance (or resistance to current flow) is important and used to calculate the maximum short circuit current which is needed for sizing, circuit breakers and fuses. It represents the amount of normal rated primary voltage which must be applied to the transformer to produce full rated load current when the secondary winding is short circuited

Regardless of type or form of the equivalent circuit chosen to represent the physical structure of the transformer it may be contains six distinct elements (series and shunt capacitances has been ignored in this paper):

- *L_m: total shunt inductance (H)*
- *R_m: total shunt resistance (kΩ)*
- *L_{eq}: total series inductance (mH)*
- *R_{eq}: total series inductance (Ω)*

A basic assumption is that each element has a constant value and there is a frequency and impedance range where each is dominate in determining the response of the transformer. The frequency behavior for the four elements and Z_m & Z_{eq} have been listed in Table 6 and illustrated in Figure 8 and 9.

TABLE 6
FREQUENCY BEHAVIOR OF IMPEDANCE ELEMENT

F (Hz)	20	40	50	60	80	100
L_m (H)	3.92	1.96	1.57	1.31	0.98	0.78
R_m (kΩ)	1.93	1.93	1.93	1.93	1.93	1.93
Z_m (kΩ)	0.39	0.39	0.39	0.39	0.39	0.39
L_{eq} (mH)	23	11	9.0	8.0	6.0	5.0
R_{eq} (Ω)	0.425	0.425	0.425	0.425	0.425	0.425
Z_{eq} (Ω)	3.26	3.26	3.26	3.26	3.26	3.26

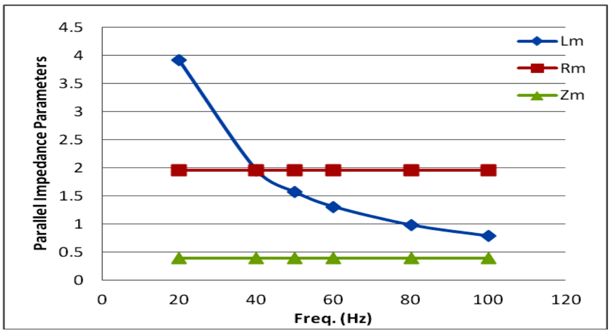

Fig. 8. Frequency Behavior of Shunt Impedance

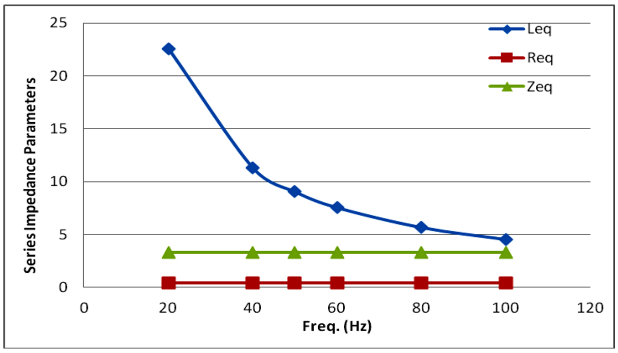

Fig. 9. Frequency Behavior of Series Impedance

Referring to these figures, it has been noticed that the series and parallel inductances (L_{eq} & L_m) decrease when the frequency increases. To reduce the inductance value, then improve the actual efficiency of the PT, the type of winding material should to be changed. While increasing the diameter of the wire in the windings decreases the series resistance. It has been also noticed that the percentage decline in the shunt inductance is about 16.5 % when using the PT at a frequency 60 Hz instead of 50 Hz. This rate becomes about 11 % in case of series inductance. These technical outputs allow the engineers to more efficiently design transformer circuits. This means that designs can be optimized prior to implementation.

V. Conclusion

The techniques used to find the parameter values of the non-ideal transformer equivalent circuit model allow the manufacturers to more efficiently design transformer circuits. As an example of the usefulness of the non-ideal equivalent circuit of the tested PT, the parameters evaluated in the lab are used to calculate two important transformer characteristic, percentage regulation and maximum efficiency.

Modeling and simulation are more accurate when the non-ideal parameters are used. Using the electrical simulation on the tested equivalent circuit yields a complete profile for the impedance behavior of the PT based on the frequency changes. The winding inductances of the PT decrease with increasing frequency, while the winding resistances are frequency-independent. It is concluded from the simulation results that the percentage decline in the shunt inductance is about 16.5 % when using the PT at a frequency 60 Hz instead of 50 Hz. This rate becomes about 11 % in case of series inductance. This means that designs can be optimized prior to implementation.

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